EVALUATION AND INTERPRETATION OF F/N-CURVES: DEVELOPMENT OF A NEW TOOL FOR TRANSPARENT AND TRACEABLE DECISION MAKING

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ABSTRACT

The evaluation of risks by f/n-curves is a graphical method to present the numbers of fatal accidents in a traffic system together with the numbers of fatalities which have occurred or which could be expected from a quantitative risk analysis [3]. The interpretation of f/n-curves, resulting from frequently performed quantitative risk assessments (QRAs) for tunnels and underground facilities, is a difficult task regarding the resulting requirements for the design or the operation of a traffic facility. This may on some occasions already be a problem for the designer himself but is a task of even higher requirements for an operating authority of such a structure, which in many cases have little experience with QRAs and the corresponding simulation model, such as CFD-Simulations for smoke expansion or escape simulations. Especially when two or more settings or configurations of safety equipment are compared to each other, the interpretation sometimes becomes rather “instinctively”, especially when the corresponding f/n-curves are interlacing and cutting across each other. In the past, some simplifications have been used for the evaluation of f/n-curves, such as the accumulated risk values. The problem is that these values mathematically do underestimate the influence of incidents with low probabilities but high damage. So-called “aversion factors” have been introduced to compensate for this problem, but often add up to the fuzziness of the whole process. The following paper describes a possible solution for this problem using a mathematical approach for comparison and decision support. The algorithm as well as the corresponding tool was developed in the course of the SOLIT² research project in Germany which is funded by the German Ministry for Economics and Technology (BMWi). In the following the authors describe the mathematical approach and show some examples of its application.

Keywords: f/n-curves, quantitative risk analysis, tunnel, safety, security

1. INTRODUCTION

1.1. General remarks

The performance oriented quantitative evaluation of safety measures for tunnels is increasingly and – to some extent –already part of our regulations and guidelines. For instance in Germany such tunnels which have a “special characteristic” beside of the standardized cases have to be assessed and evaluated with a quantitative risk assessment under the regulations of the German “RABT”-guideline (German Guideline for equipment and operation of road tunnels based on EU-directive No.), [8]. In case dangerous goods are transported through a tunnel, such assessments have become mandatory throughout the whole European Community.

With the results of such assessments already at hand the discussion usually starts about the conclusion to be drawn from the corresponding f/n-curves and particularly regarding the shape of the graph. If the assessment is carried out against a specific benchmark, such as a risk-acceptance curve for ADR-assessments (Accord européen relatif au transport international des marchandises Dangereuses par Route), the evaluation process is rather easy if the shape of the graph does (or does not) interlace or reach beyond the respective acceptance curve. It is
also very easy, if different curves for alternatives solutions – for instance in case of a comparison of different technical settings or safety measures – show high levels of variation when compared with each other. But on some occasions the applied and competing mitigation measures produce similar performances within the chosen scenarios and boundary conditions leading to a nearly similar performance within the assessment. In this case it might become very difficult to identify the optimal curve and thereby the optimal measure or configuration for the specific structure or tunnel.

In the past simplifications and workarounds were developed for such cases, with the accumulated risk value – the total weight of the curve (surface integral of the curve) – being the most prominent one. Main problem with the application of especially this value is that it mathematically underestimates incidents with low probabilities but very high damages within the summation. Therefore so called “aversion-factors”, especially developed to provide more of an equilibrium regarding the weighting of highly and lowly probable incidents, have been introduced and implemented from other sciences. Unfortunately, these factors do add to the overall fuzziness of the result at hand, since there is no general approach for developing such factors [7]. In theory such a factor can be defined and applied in an arbitrary way by the author of a study. Sensitivity analyses regarding the influence of aversion factors are also lacking within the scientific community. Thereby the tracing of a result as well as gaining transparency regarding the assessment and evaluation process becomes rather difficult, especially for operating authorities which may have little experience with quantitative risk analysis.

1.2. The project SOLIT²

In 2009 the German Federal Ministry for Economics and Technology funded the research project SOLIT² (Safety of life in tunnels 2). As one of the project’s main goals the quantification of a compensation potential of fixed fire fighting systems in traffic tunnels was targeted in comparison to other safety measures by a German consortium, consisting of FOGTEC, BUNG – Engineers, TÜV South, STUVA and the Institute for Tunnelling and Construction Management at Ruhr-University Bochum (RUB-TLB). The idea was to find a setting of safety measures, possibly including a fixed fire fighting system, which provides an equal level of risks compared to typical road tunnel equipment settings following the regulations of the German RABT, while requiring lower amount costs for investment, operation and maintenance. Alternatively, a setting was targeted that provides a lower level of risks, while requiring the same amount of costs for investment and maintenance. Hereby RUB-TLB was assigned with the development of a lifecycle costing model for tunnel equipment enabling its user to carry out the respective comparison in terms of costs and investments. Additionally, a mathematical solution was developed for the aforementioned decision support problem. The results of this development are described in the following.

2. DEVELOPMENT OF AN ALTERNATIVE APPROACH FOR THE EVALUATION OF QUANTITATIVE RISK ASSESSMENTS

2.1. Requirements for an evaluation method

With the general remarks in mind some basic assumptions can be stated regarding the development of a possible solution of the decision problem at hand:

- A corresponding algorithm has to deliver a procedure that can easily be applied by any user for comparison of specific results
- The algorithm has to deliver a reproducible and transparent procedure
- Further, the algorithm has to be executed with the help of attributes so that the evaluation can be carried out with project oriented criteria
Multi criteria decision models generally meet these requirements while offering the opportunity to describe and to analyze complex decision situations [2]. By evaluating all advantages and disadvantages in a prior study [13][14][15], the choice for an evaluation method was made in favor of the Analytic Hierarchy Process (AHP). The AHP is suitable for a precise structure of complex decision problems. The method is based on decision relevant alternatives and goals and considers both, qualitative and quantitative data. For practical use, the method includes a clear structure. According to [1], the AHP can be easily applied, the use for single persons and groups as well, the advancement of agreement and consensus, and finally the communication and transparency of results.

2.2. The Analytic Hierarchy Process (AHP)

The AHP was developed by Thomas Saaty in the USA in the 1970s [9][10][11][12]. It is characterized by the three main parts: analytical procedure, hierarchical structure and a processual decision [5][17]. Analytic procedure means that the method is working with mathematical-logical functions which are comprehensible for all project participants. A hierarchical structure has to be applied to the decision problem so that it can be split into different levels of comparison. The process-related character allows the method to be restarted as many times as needed in order to reproduce decisions or to describe the whole decision making process. Furthermore, it is possible to imply quantitative and qualitative information during the decision process.

For a meaningful evaluation result, different information has to be weighted in order to show the significance of the decision. For the pair and alternative comparison Saaty introduces a 9-value-scale [4]. This scale includes also the use of reciprocal scale values. E.g. if one element is 3 times more important than another element it means that the other element possesses the value 1/3. Due to the fact that those pair comparisons are often made in a subjective way, it might be possible that they are inconsistent. For instance, if criterion A is three times more important than criterion B, and B is two times more important than C, the decision maker could evaluate criterion A three times more important than C (whereas it has to be six times). In that case, the made evaluation is not correct and would lead to a wrong result. But to a certain very limited extent inconsistencies are allowed and do not endanger the whole decision [16].

For checking consistencies, Saaty defines the consistency index (CI) and the consistency ratio (CR). With the help of the eigenvalue-method it is possible to calculate the inconsistency and to detect wrong comparisons. The reference point given by Saaty for CR is 0.1. If the value of 0.1 will be exceeded, the decision process is regarded to be inconsistent so that the logic and interpretability of the results are not given anymore. The decision maker then has to correct the correlating mistake and to evaluate the whole process again. For providing a traceable and transparent decision, a sensitivity analysis then has to be carried out. The main goal of this analysis is to show the influence of weight changes (read: prioritization of specific criterions) which may lead to a change in the ranking of the alternatives. This analysis is a very effective tool to analyze the stability of results, especially when one alternative is prioritized in the result of an AHP evaluation by narrow margin. For the fundamental mathematical procedure the reader is referred to fundamental literature, such as [9][10][11][12].

2.3. Analysis of f/n-curves using the Analytic Hierarchy Process

In the following the authors will show the theoretical approach of such an application. As already stated, it is possible to estimate the accumulated risk value for typical f/n-curves based on the surface integral. In consideration of the shown Analytic Hierarchy Process and its algorithm the authors investigate how single areas of an f/n-curve could be weighted stronger or weaker for the comparison and the identification of the most ideal curve and
thereby the most ideal technical setting. In other words: By applying AHP to the comparison of f/n-curves a decision maker is enabled to analyze specific areas of different curves for a deeper investigation of the risk based f/n-diagram. To simplify the description of the overall approach, the authors will refer to the curve or the diagram as a whole. Naturally all mathematical calculations are carried out with the original data regarding probabilities and corresponding damage, gained from the underlying quantitative risk analysis.

2.3.1. Hierarchical Structure of the Decision Problem

First, in consideration of the AHP and its boundary conditions a hierarchical structure has to be developed. For that, in the first level “single risk areas” are defined by splitting the f/n-curve into several areas for evaluation. Of course, it is possible to divide every single area into subareas if a refinement becomes necessary, creating further sublevels. An example of a QRA-related AHP-hierarchy is shown in figure 1.

2.3.2. Calculation of the collective risk values of all scenarios

In the next step, the accumulated risk values $R_i$ of all f/n-curves have to be calculated. The reason is that, when using the AHP input data has to be normalized, so a decision maker could get a ranking of the alternatives/criteria (1). Finally, the sum of all accumulated risk values has to be calculated (2).

$$w_{R_i} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}} \quad (1)$$

$$R_G = \sum_{i=1}^{n} R_i \quad (2)$$

2.3.3. Definition of single risk areas

Then the f/n-curve has to be chosen which includes the theoretical highest damage value $N_{k,max}$. The lowest and highest damage values present the limits for the definition of the single risk areas. Within these limits the decision maker splits the f/n-curve into several areas, further called “Risk Areas” ($N_{RA_j}$) (3) which are oriented according to the damage value $N_{k,max}$. All further f/n-curves have to be split accordingly.

Number of Risk Areas: NRA  

$$N_{RA_j} = \frac{N_{k,max}}{N_{RA}} \quad (3)$$

In figure 2 it is shown how a single f/n-curve can be splitted here into 4 NRA’s.

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2.3.4. Calculation of single expected risk values

After all areas are defined the accumulated risk values of every single area have to be calculated. For an appropriate application of the AHP-method it is necessary to weight these areas because of the imminent pair-wise comparison for identifying the most important risk area. The actual weight has to be defined in consideration of the calculated surface sums. For the local weight of every single area the accumulated risk values of all f/n-curves have to be summed up ($R_{f/N}$) (2). In a second step all accumulated risk values for every single area of all f/n-curves ($R_{RA_j}(R_i)$) (4) have to be evaluated whereas $W_{mn}$ is the probability, $H_m$ the frequency and $A_{mn}$ the fatality of an event. The last step is described by the division of the summed risk values for the single areas through the sum of all expected risk values ($WE_{N_{RA_j}}$) (5). This mathematical procedure has to be done for every area, so that a local normalized weight can be calculated.

$$R_{RA_j}(R_i) = H_m \cdot W_{mn} \cdot A_{mn} \quad (4)$$

$$WE_{N_{RA_j}} = \frac{\sum_{i=1}^{n} R_{RA_j}(R_i)}{\sum_{i} R_i} \quad (5)$$

2.3.5. Weight of the scenarios

Next, the quantitative weighting of the scenarios with regard to the single risk areas has to be carried out. For that, the accumulated risk values of every single area ($R_{RA_j}(R_i)$) are compared in reference to the scenarios (6).

The ratio has to be calculated with reciprocal values following the principle: The higher a value the lower its benefit.

$$W_{RA_j}(R_i) = \frac{1}{\frac{1}{R_{RA_1}(R_i)} + \frac{1}{R_{RA_2}(R_i)} + \ldots + \frac{1}{R_{RA_n}(R_i)}} \quad (6)$$
2.3.6. Calculation of the total weight

With the local weights of the main-criteria the global weight has to be calculated (same for the sub-criteria in a next step). Doing so every minor level of criteria has to be multiplied with the local weight of the superior level. The formula for the calculation of the global weight for an element i \((w_{rel}(i))\) for the \(n^{th}\) hierarchical level is:

\[
w_{rel}(i) = w_n \cdot w_{n-1}\]

(7)

Finally, the local alternative weights are multiplied with the global weights of superior criterions so that the decision maker is getting global alternative weights. With a final summation of the global alternative weights per alternative the preference index \((w)\) can be estimated, which describes the importance of every single alternative.

2.4. Example of an application of the Analytic Hierarchy Process

2.4.1. General Remarks

Within the previous chapters the rather theoretical approach of applying AHP to the identification of a preferred solution was described. This approach alone neither clarifies the mentioned decision problem nor does it help the decision maker to trace his decision or to make up for more transparency. The benefit of using this approach becomes obvious when applied to a realistic scenario. Therefore a simple example is created as follows. We assume that for a specific tunnel under a specific scenario three different configurations of safety measures are compared with each other. In Figure 3 three possible \(f/n\)-curves as a result of a previously conducted quantitative risk assessment are shown.

![f/n-curves of three different tunnel safety settings](image)

**Fig. 3:** \(f/n\)-curves of three different tunnel safety settings

Again, one has to keep in mind that all calculations are carried out with the underlying data, but that the authors are referring to the resulting diagram to make the process of application and evaluation more transparent. Also, one has to regard that the authors chose a rather obvious disparity between the different results to show the possibilities with the application.
2.4.2. Evaluation of the results using AHP

As typical for the evaluation process, maximum damages as well as accumulated risk values are observed in the beginning. Although configuration 2 has the highest damage value ($N_2 = 100$) configuration 1 and 3 display a higher accumulated risk value, so that configuration 2 might be preferred. As the AHP is applied to the data of the QRA at hand, the total weights of all configurations have to be calculated. For this example the decision maker created four risk areas (figure 3). The results are shown in table 1, as well as the corresponding accumulated risk values.

<table>
<thead>
<tr>
<th>Tab. 1: Calculation of the total weight $W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Configuration 1</td>
</tr>
<tr>
<td>$R_1 = 0.613$</td>
</tr>
<tr>
<td>Local weight</td>
</tr>
<tr>
<td>Risk area 1</td>
</tr>
<tr>
<td>Risk area 2</td>
</tr>
<tr>
<td>Risk area 3</td>
</tr>
<tr>
<td>Risk area 4</td>
</tr>
<tr>
<td>Total weight</td>
</tr>
<tr>
<td>= 17.98%</td>
</tr>
</tbody>
</table>

According to Tab. 1 we assume configuration 2 as the best alternative with a total weight of about 62%. This result is nearly equal to the result if the accumulated risk values are taken into account ($R_2=0.180$). The advantage here is that the whole decision problem is now transferred into a hierarchical structure. By doing this the results can now be analyzed with the help of a sensitivity analysis. The goal of such an analysis is to check made decision by changing the individual weights of the different criterions, respectively risk areas. To exemplify the possibilities of such an analysis Figures 4 and 5 display the sensitivity analysis for the risk areas 3 and 4.

**Fig.4:** Sens.-analysis of risk area 3

**Fig.5:** Sens.-analysis of risk area 4

Here it is noticeable that a higher importance of the corresponding areas would induce a change of the overall ranking of alternatives. In other words: If the decision maker would decide to attach more importance to higher damages (raise the importance of areas 3 and 4) then configurations 1 and 3 have to be preferred since their performance is much better within the corresponding risk areas. That said such an analysis enables the user to compare different configurations with keeping the focus on specific areas of damage, but without having to use simplifications that modify the results mathematically. The input data remains clean and without additional fuzziness so that the center of the decision making process can now be
moved around (laying the focus on highly probable incidents vs. incidents with low probability of occurrence but dramatically high outcome).

Last but not least, each criterion is weighted to 100% in the following step. Now, the calculated total weight states that within risk areas 1 and 2 configuration 2 becomes the best alternative, while within risk areas 3 and 4 configuration 3 respectively 2 is the best alternative to chose.

![Fig.6: Performance-Analysis for the first level](image)

3. **CONCLUSION AND OUTLOOK**

The analysis of quantitative risk assessments includes complex decision situations which require different perspectives as well as deepened knowledge of the situation at hand and the underlying methods and procedures. The present article illustrates the possibility for a new approach of evaluating the corresponding f/n-curves especially regarding difficult situations of evaluation and decision making. The mathematical procedure of the AHP allows for comprehensible, reproducible and transparent choices, without the need for simplifications and additional factors that add up to the uncertainty of the result. At a first glance the algorithm seems to produce more complexity for the decision problem. But due to this strictly mathematical approach it can be transformed into easy-to-handle software-tools. Currently the authors are working on such a tool as a further step of development. It will allow a flexible and individual adaptation of the evaluation hierarchy for a specific project. Results are expected within the next months.

Furthermore, AHP delivers the possibility to implement other criterions in an equal fashion, such as lifecycle costs or structural assessment. At present the development of such a decision model is done by the authors as part of the work within the SOLIT²-project.

4. **REFERENCES**


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