Kittel Problem 1.3

Show that the $c/a$ ratio for an ideal hexagonal close-packed structure is $\sqrt{8/3} = 1.633$. If $c/a$ is significantly larger than this value, the crystal structure may be thought of as composed of planes of closely packed atoms, the planes being loosely packed.

$h$...height of the equilateral triangle

$a$...lattice constant

$$h = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \sqrt{1 - \frac{1}{4}} * a = \sqrt{\frac{3}{4}} * a$$

Distance $x$ from an atom to the middle of the triangle:

$$x = \frac{2}{3} * h = \frac{2}{3} * \sqrt{\frac{3}{4}} * a$$

$$x = \frac{1}{\sqrt{3}} * a$$

$$\frac{c}{2} = \sqrt{a^2 - x^2} = \sqrt{a^2 - \frac{1}{3}a^2} = \sqrt{\frac{2}{3}} * a$$

$$\frac{c}{a} = \sqrt{\frac{8}{3}}$$