Extrinsic semiconductors

Extrinsic semiconductors

Dopants

Donors

Examples: P, As in Si.

Mobile negative electrons

Fixed positive donors

Acceptors

Examples: B, Ga in Si.

Mobile positive holes

Fixed negative acceptors

Charge Neutrality

$$n + N_A^- = p + N_D^+$$

$$n = N_c(T)\exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$p = N_v(T)\exp\left(\frac{E_v - E_F}{k_B T}\right)$$

Ionized donors and acceptors

For $E_v + 3k_BT < E_F < E_c\text{- } 3k_BT$ Boltzmann approximation

$$N_D^+ = \frac{N_D}{1+2\exp\left(\frac{E_F - E_D}{k_BT}\right)}$$

$$N_A^- = \frac{N_A}{1+4\exp\left(\frac{E_A - E_F}{k_BT}\right)}$$

4 for materials with light
holes and heavy holes (Si)
2 otherwise

N_D = donor density cm^{-3}

$N_D{}^+$ = ionized donor density cm^{-3}

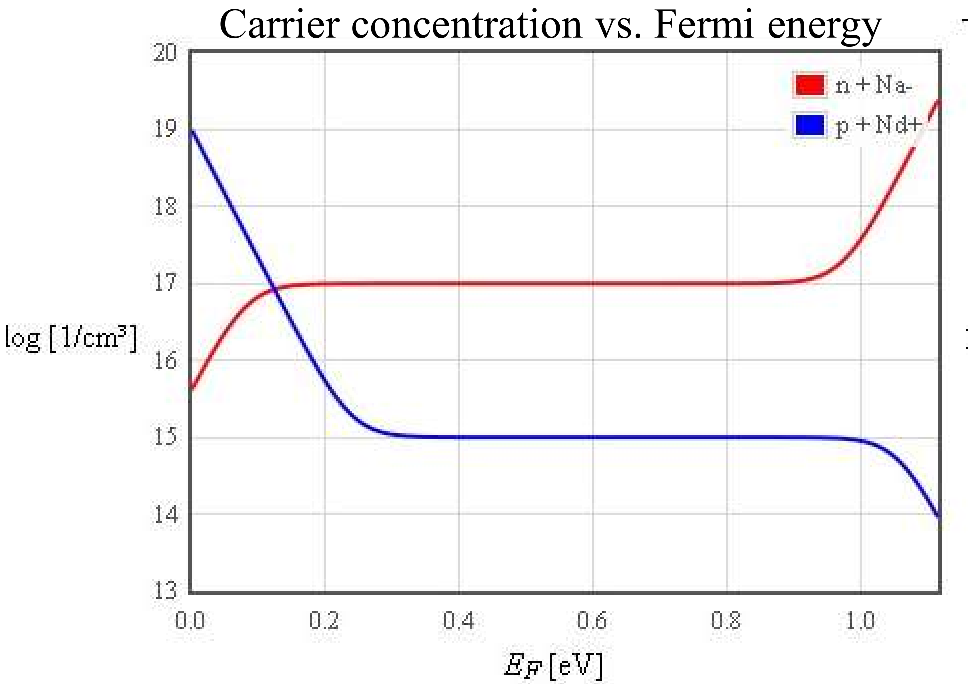
N_A = donor density cm^{-3}

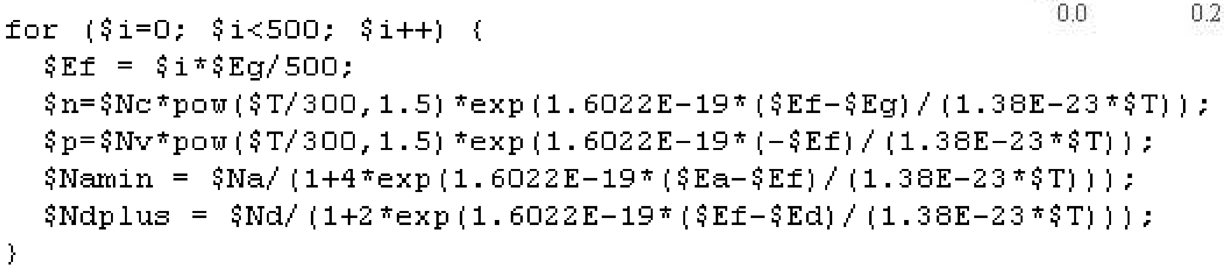
$N_A{}^-$ = ionized donor density cm^{-3}

Mostly, $N_D{}^+ = N_D$ and $N_A{}^- = N_A$

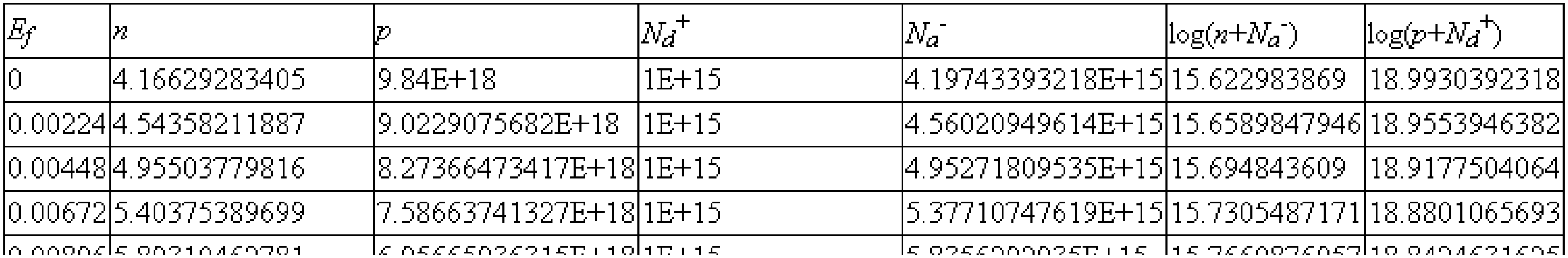
Charge neutrality

$$n + N_A^- = p + N_D^+$$

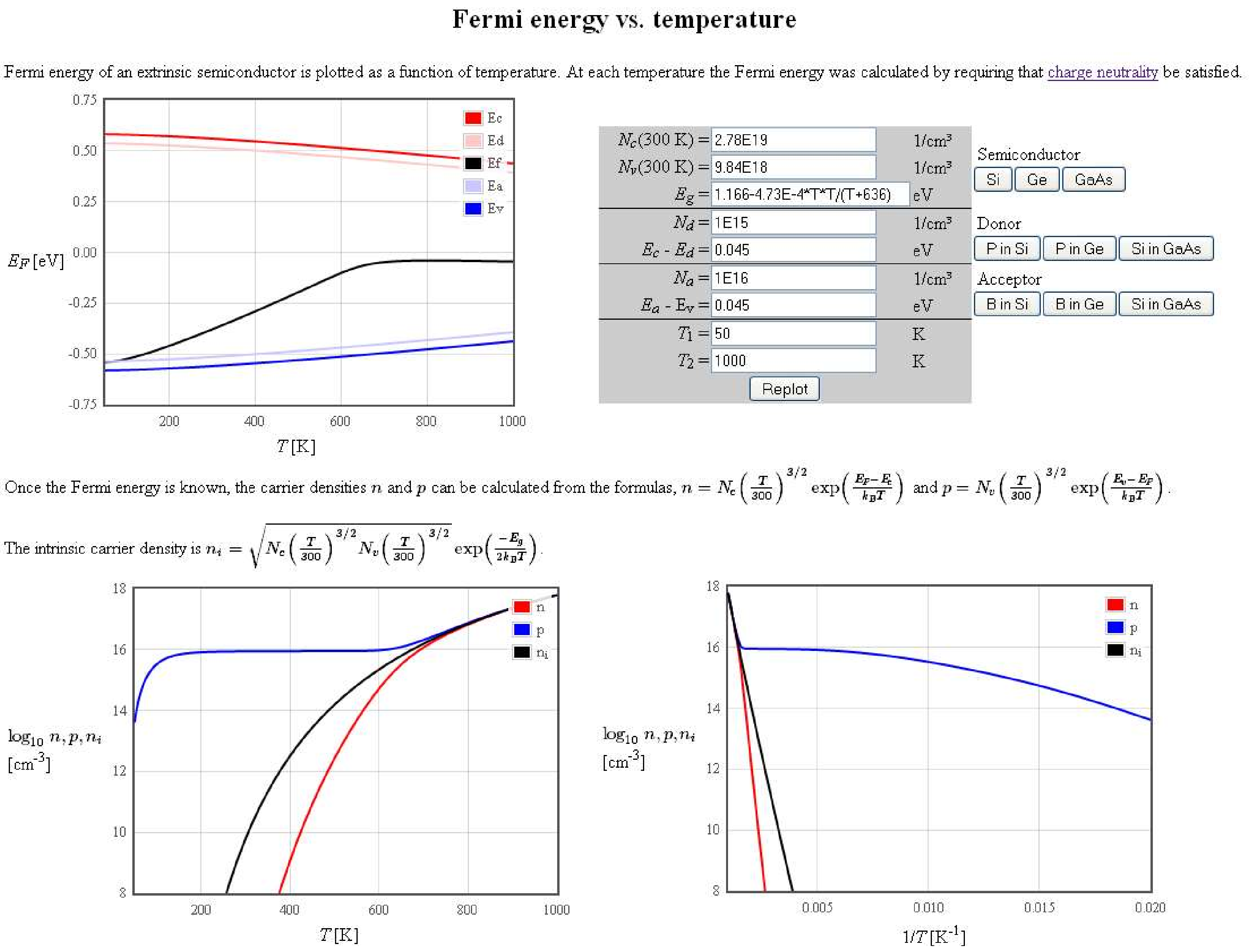




```
for ($i=0; $i<500; $i++) {
  $Ef = $i*$Eg/500;
  $n=$Nc*pow($T/300,1.5)*exp(1.6022E-19*($Ef-$Eg)/(1.38E-23*$T));
  $p=$Nv*pow($T/300,1.5)*exp(1.6022E-19*(-$Ef)/(1.38E-23*$T));
  $Namin = $Na/(1+4*exp(1.6022E-19*($Ea-$Ef)/(1.38E-23*$T)));
  $Ndplus = $Nd/(1+2*exp(1.6022E-19*($Ef-$Ed)/(1.38E-23*$T)));
}
```



E_f	n	p	N_d^+	N_a^-	log(n+N_a^-)	log(p+N_d^+)
0	4.16629283405	9.84E+18	1E+15	4.19743393218E+15	15.622983869	18.9930392318
0.00224	4.54358211887	9.0229075682E+18	1E+15	4.56020949614E+15	15.6589847946	18.9553946382
0.00448	4.95503779816	8.27366473417E+18	1E+15	4.95271809535E+15	15.694843609	18.9177504064
0.00672	5.40375389699	7.58663741327E+18	1E+15	5.37710747619E+15	15.7305487171	18.8801065693



Fermi energy vs. temperature

Fermi energy of an extrinsic semiconductor is plotted as a function of temperature. At each temperature the Fermi energy was calculated by requiring that charge neutrality be satisfied.

N_c(300 K) = 2.78E19 1/cm³

N_v(300 K) = 9.84E18 1/cm³

E_g = 1.166-4.73E-4*T*T/(T+636) eV

N_d = 1E15 1/cm³

E_c - E_d = 0.045 eV

N_a = 1E16 1/cm³

E_a - E_v = 0.045 eV

T_1 = 50 K

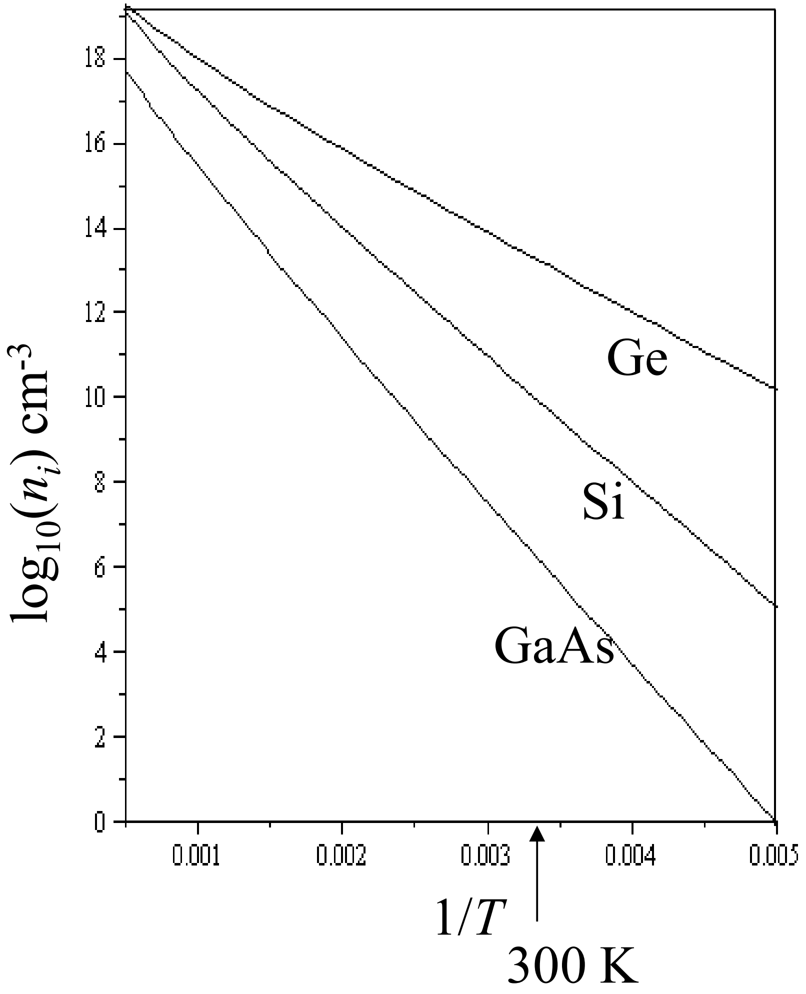
T_2 = 1000 K

Once the Fermi energy is known, the carrier densities n and p can be calculated from the formulas, $n = N_c\left(\frac{T}{300}\right)^{3/2}\exp\left(\frac{E_F-E_c}{k_BT}\right)$ and $p = N_v\left(\frac{T}{300}\right)^{3/2}\exp\left(\frac{E_v-E_F}{k_BT}\right)$.

The intrinsic carrier density is $n_i = \sqrt{N_c\left(\frac{T}{300}\right)^{3/2}N_v\left(\frac{T}{300}\right)^{3/2}}\exp\left(\frac{-E_g}{2k_BT}\right)$.

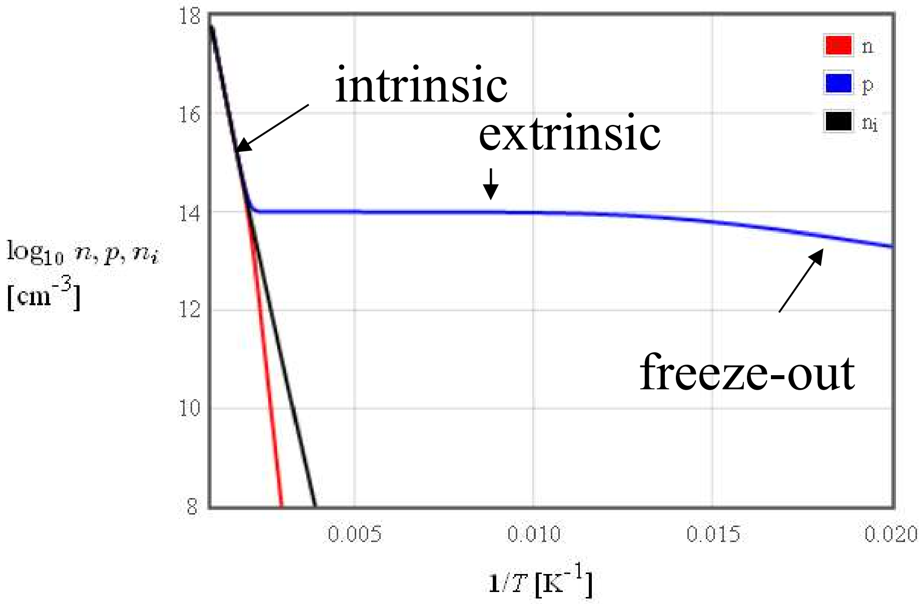
http://lamp.tu-graz.ac.at/~hadley/psd/L4/eftplot.html

Intrinsic semiconductors



$$n_i = \sqrt{N_v N_c}\,\exp\left(-\frac{E_g}{2k_B T}\right)$$

Extrinsic semiconductors



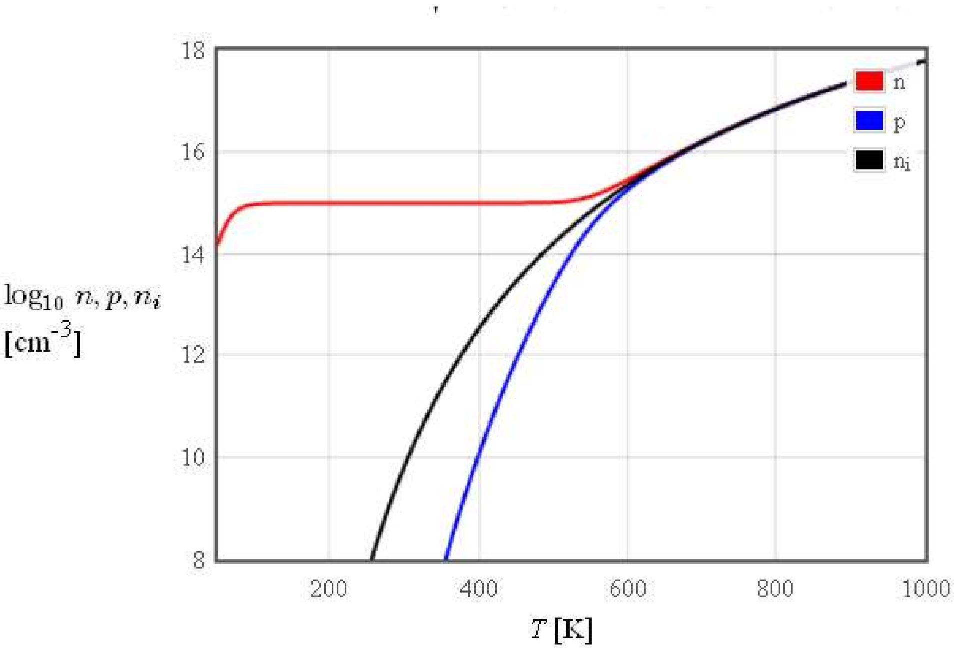
At high temperatures, extrinsic semiconductors have the same temperature dependence as intrinsic semiconductors.

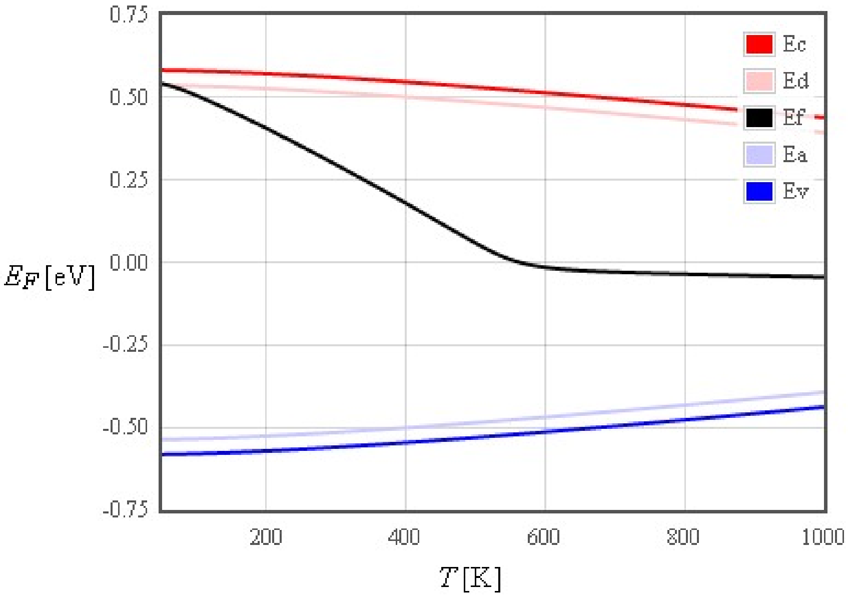
n-type (extrinsic)

$$n = N_D = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$\boxed{E_F = E_c - k_B T \ln\left(\frac{N_c}{N_D}\right)}$$

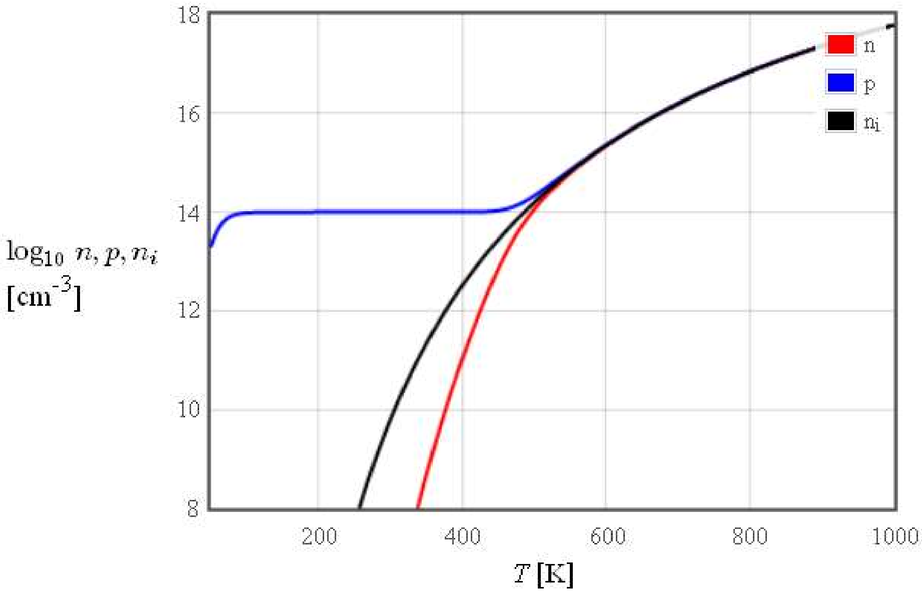
For n-type, n ~ density of donors,
$p = n_i^2/n$



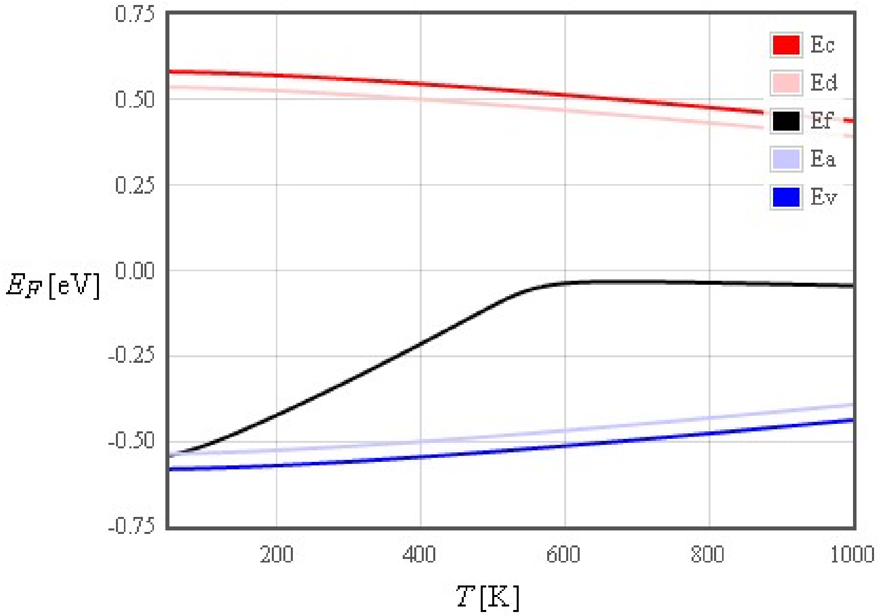


p-type (extrinsic)

$$p = N_A = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$



$$\boxed{E_F = E_v + k_B T \ln\left(\frac{N_v}{N_A}\right)}$$



For p-type, p ~ density of acceptors,
$n = n_i^2/p$

Intrinsic / Extrinsic

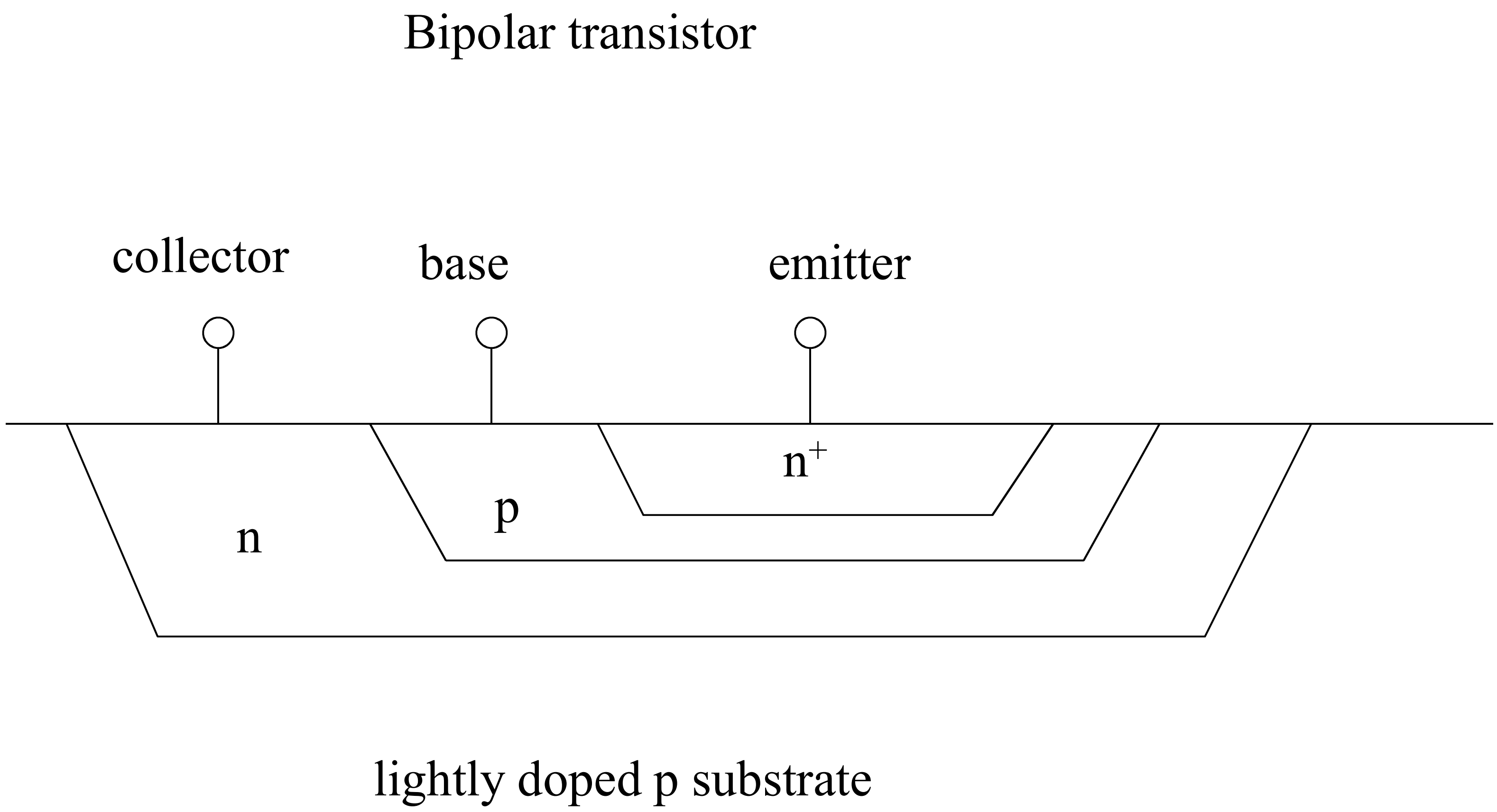
Intrinsic: $n = p$

Conductivity strongly temperature dependent near room temperature

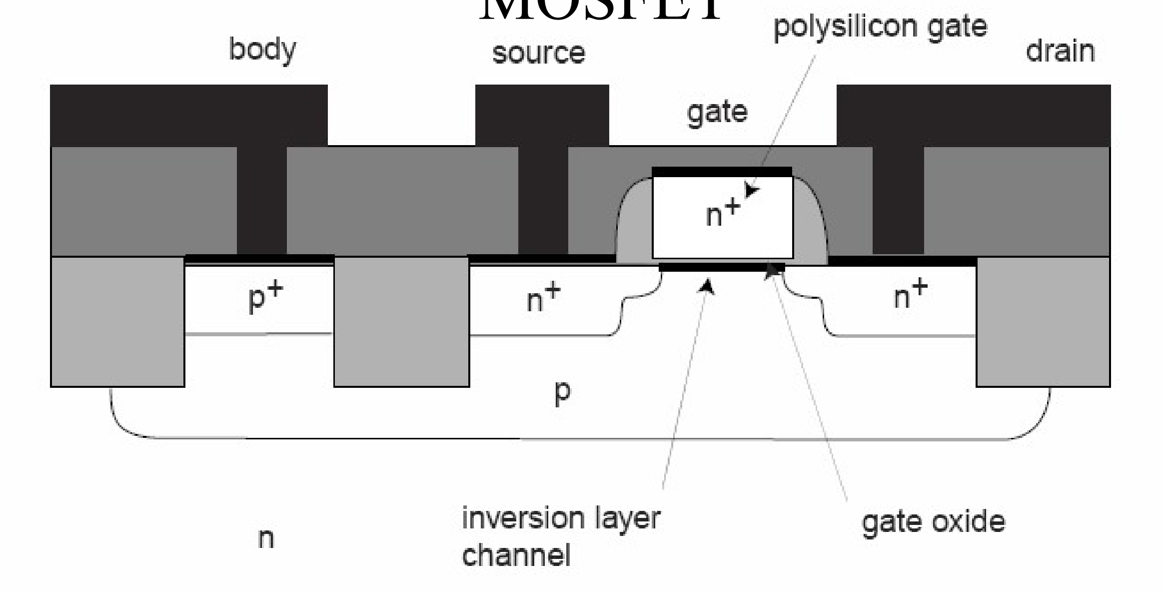
Extrinsic: $n \neq p$

Conductivity almost temperature independent at room temperature

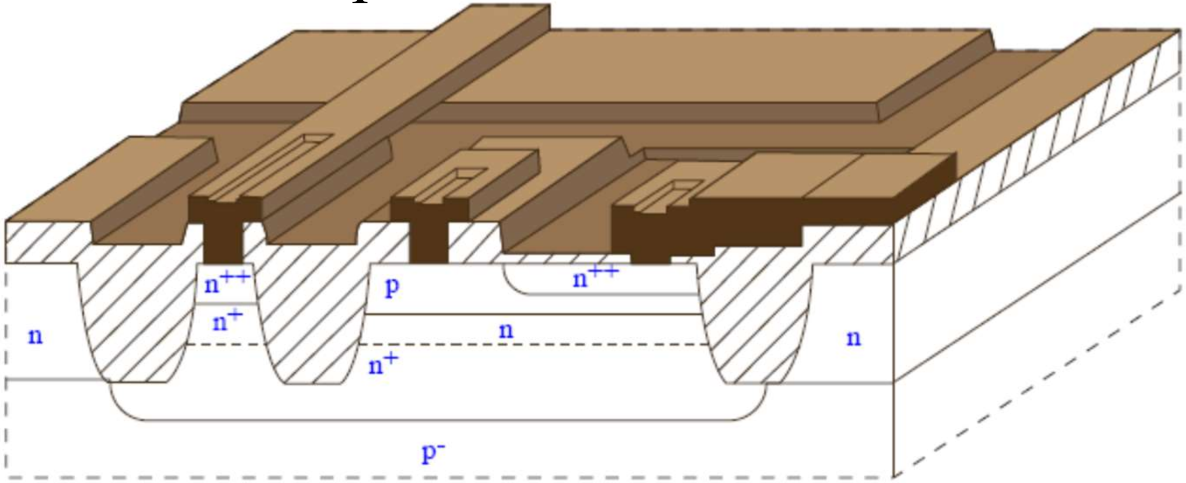
Why dope with donors AND acceptors?



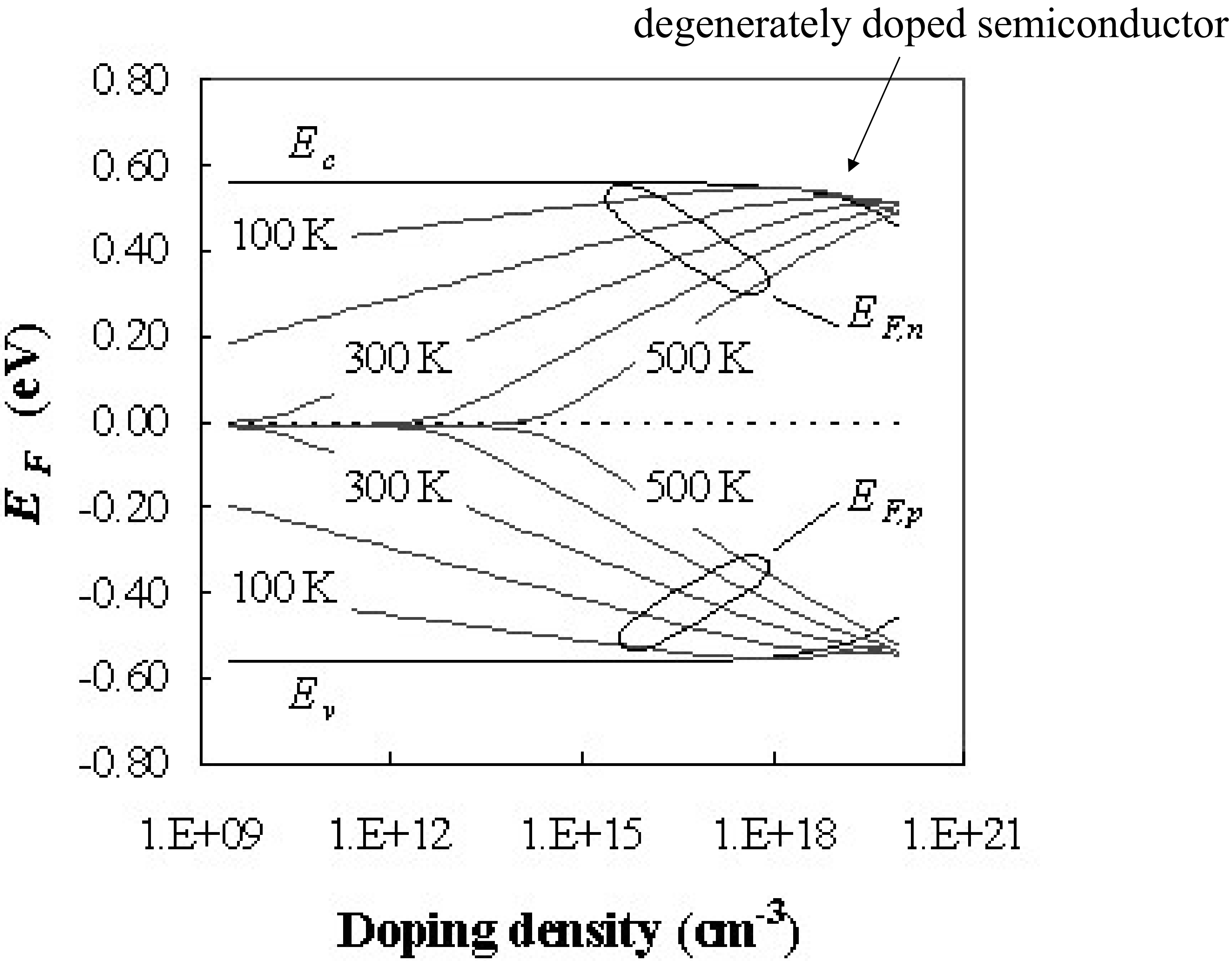
MOSFET



Bipolar Junction Transistor



Oxide isolated integrated BJT - a modern process


degenerately doped semiconductor
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
-0.80
E_c
E_v
100 K
300 K
500 K
300 K
500 K
100 K
$E_{F,n}$
$E_{F,p}$
1.E+09
1.E+12
1.E+15
1.E+18
1.E+21
E_F (eV)
Doping density (cm^{-3})

Degenerate semiconductor

Heavily doped semiconductors are called degenerately doped

$N_D > 0.1\ N_c$ -> E_F in the conduction band

$N_A > 0.1\ N_v$ -> E_F in the valence band

Heavy doping narrows the band gap

The Boltzmann approximation is not valid

Degenerate semiconductors = metal

Carrier transport

Carrier Transport

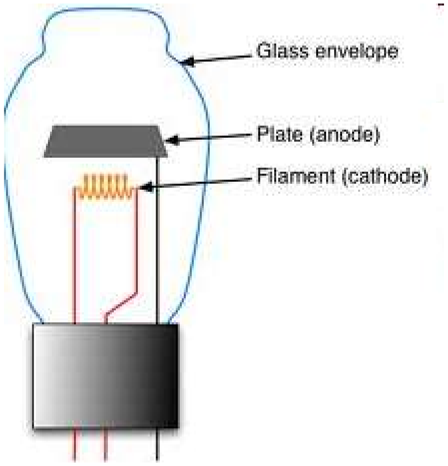
Ballistic transport
Drift
Diffusion
Tunneling

Ballistic transport

$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{d\vec{v}}{dt}$$

$$\vec{v} = \frac{-e\vec{E}t}{m} + \vec{v}_0$$

$$\vec{x} = \frac{-e\vec{E}t^2}{2m} + \vec{v}_0 t + \vec{x}_0$$



Electrons moving in an electric field follow parabolic trajectories like a ball in a gravitational field.

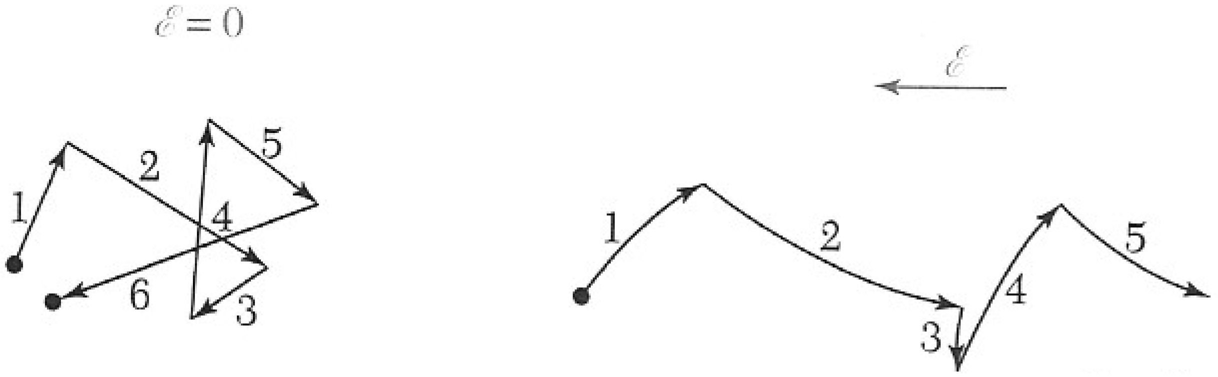
Drift

The electrons scatter and change direction after a time τ_{sc}.

Classical equipartition: $\frac{1}{2}mv_{th}^2 = \frac{3}{2}k_BT$

At 300 K, $v_{th} \sim 10^7$ cm/s.

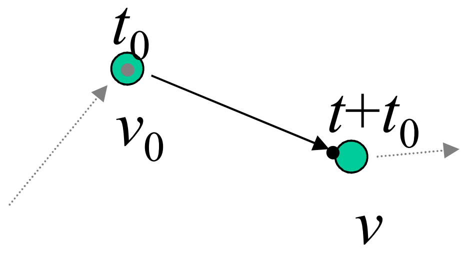
mean free path: $\ell = v_{th}\tau_{sc} \sim 10$ nm ~ 200 atoms



Drift (diffusive transport)

$$\vec{F} = -e\vec{E} = m^*\vec{a} = m^*\frac{d\vec{v}}{dt}$$

time between two collisions



$$\vec{v} = \vec{v}_0 - \frac{e\vec{E}}{m^*}(t - t_0)$$

$<v_0> = 0$ $\quad$ $<t - t_0> = \tau_{sc}$

$$\vec{v}_d = \frac{-e\vec{E}\tau_{sc}}{m^*} = \frac{-e\vec{E}\ell}{m^* v}$$

drift velocity: $\overrightarrow{v}_{d,n} = -\mu_n \overrightarrow{E}$ $\quad$ $\overrightarrow{v}_{d,p} = \mu_p \overrightarrow{E}$

Drift

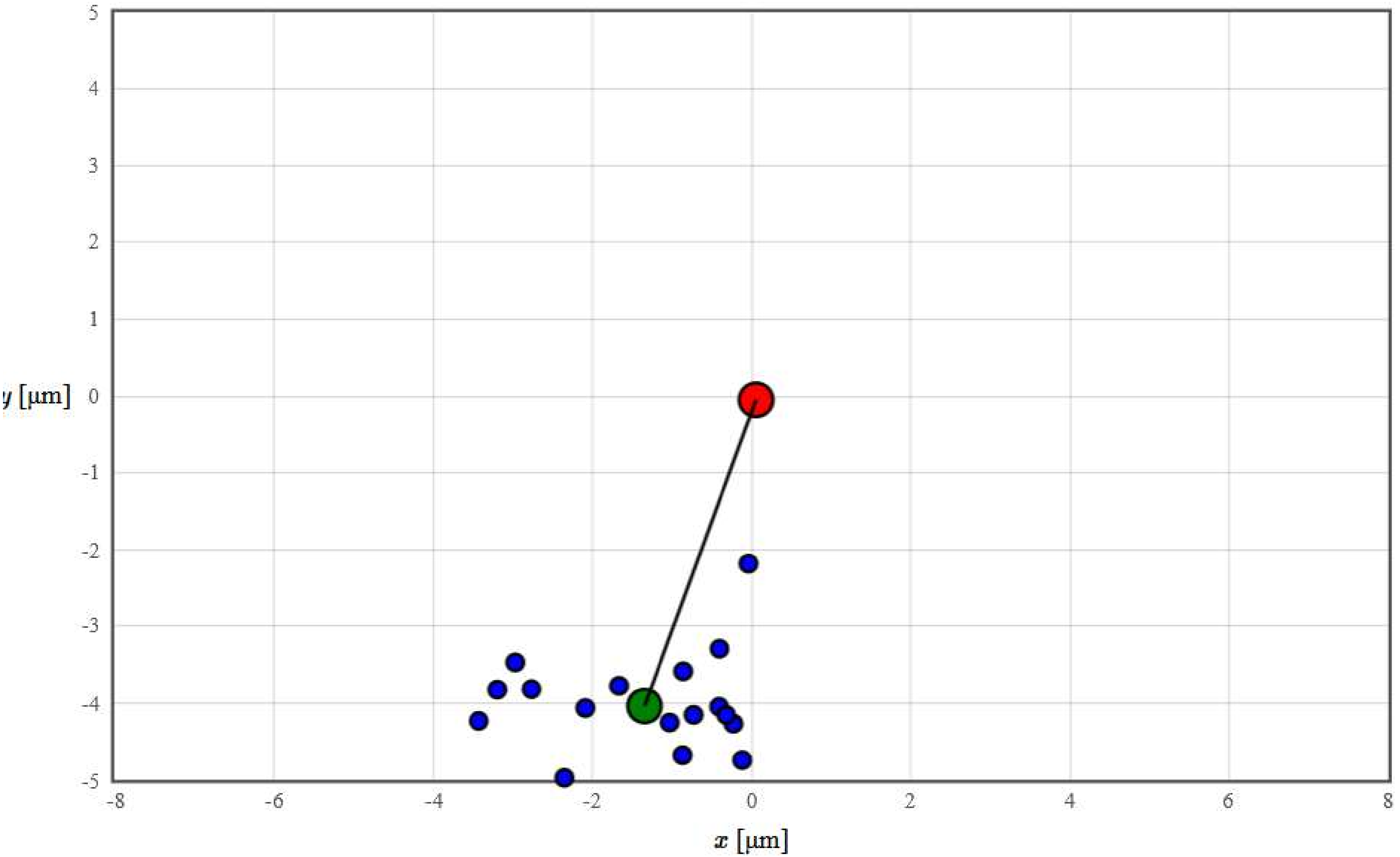
drift velocity: $\overrightarrow{v}_{d,n} = -\mu_n \overrightarrow{E}$ $\overrightarrow{v}_{d,p} = \mu_p \overrightarrow{E}$

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = \left(ne\mu_n + pe\mu_p\right)\vec{E} = \sigma\vec{E}$$ (Ohm's law)

$$\mu = \frac{-e\tau_{sc}}{m^*} = \frac{-e\ell}{m^* v}$$

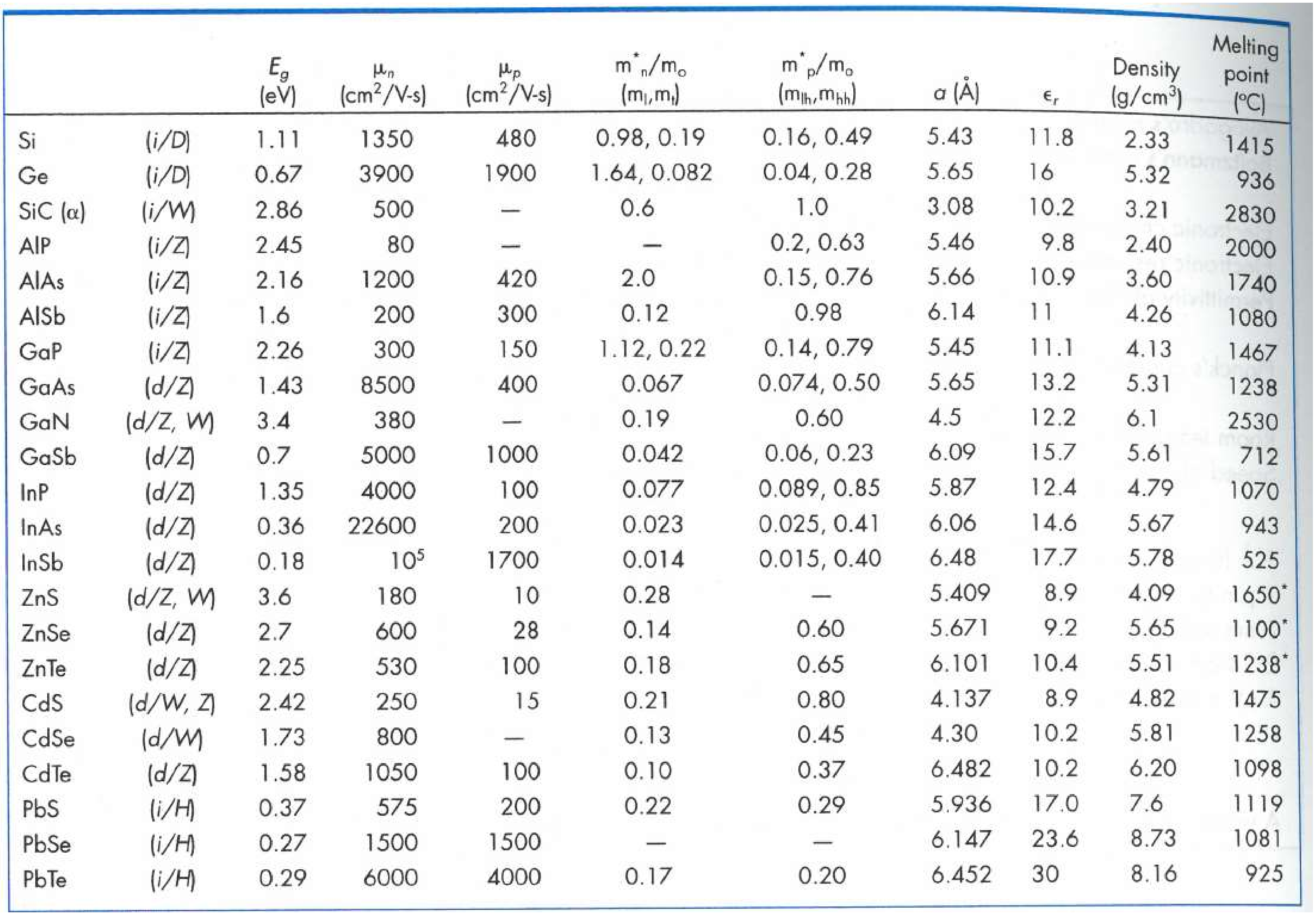
for Si: $\mu_n = 1350\ \mathrm{cm}^2/\mathrm{Vs}$
$\mu_p = 450\ \mathrm{cm}^2/\mathrm{Vs}$

For $E = 1000$ V/cm $v_d = 10^6$ cm/s



http://lampx.tugraz.at/~hadley/psd/L5/drude.php

Drift



		E_g (eV)	μ_n ($cm^2/V\text{-}s$)	μ_p ($cm^2/V\text{-}s$)	m^*_n/m_o (m_l, m_t)	m^*_p/m_o (m_{lh}, m_{hh})	a (Å)	ϵ_r	Density (g/cm^3)	Melting point (°C)
Si	(*i*/*D*)	1.11	1350	480	0.98, 0.19	0.16, 0.49	5.43	11.8	2.33	1415
Ge	(*i*/*D*)	0.67	3900	1900	1.64, 0.082	0.04, 0.28	5.65	16	5.32	936
SiC (α)	(*i*/*W*)	2.86	500	—	0.6	1.0	3.08	10.2	3.21	2830
AlP	(*i*/*Z*)	2.45	80	—	—	0.2, 0.63	5.46	9.8	2.40	2000
AlAs	(*i*/*Z*)	2.16	1200	420	2.0	0.15, 0.76	5.66	10.9	3.60	1740
AlSb	(*i*/*Z*)	1.6	200	300	0.12	0.98	6.14	11	4.26	1080
GaP	(*i*/*Z*)	2.26	300	150	1.12, 0.22	0.14, 0.79	5.45	11.1	4.13	1467
GaAs	(*d*/*Z*)	1.43	8500	400	0.067	0.074, 0.50	5.65	13.2	5.31	1238
GaN	(*d*/*Z*, *W*)	3.4	380	—	0.19	0.60	4.5	12.2	6.1	2530
GaSb	(*d*/*Z*)	0.7	5000	1000	0.042	0.06, 0.23	6.09	15.7	5.61	712
InP	(*d*/*Z*)	1.35	4000	100	0.077	0.089, 0.85	5.87	12.4	4.79	1070
InAs	(*d*/*Z*)	0.36	22600	200	0.023	0.025, 0.41	6.06	14.6	5.67	943
InSb	(*d*/*Z*)	0.18	10^5	1700	0.014	0.015, 0.40	6.48	17.7	5.78	525
ZnS	(*d*/*Z*, *W*)	3.6	180	10	0.28	—	5.409	8.9	4.09	1650*
ZnSe	(*d*/*Z*)	2.7	600	28	0.14	0.60	5.671	9.2	5.65	1100*
ZnTe	(*d*/*Z*)	2.25	530	100	0.18	0.65	6.101	10.4	5.51	1238*
CdS	(*d*/*W*, *Z*)	2.42	250	15	0.21	0.80	4.137	8.9	4.82	1475
CdSe	(*d*/*W*)	1.73	800	—	0.13	0.45	4.30	10.2	5.81	1258
CdTe	(*d*/*Z*)	1.58	1050	100	0.10	0.37	6.482	10.2	6.20	1098
PbS	(*i*/*H*)	0.37	575	200	0.22	0.29	5.936	17.0	7.6	1119
PbSe	(*i*/*H*)	0.27	1500	1500	—	—	6.147	23.6	8.73	1081
PbTe	(*i*/*H*)	0.29	6000	4000	0.17	0.20	6.452	30	8.16	925

Solid state electronic devices, Streetman and Banerjee

$$\vec{v}_{d,n} = -\mu_n \vec{E} \qquad \vec{v}_{d,p} = \mu_p \vec{E}$$

$$\vec{j} = -ne\vec{v}_{d,n} + pe\vec{v}_{d,p} = \left(ne\mu_n + pe\mu_p\right)\vec{E} = \sigma\vec{E}$$

Matthiessen's rule

$$\frac{1}{\tau_{sc}} = \frac{1}{\tau_{sc,lattice}} + \frac{1}{\tau_{sc,impurity}}$$

mostly temperature independent

phonons, temperature dependent

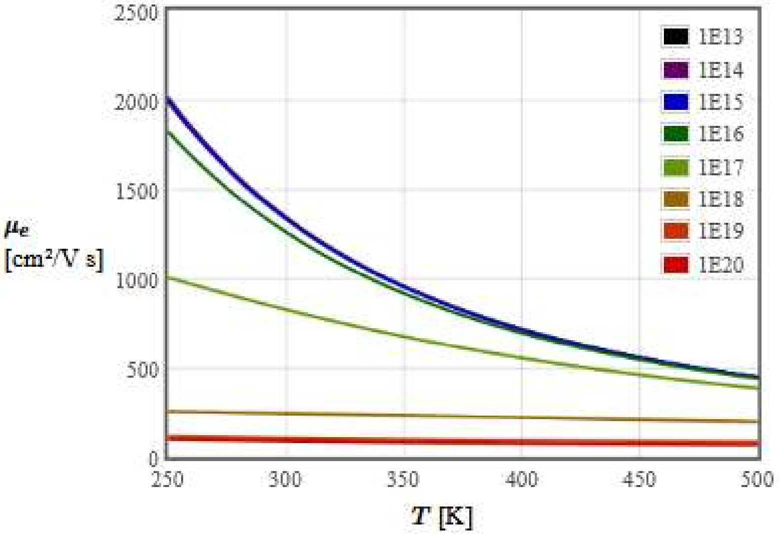
$$\frac{1}{\mu} = \frac{1}{\mu_{lattice}} + \frac{1}{\mu_{impurity}}$$

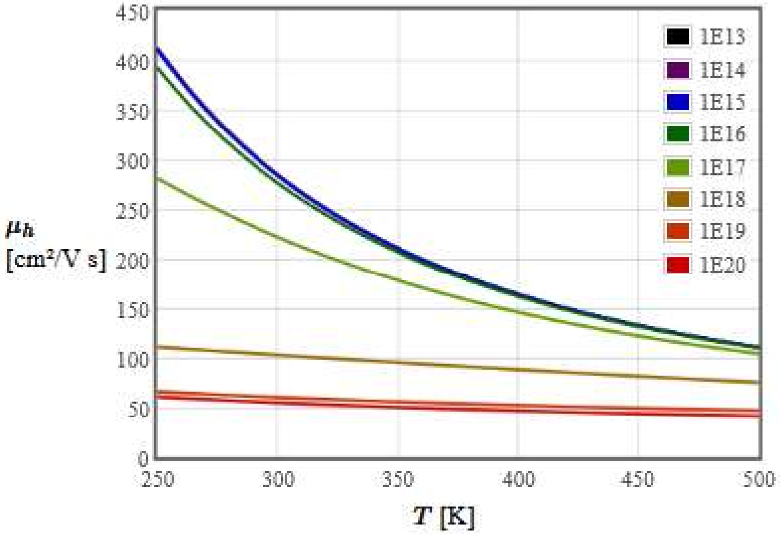
$$\sigma = \frac{1}{\rho} = ne\mu_n + pe\mu_p$$

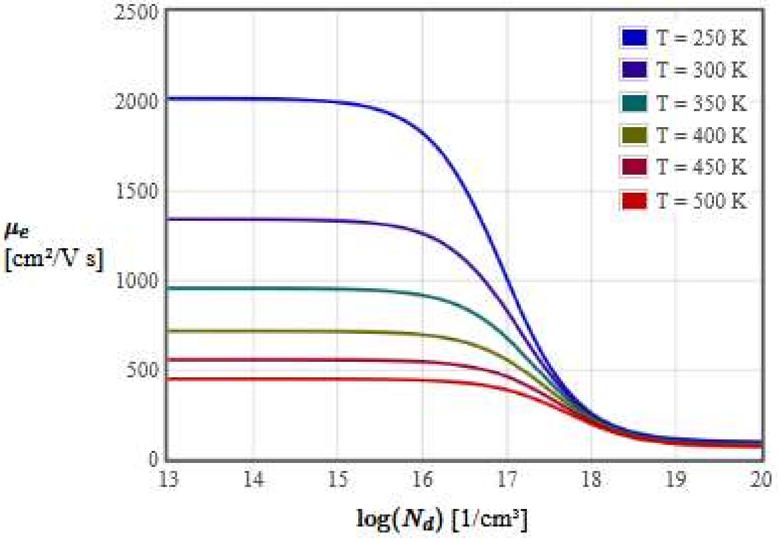
doping increases the conductivity
by increasing the carrier density
but decreases the mobility

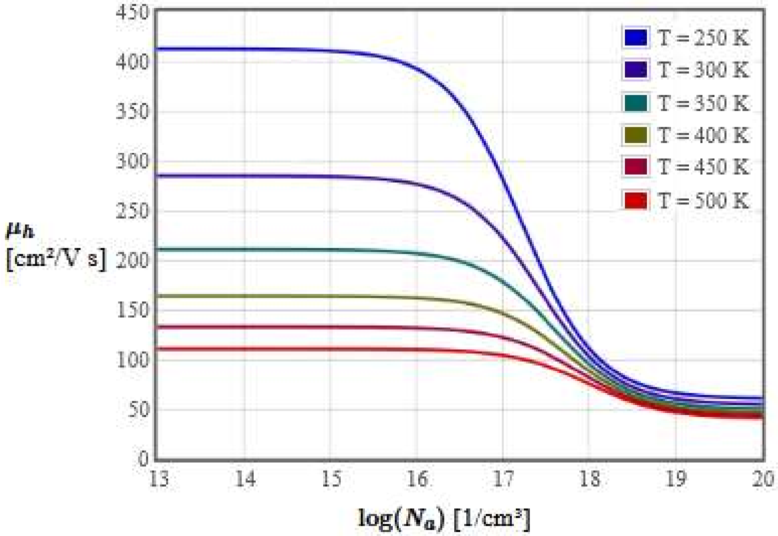
$$\mu_e = 88\left(\frac{T}{300}\right)^{-0.57} + \frac{7.4 \times 10^8 T^{-2.33}}{1 + 0.88\left[\frac{N_d}{1.26\times10^{17}\left(\frac{T}{300}\right)^{2.4}}\right]\left(\frac{T}{300}\right)^{-0.146}} \quad \mathrm{cm^2/V\ s},$$

$$\mu_h = 54.3\left(\frac{T}{300}\right)^{-0.57} + \frac{1.36 \times 10^8 T^{-2.33}}{1 + 0.88\left[\frac{N_a}{2.35\times10^{17}\left(\frac{T}{300}\right)^{2.4}}\right]\left(\frac{T}{300}\right)^{-0.146}} \quad \mathrm{cm^2/V\ s}.$$



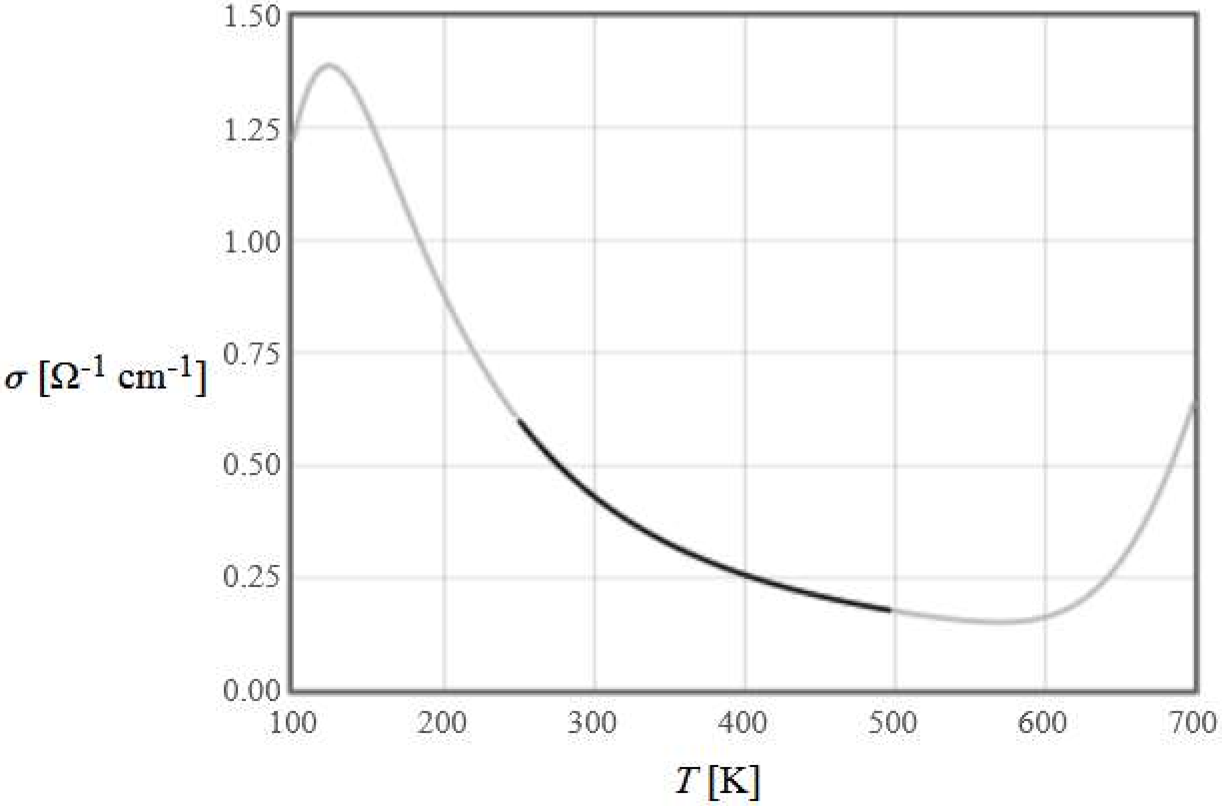






Conductivity of Silicon

$$\sigma = ne\mu_n + pe\mu_p$$



$N_A = 10^{16}$ 1/cm^3

http://lampx.tugraz.at/~hadley/psd/L4/conductivity.php

Ballistic transport in transistors

The mean free path ~100 nm > gate length ~ 20 nm

v not proportional to *E*

$$\xcancel{\vec{v} = \mu\vec{E}}$$

j not proportional to *E*

$$\xcancel{\vec{j} = \sigma\vec{E}}$$

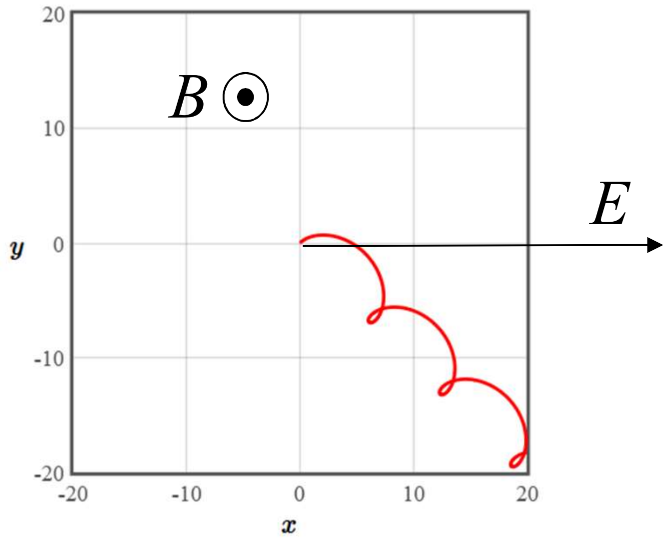
nonlocal response

Electrons bend in a magnetic field like they do in vacuum.

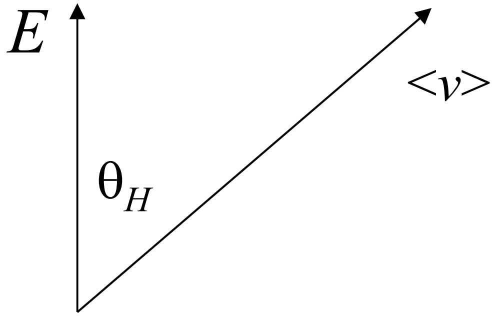
Crossed *E* and *B* fields

Ballistic transport

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v} \times \vec{B}\right)$$



Diffusive transport



Hall angle: $$\theta_H = \tan^{-1}\left(-\frac{eB_z\tau_{sc}}{m^*}\right)$$

http://lampx.tugraz.at/~hadley/physikm/script/magnetism/constant_EB.en.php

Magnetic field (diffusive transport)

$$\vec{F} = m\vec{a} = -e\vec{E} = e\frac{\vec{v}_d}{\mu}$$

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v}_d \times \vec{B}\right) = e\frac{\vec{v}_d}{\mu}$$

If B is in the z-direction, the three components of the force are

$$-\mu\left(E_x + v_{dy}B_z\right) = v_{dx}$$

$$-\mu\left(E_y - v_{dx}B_z\right) = v_{dy}$$

$$-\mu E_z = v_{dz}$$

Magnetic field

$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

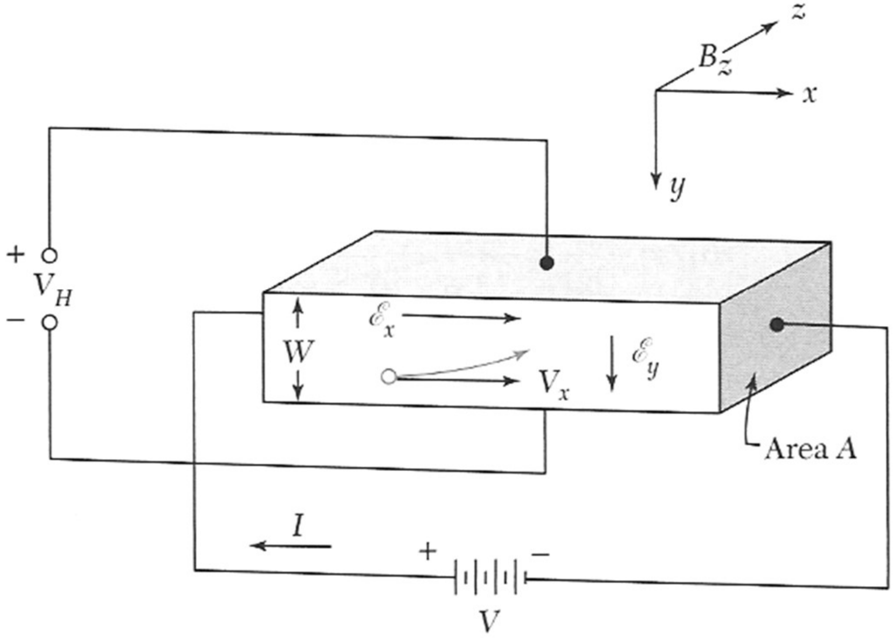
$$v_{d,z} = -\mu E_z$$

If $E_y = 0$,

$$v_{d,y} = -\mu B_z v_{d,x}$$

$$\tan \theta_H = -\mu B_z$$

The Hall Effect (diffusive regime)



$$v_{d,x} = -\mu E_x - \mu B_z v_{d,y}$$

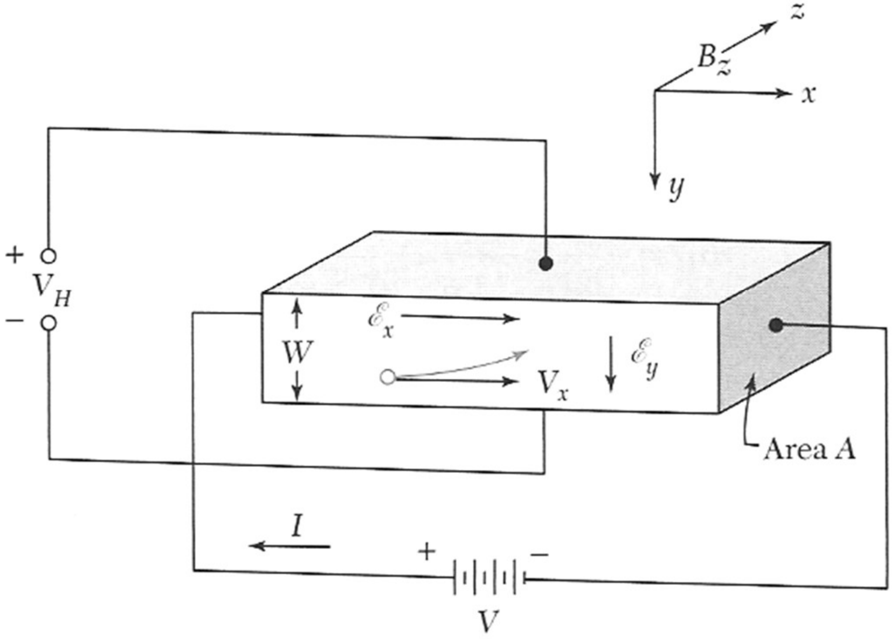
$$v_{d,y} = -\mu E_y + \mu B_z v_{d,x}$$

$$v_{d,z} = -\mu E_z$$

If $v_{d,y} = 0$,

$E_y = v_x B_z = V_H/W = R_H j_x B_z$ V_H = Hall voltage, R_H = Hall Constant

The Hall Effect (diffusive regime)



$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$

$E_y = v_x B_z = V_H/W = R_H j_x B_z$ V_H = Hall voltage, R_H = Hall Constant

$R_H = v_x/j_x$

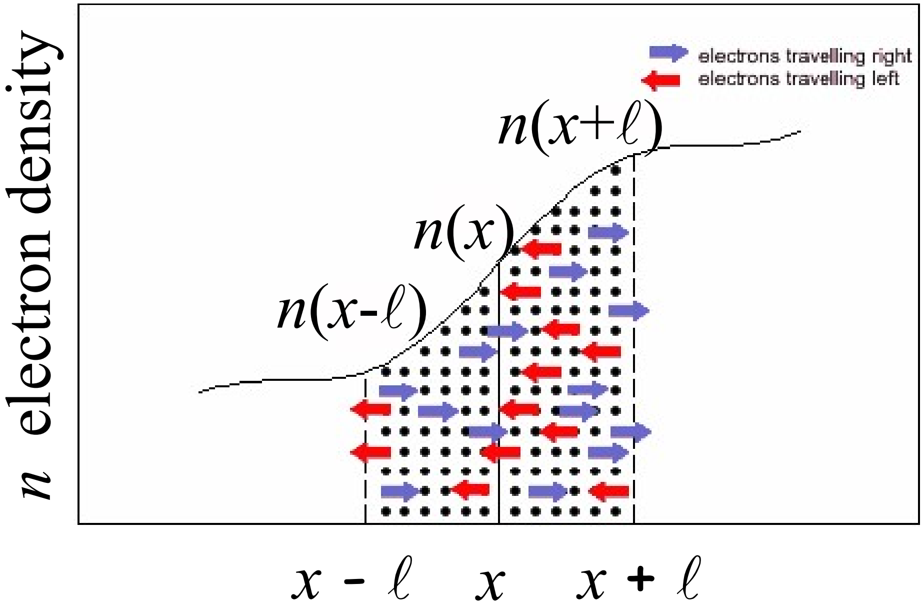
$v_x = - j_x/ne$ for n-type $v_x = j_x/pe$ for p-type

$R_H = - 1/ne$ for n-type $R_H = 1/pe$ for p-type

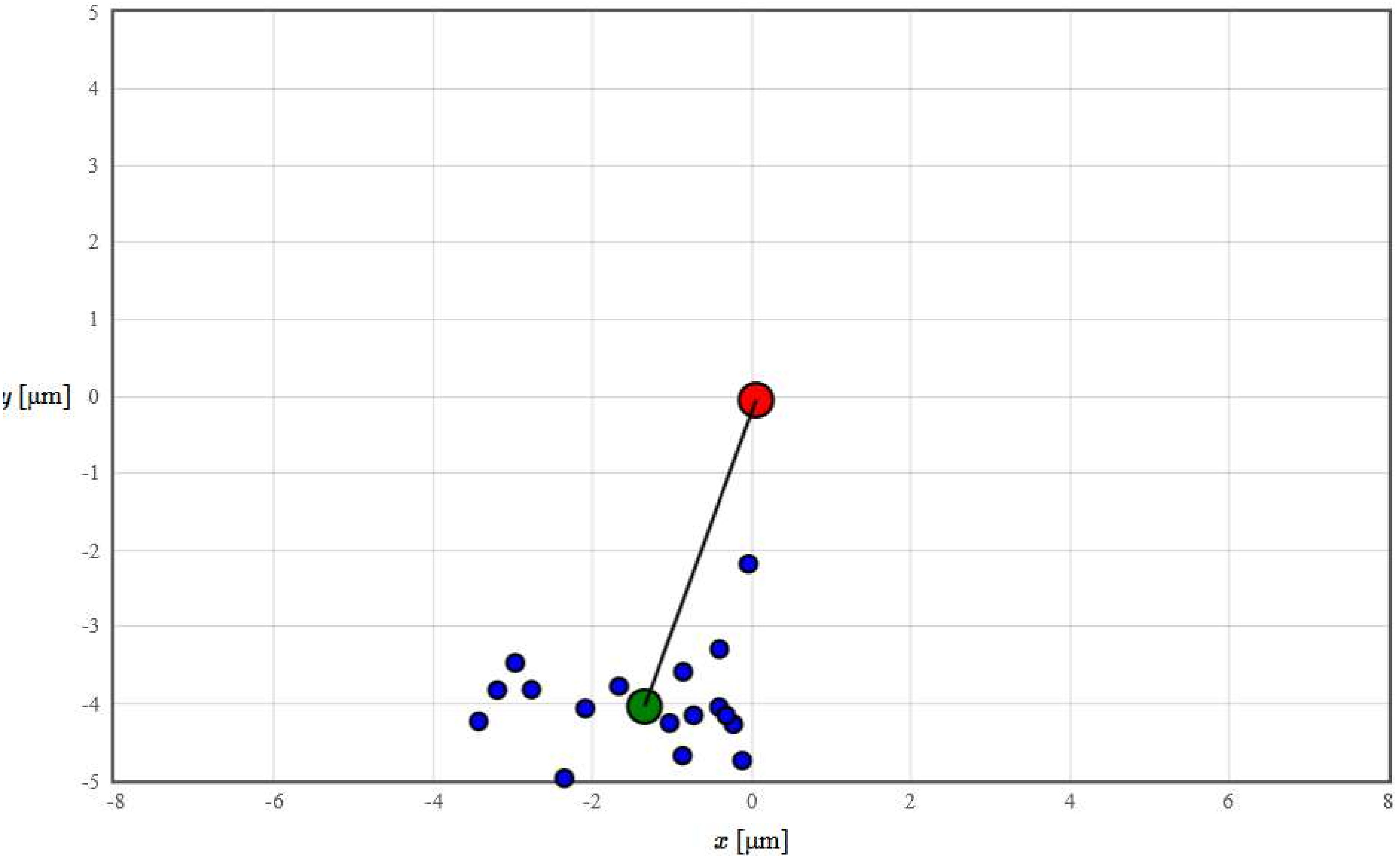
Diffusion

$$j_{n,diff} = \left|e\right| D_n \frac{dn}{dx}$$

$$j_{p,diff} = -\left|e\right| D_p \frac{dp}{dx}$$

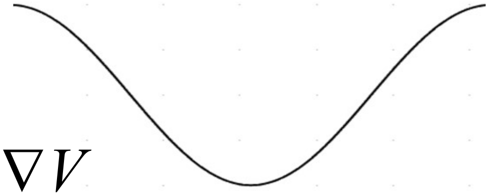


Diffusion is from high concentration to low concentration.



http://lampx.tugraz.at/~hadley/psd/L5/drude.php

Einstein relation



$$\vec{E} = -\nabla V$$

$$n = A\exp\left(\frac{-eV}{k_B T}\right)$$

Boltzmann factor

In equilibrium, drift = diffusion

$$-en\mu\vec{E} + eD\nabla n = 0$$

$$\nabla n = -\frac{e}{k_B T} A\exp\left(\frac{-eV_{pot}}{k_B T}\right)\nabla V = -\frac{ne}{k_B T}\nabla V = \frac{ne\vec{E}}{k_B T}$$

$$-en\mu\vec{E} + eD\frac{ne\vec{E}}{k_B T} = 0$$

$$\boxed{D = \frac{\mu k_B T}{e}}$$

Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen