Carrier transport

Current Density Equations

Drift

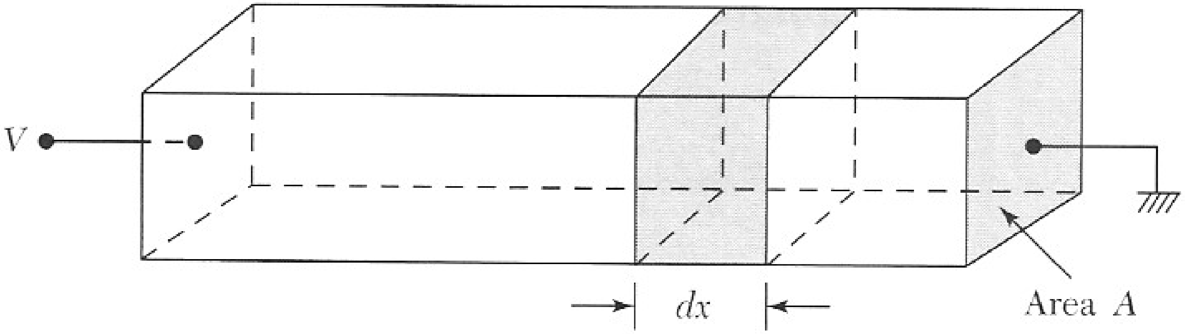
Diffusion

$$\vec{j}_n = ne\mu_n\vec{E} + eD_n\nabla n$$
$$\vec{j}_p = pe\mu_p\vec{E} - eD_p\nabla p$$

$$\vec{j}_{total} = \vec{j}_n + \vec{j}_p$$

Because of the currents, the electron and hole concentrations change.

Continuity equations

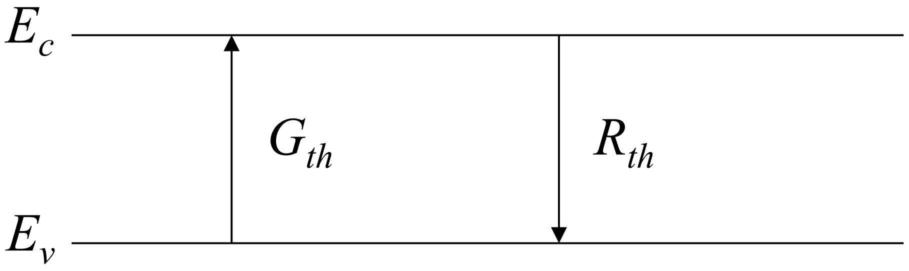


$$\frac{\partial p}{\partial t} = -\frac{1}{e}\nabla\cdot\vec{j}_p + G_p - R_p$$

$$\frac{\partial n}{\partial t} = \frac{1}{e}\nabla\cdot\vec{j}_n + G_n - R_n$$

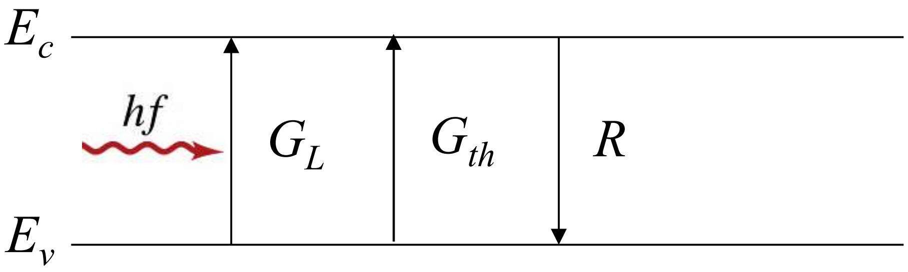
j_n and j_p consist of drift and diffusion terms

Generation and Recombination



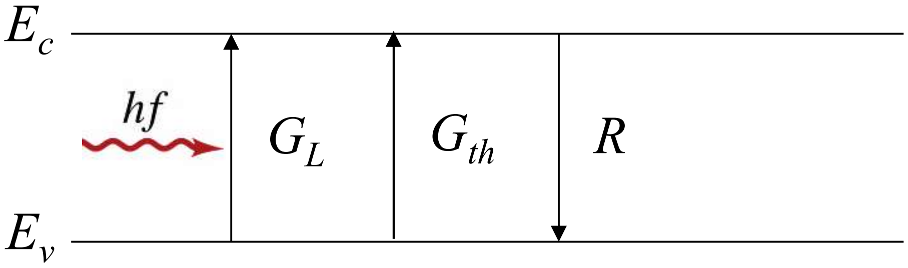
thermal
equilibrium

Shining light on a semiconductor or injecting electrons or holes from a contact can result in a **non-equilibrium** distribution $np \neq n_i^2$



non equilibrium

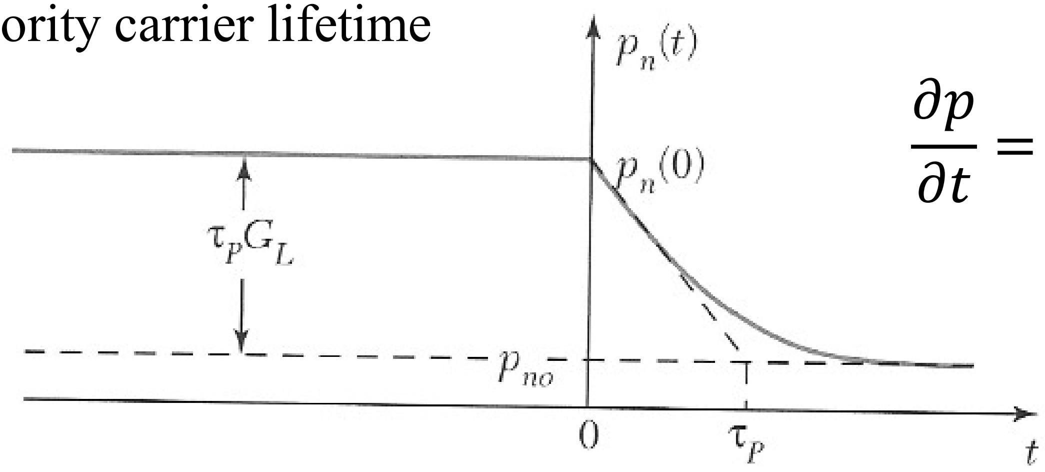
Recombination



$$R - R_{th} = \frac{p_n - p_{n0}}{\tau_p}$$

Recombination rate is limit by the density of minority carriers.
The majority carriers have to find a minority carrier to recombine.

p_n (or n_p) = minority carrier concentration
p_{n0} (or n_{p0}) = equilibrium minority carrier concentration
τ_p = minority carrier lifetime



$$\frac{\partial p}{\partial t} = \frac{p_n - p_{n0}}{\tau_p}$$

minority carrier lifetimes

p-type

$$n_p(t) = n_{excess} \exp(-t/\tau_n) + n_{p0}$$

minority carrier lifetimes

n-type

$$p_n(t) = p_{excess} \exp(-t/\tau_p) + p_{n0}$$

$$np = n_i^2$$

Continuity equations

$$\frac{\partial n}{\partial t} = \frac{1}{e}\nabla \cdot \vec{j}_n + G_n - R_n$$

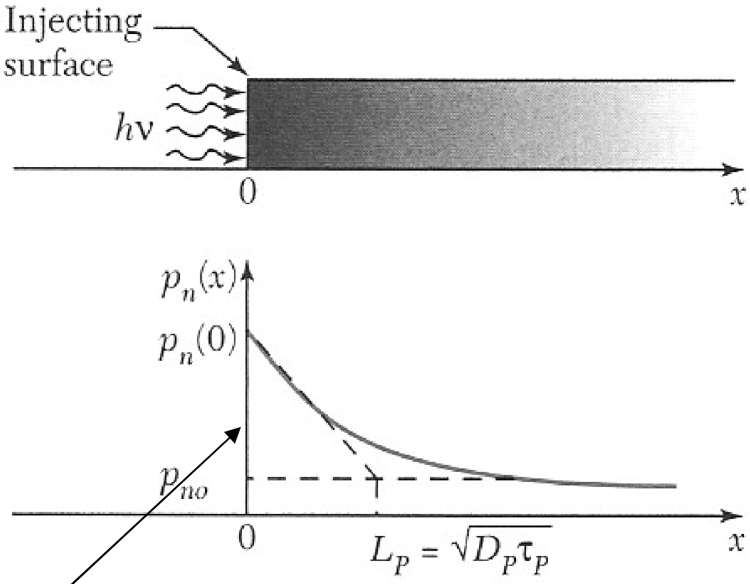
drift: $\vec{j}_n = -ne\mu_n\vec{E}$ $\qquad \nabla \cdot \vec{j}_n = -en\mu_n\nabla \cdot \vec{E} - e\nabla n\mu_n\vec{E}$

diffusion: $\vec{j}_{n,diff} = |e|D_n\nabla n$ $\qquad \nabla \cdot \vec{j}_{n,diff} = |e|D_n\nabla^2 n$

$$\boxed{\begin{aligned} \frac{\partial n}{\partial t} &= n\mu_n\nabla \cdot \vec{E} + \nabla n\mu_n\vec{E} + D_n\nabla^2 n + G_n - \frac{n-n_0}{\tau_n} \\ \frac{\partial p}{\partial t} &= -p\mu_p\nabla \cdot \vec{E} - \nabla p\mu_p\vec{E} + D_p\nabla^2 p + G_p - \frac{p-p_0}{\tau_p} \end{aligned}}$$

Diffusion Length

n-type



Steady state

$$\frac{\partial p_n}{\partial t} = \overset{\downarrow}{0} = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p}$$

$$p_n(x) = p_{n0} + \left(p_n(0) - p_{n0} \right) \exp\left(\frac{-x}{L_p} \right)$$

Generation only occurs at the surface. There the minority carrier density is $p_n(0)$.

Diffusion Length

$$0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau_p} \quad \Leftarrow \quad p_n(x) = p_{n0} + \left(p_n(0) - p_{n0}\right)\exp\left(\frac{-x}{L_p}\right)$$

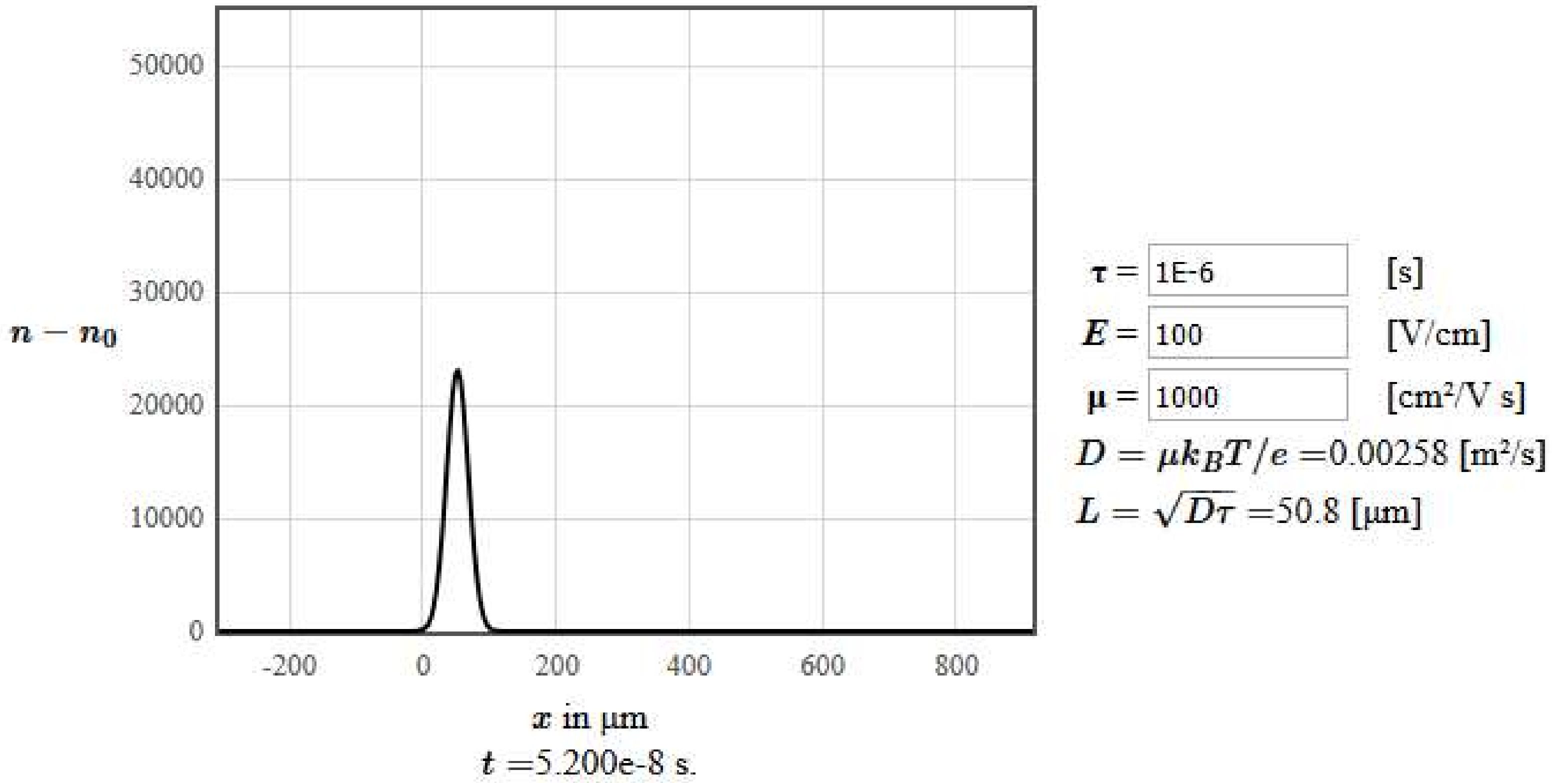
$$0 = \frac{D_p\left(p_n(0) - p_{n0}\right)}{L_p^2}\exp\left(\frac{-x}{L_p}\right) - \frac{\left(p_n(0) - p_{n0}\right)}{\tau_p}\exp\left(\frac{-x}{L_p}\right)$$

$$L_p = \sqrt{D_p \tau_p}$$

diffusion length,
typically microns

Haynes Shockley experiment

$$n_p(x,t) = \frac{n_{generated}}{\sqrt{4\pi D_n t}} \exp\left(-\frac{(x-\mu_n E t)^2}{4 D_n t}\right) \exp\left(-\frac{t}{\tau_n}\right) + n_{p0}$$



t =5.200e-8 s.

τ = 1E-6 [s]

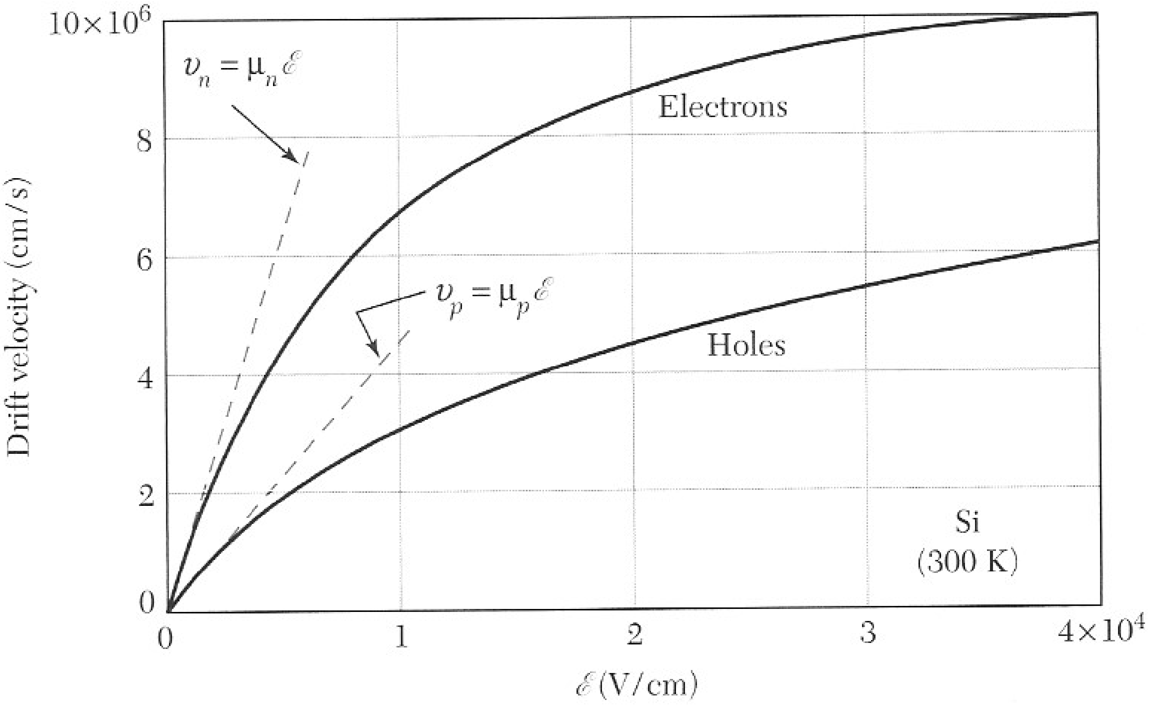
E = 100 [V/cm]

μ = 1000 [cm²/V s]

$D = \mu k_B T/e$ =0.00258 [m²/s]

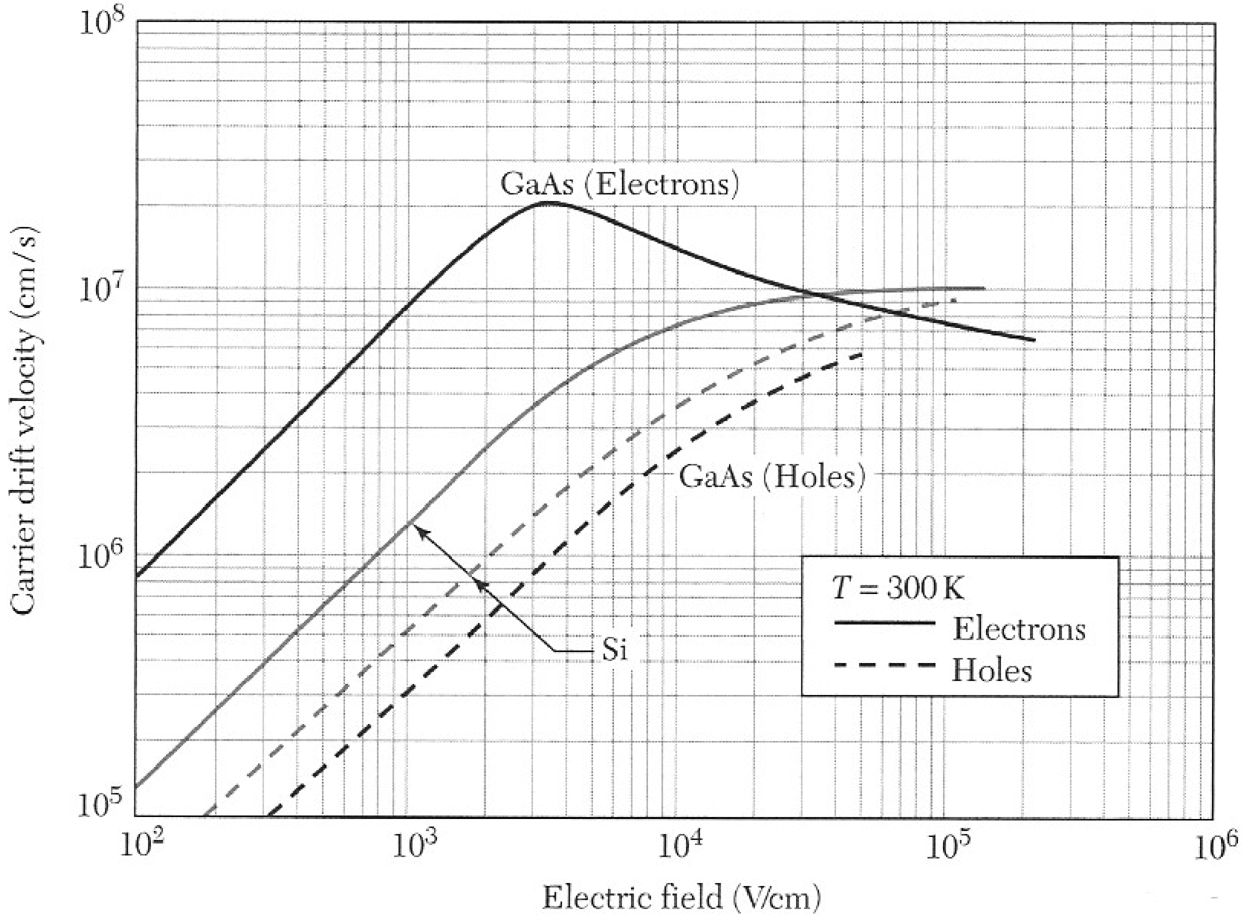
$L = \sqrt{D\tau}$ =50.8 [μm]

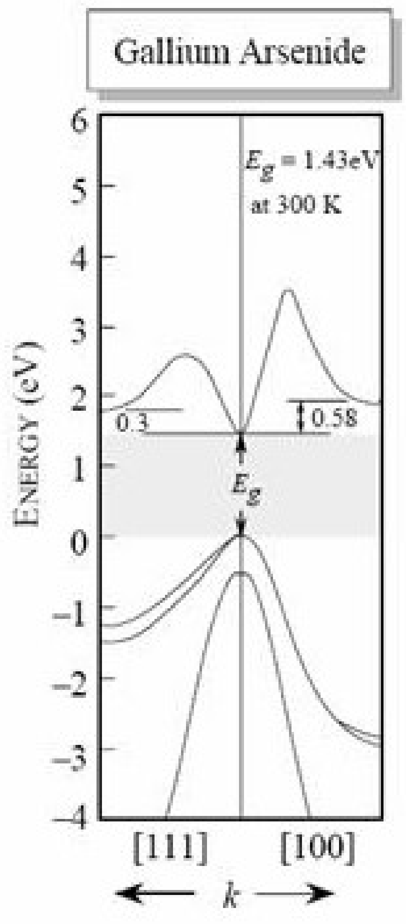
High Fields



Silicon

High Fields



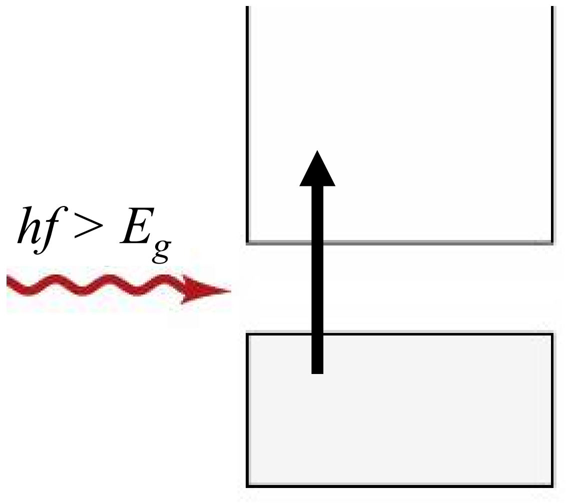


GaAs

Impact ionization

Carriers are accelerated to an energy above the gap before they scatter. They generate more electron-hole pairs. This results in an avalanche breakdown of the device.

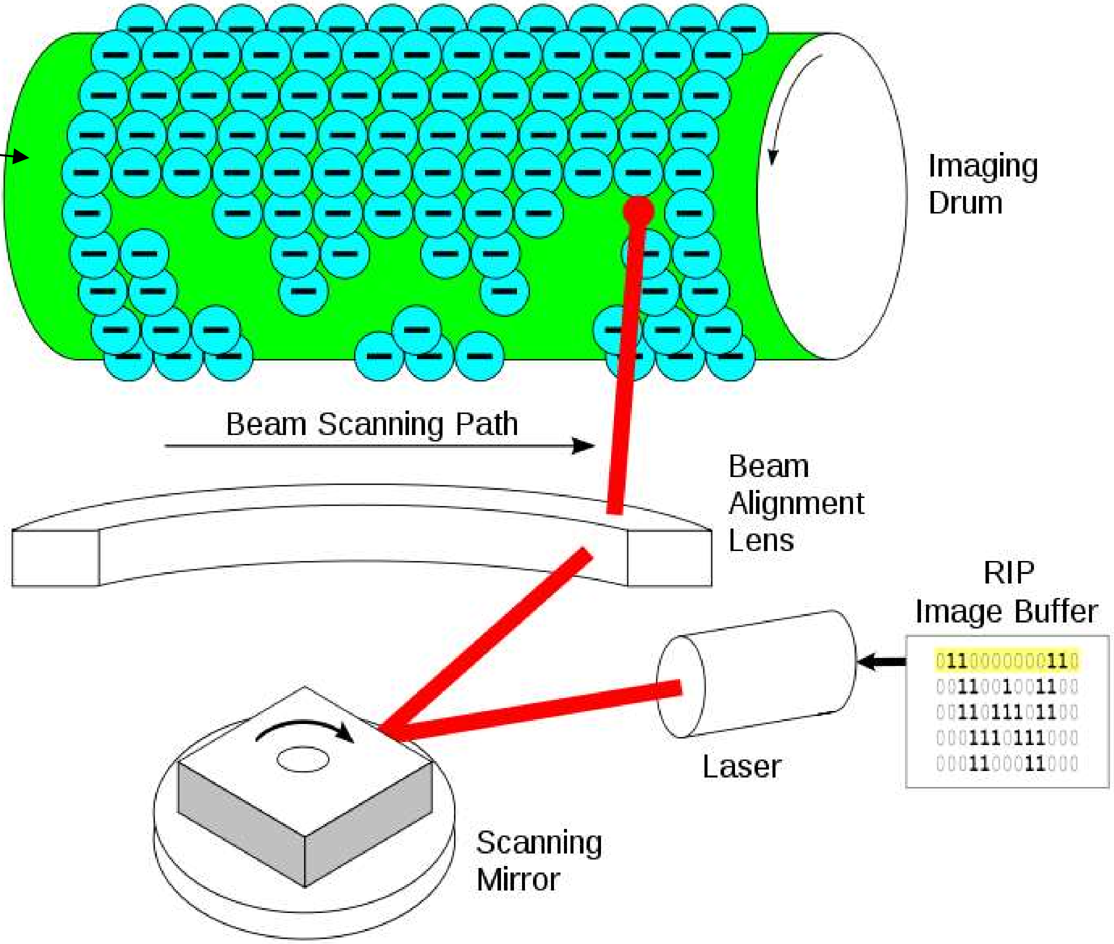
Photoconductivity



$$\sigma = ne\mu_n + pe\mu_p$$

Light increases the conductivity of a semiconductor.

Laser printer



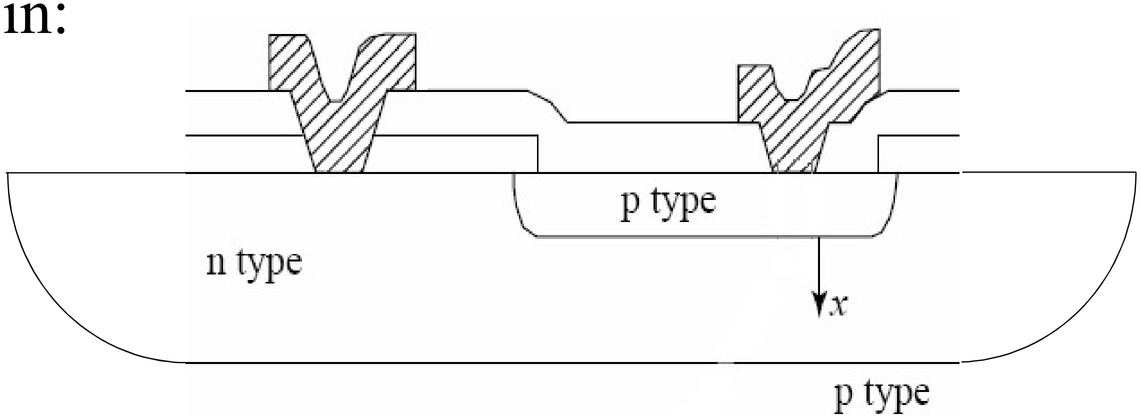
semiconducting
photoconductor

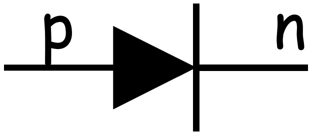
pn - Junctions

pn junctions

pn junctions are found in:

- diodes
- solar cells
- LEDs
- isolation
- JFETs
- bipolar transistors
- MOSFETs
- Lasers diodes

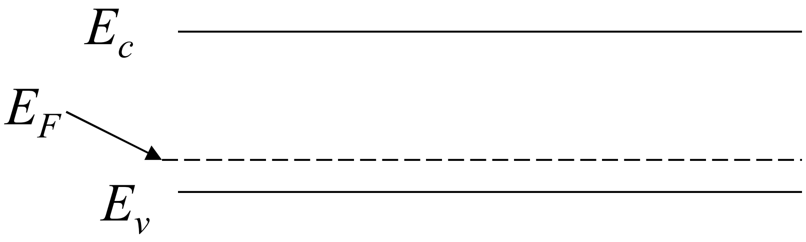




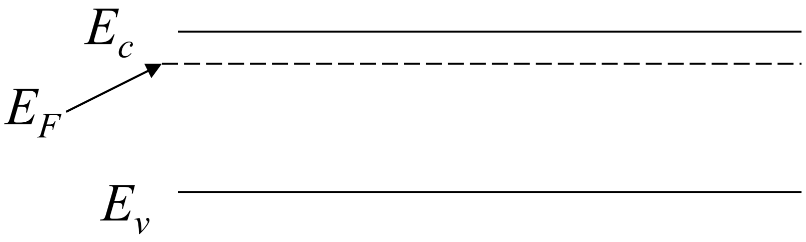
pn junction

isolated semiconductors

p-type



n-type



$$E_F = E_v + k_B T \ln\left(\frac{N_v}{N_A}\right)$$

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

valid for both n and p doping

$$E_F = E_c - k_B T \ln\left(\frac{N_c}{N_D}\right)$$

pn junction

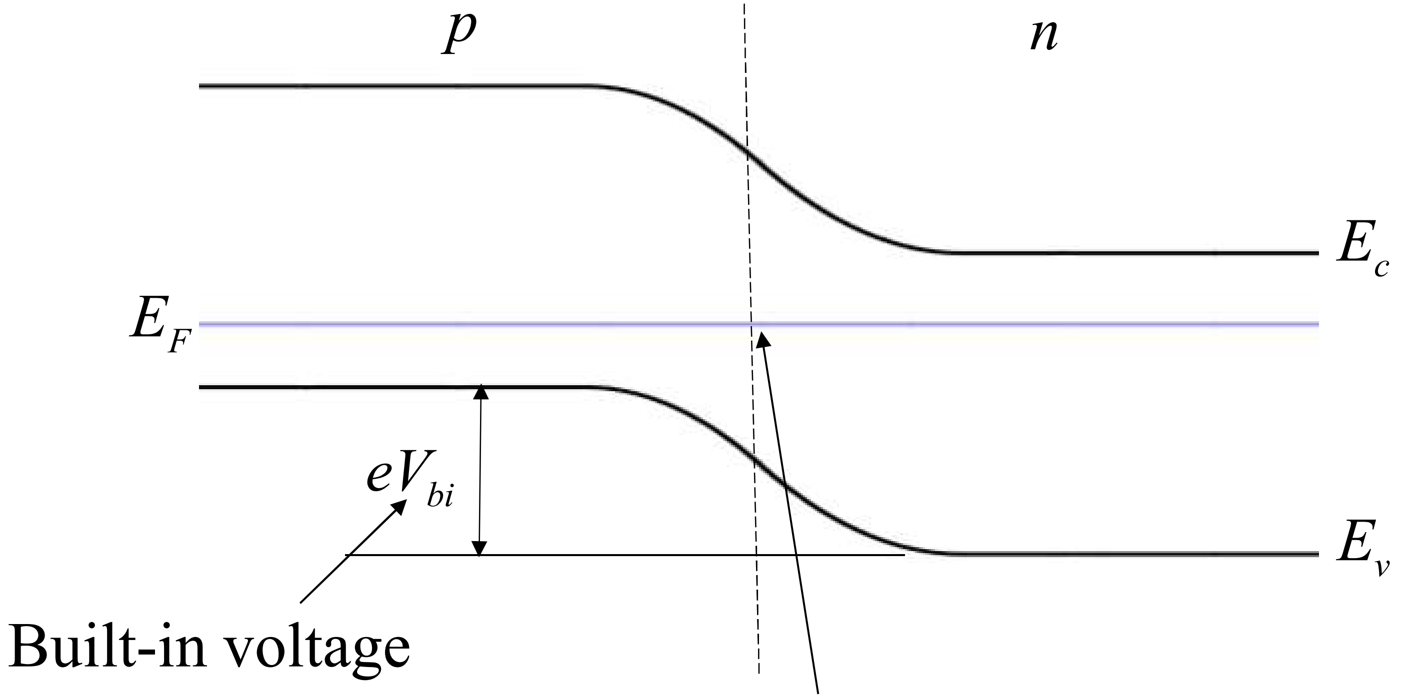
semiconductors in contact
electrons flow from n to p

$$n = N_c \exp\left(\frac{\boxed{E_F - E_c}}{k_B T}\right) \approx N_D$$

$$p = N_v \exp\left(\frac{\boxed{E_v - E_F}}{k_B T}\right) \approx N_A$$

pn junction

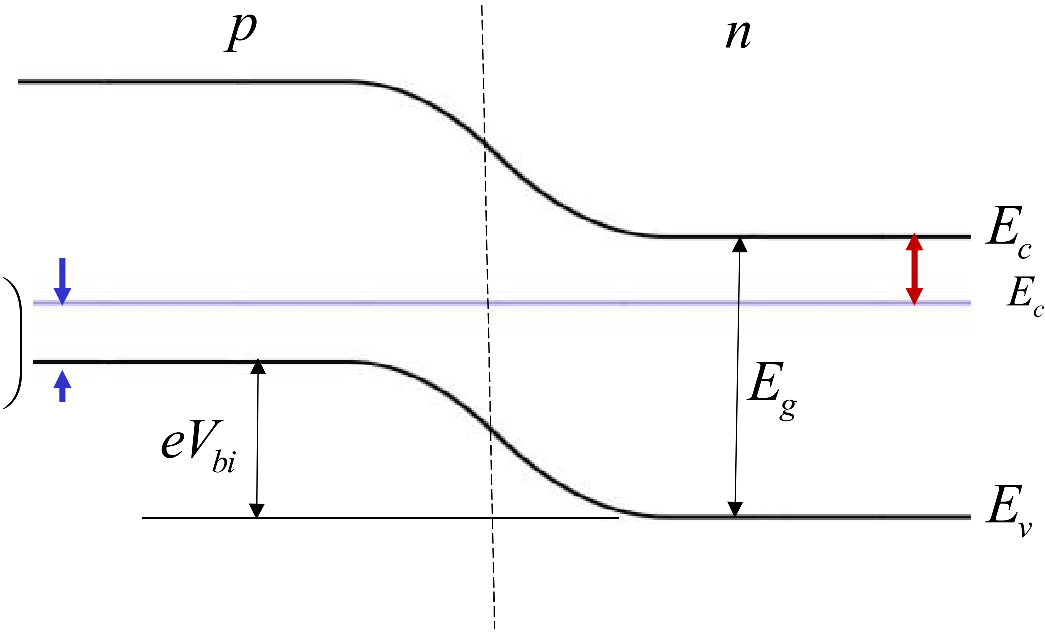
semiconductors in contact



Intrinsic semiconductor

Abrupt junction: the doping changes abruptly from p to n

Built-in voltage V_{bi}



$$E_F - E_{vp} = k_B T \ln\left(\frac{N_v}{N_A}\right)$$

$$E_{cn} - E_F = k_B T \ln\left(\frac{N_c}{N_D}\right)$$

$$eV_{bi} = E_g - \boxed{k_B T \ln\left(\frac{N_c}{N_{D,n} - N_{A,n}}\right)} - \boxed{k_B T \ln\left(\frac{N_v}{N_{A,p} - N_{D,p}}\right)}$$

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c N_v}{\left(N_{D,n} - N_{A,n}\right)\left(N_{A,p} - N_{D,p}\right)}\right)$$

V_{bi}

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_c N_v}{\left(N_{D,n} - N_{A,n}\right)\left(N_{A,p} - N_{D,p}\right)}\right)$$

$$n_i^2 = N_v N_c \exp\left(\frac{-E_g}{k_B T}\right) \qquad E_g = -k_B T \ln\left(\frac{n_i^2}{N_v N_c}\right)$$

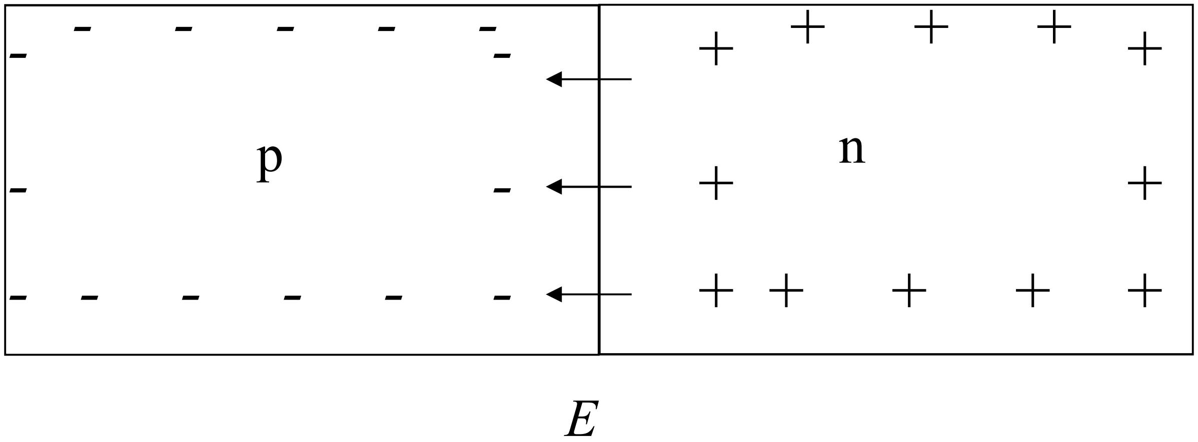
$$eV_{bi} = k_B T \ln\left(\frac{\left(N_{D,n} - N_{A,n}\right)\left(N_{A,p} - N_{D,p}\right)}{n_i^2}\right)$$

for $N_{D,n}$ - $N_{A,n} = N_D$ and $N_{A,p}$ - $N_{D,p} = N_A$

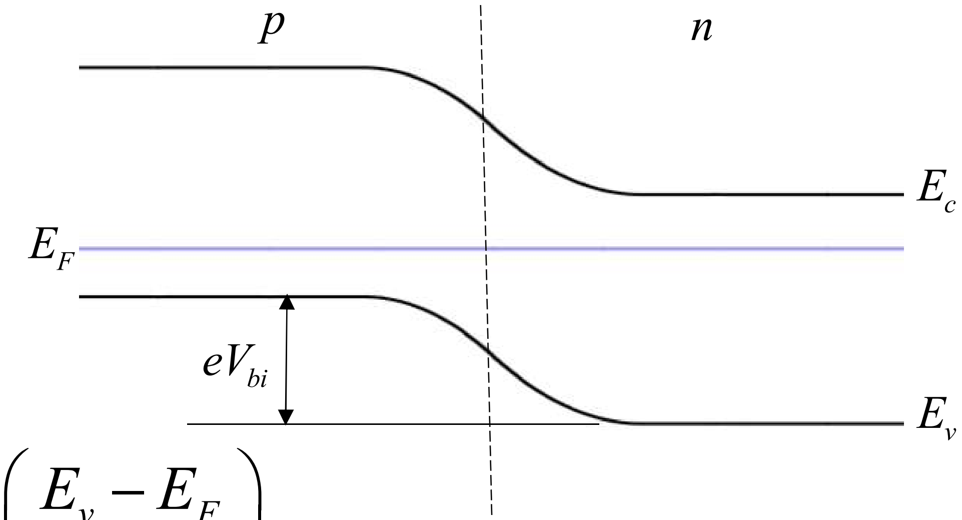
$$\boxed{eV_{bi} = k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right)}$$

V_{bi}

Can V_{bi} perform work?



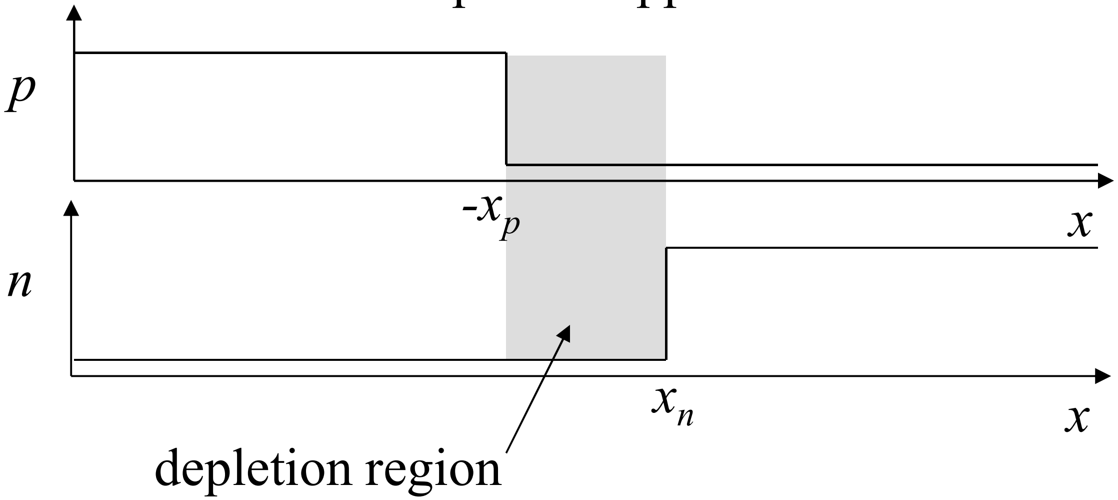
p and n profiles



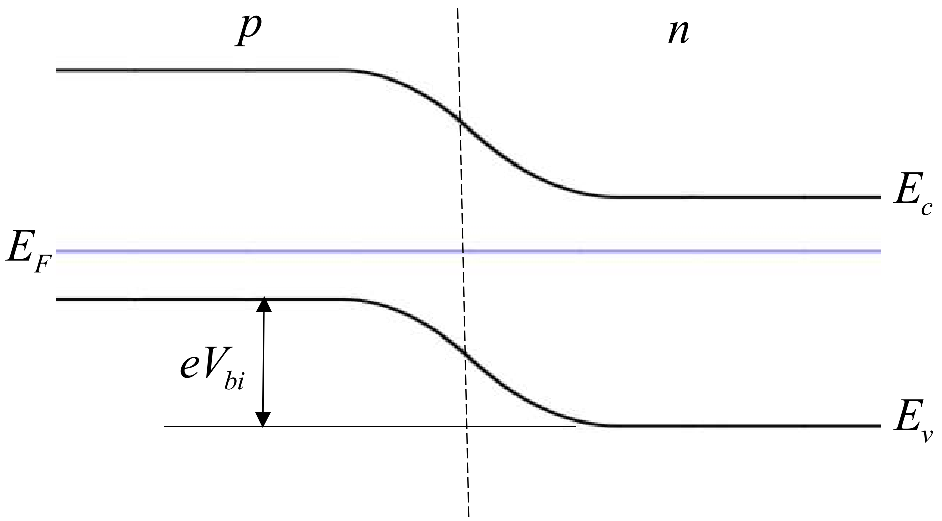
$$p = N_v \exp\left(\frac{E_v - E_F}{k_B T}\right)$$

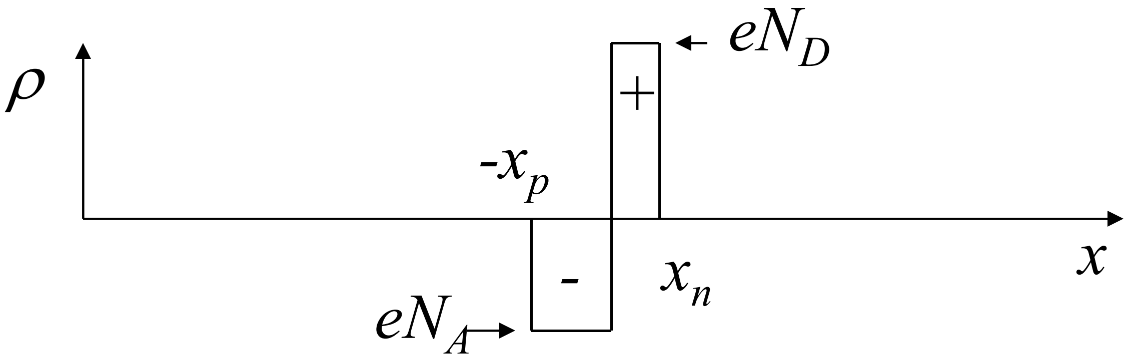
$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

depletion approximation



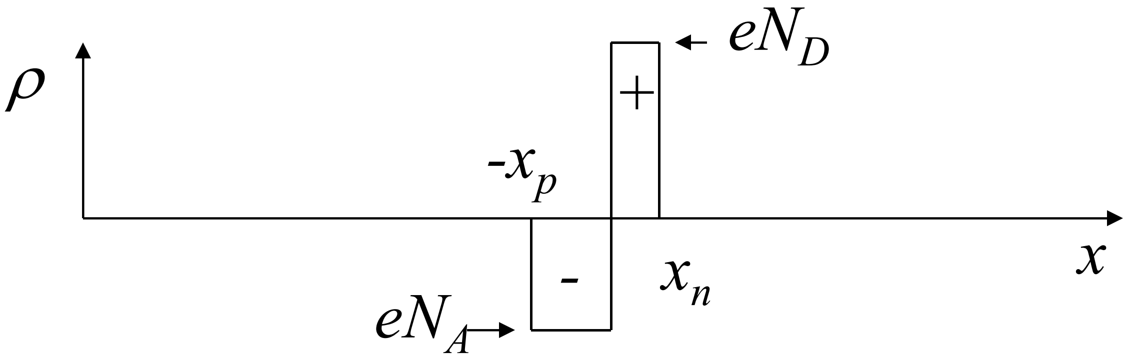
space charge



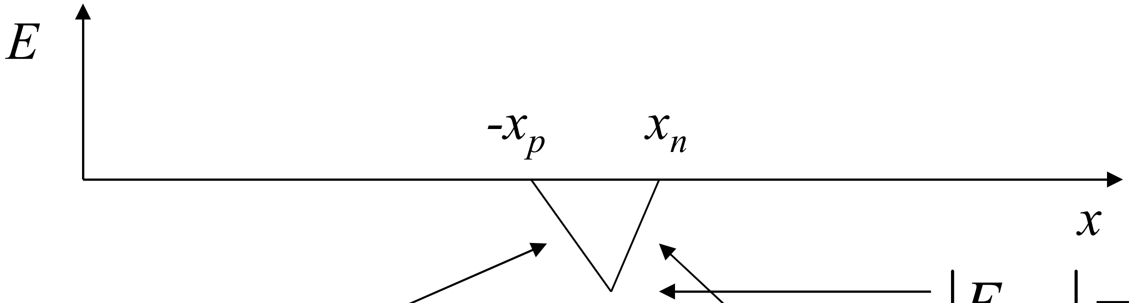


$$N_A x_p = N_D x_n$$

electric field



Gauss's law $\nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon}$ in 1-D is $\dfrac{dE}{dx} = \dfrac{\rho}{\varepsilon}$



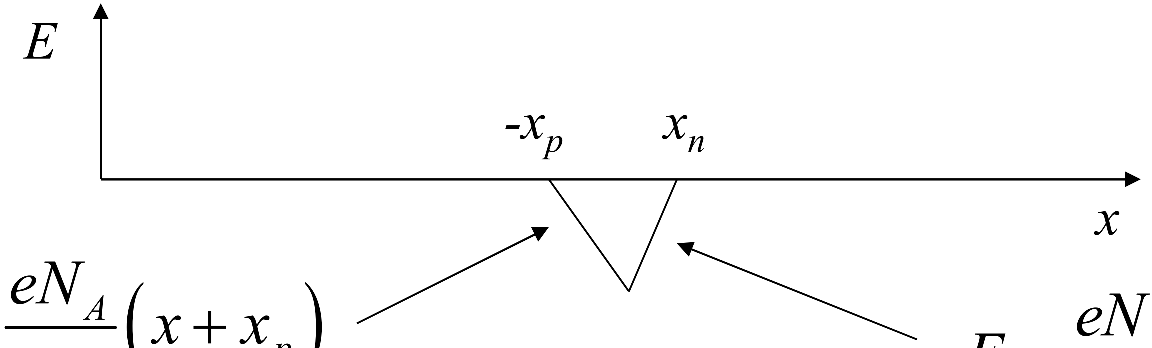
$$E = -\frac{eN_A}{\varepsilon}\left(x + x_p\right)$$

$$\left|E_{\max}\right| = \frac{eN_D x_n}{\varepsilon} = \frac{eN_A x_p}{\varepsilon}$$

$$E = \frac{eN_D}{\varepsilon}\left(x - x_n\right)$$

E pushes the holes towards p and the electrons towards n

potential



$$E = -\frac{eN_A}{\varepsilon}\left(x + x_p\right)$$

$$E = \frac{eN_D}{\varepsilon}\left(x - x_n\right)$$

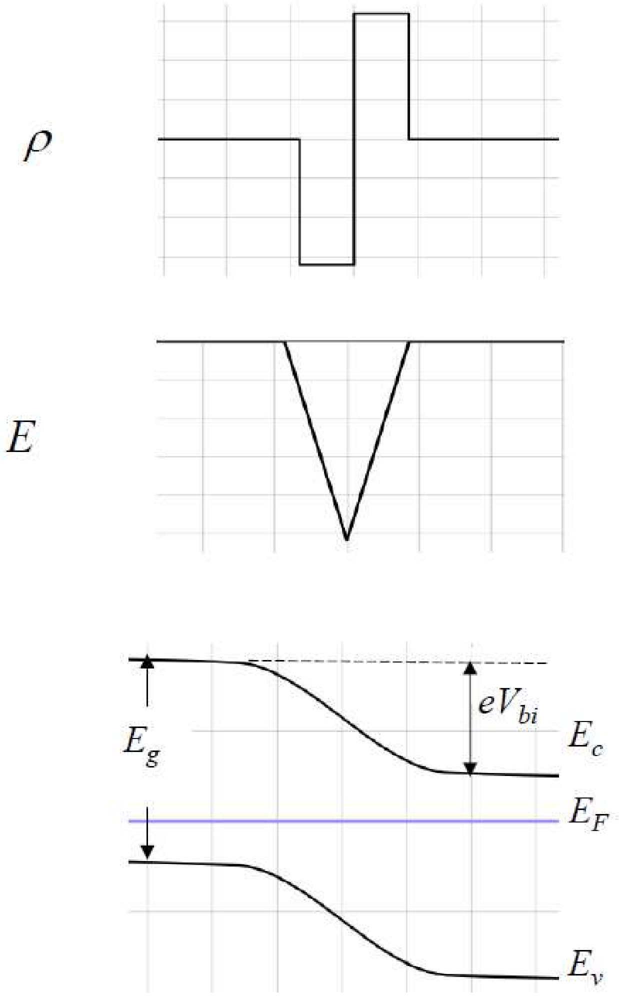
$$\frac{dV}{dx} = -E$$

$$V = \frac{eN_A}{\varepsilon}\left(\frac{x^2}{2} + xx_p\right) \qquad -x_p > x > 0$$

$$V = \frac{-eN_D}{\varepsilon}\left(\frac{x^2}{2} - xx_n\right) \qquad 0 > x > x_n$$

$$V(-x_p) = \frac{-eN_A}{2\varepsilon}x_p^2 \qquad V(0) = 0 \qquad V(x_n) = \frac{eN_D}{2\varepsilon}x_n^2$$

abrupt pn junction

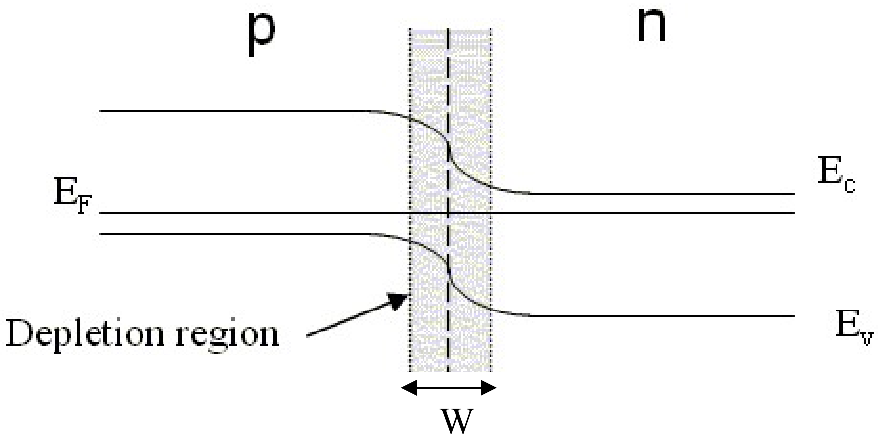


$$V_{bi} = \frac{k_B T}{e} \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$$= \frac{eN_A x_p^2}{2\varepsilon} + \frac{eN_D x_n^2}{2\varepsilon}$$

http://lampx.tugraz.at/~hadley/psd/L6/abrupt.html

Depletion width



$$V_{bi} = \frac{k_B T}{e} \ln\left(\frac{N_D N_A}{n_i^2}\right) = \frac{eN_A x_p^2}{2\varepsilon} + \frac{eN_D x_n^2}{2\varepsilon}$$

$$N_A x_p = N_D x_n = N_D\left(W - x_p\right) = N_A\left(W - x_n\right)$$

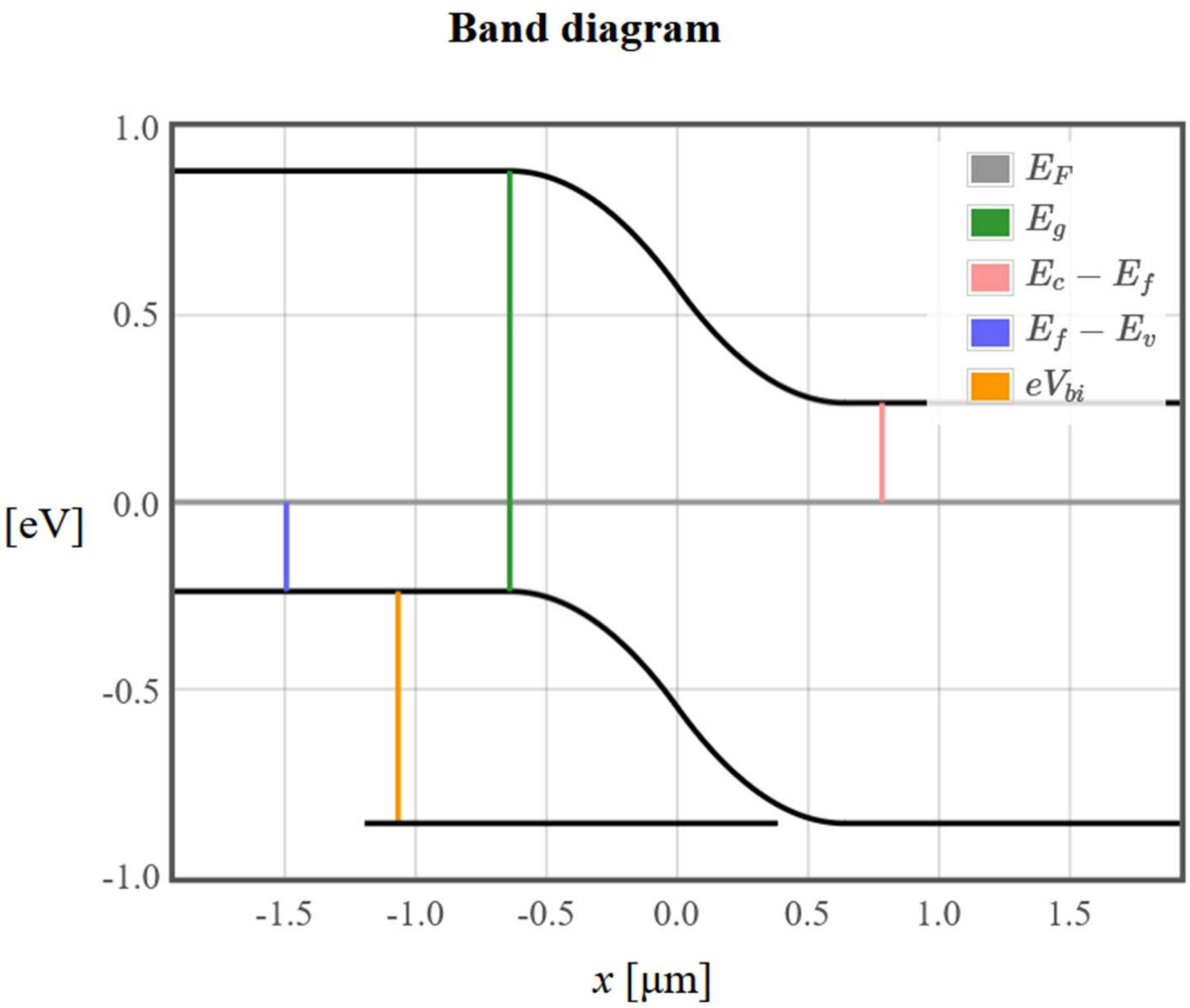
$$x_p = \frac{N_D W}{N_A + N_D} \qquad x_n = \frac{N_A W}{N_A + N_D}$$

$$V_{bi} = \frac{e}{2\varepsilon}\frac{N_D N_A}{N_D + N_A} W^2$$

$$W = \sqrt{\frac{2\varepsilon\left(N_D + N_A\right)V_{bi}}{eN_D N_A}}$$

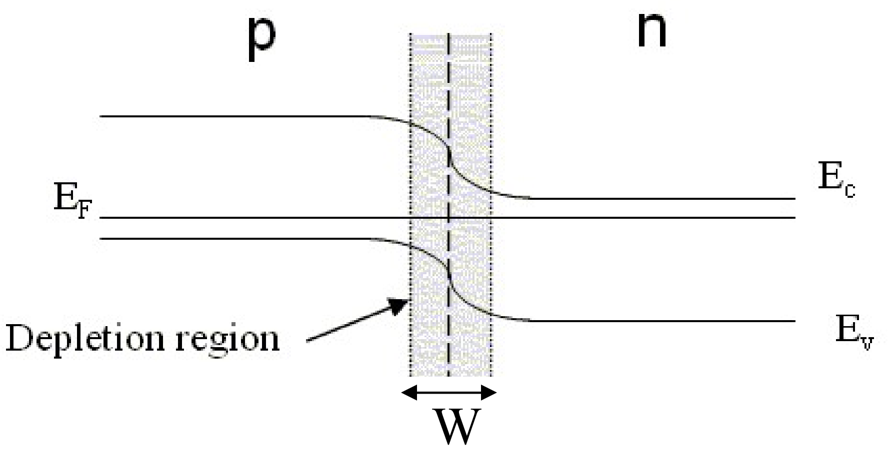
light doping => wide depletion width

Depletion width



http://lampz.tugraz.at/~hadley/psd/L6/Vbi.html

Depletion width



$V_{bi} \sim 1\text{V}$

$W \sim 10$ nm - 10 µm

$E_{max} \sim 10^4$ V/cm

The electric field pushes the electrons towards the n-region and the holes towards the p-region.

Diffusion sends electrons towards the p-region and holes towards the n-region.

Abrupt pn junctions in the depletion approximation

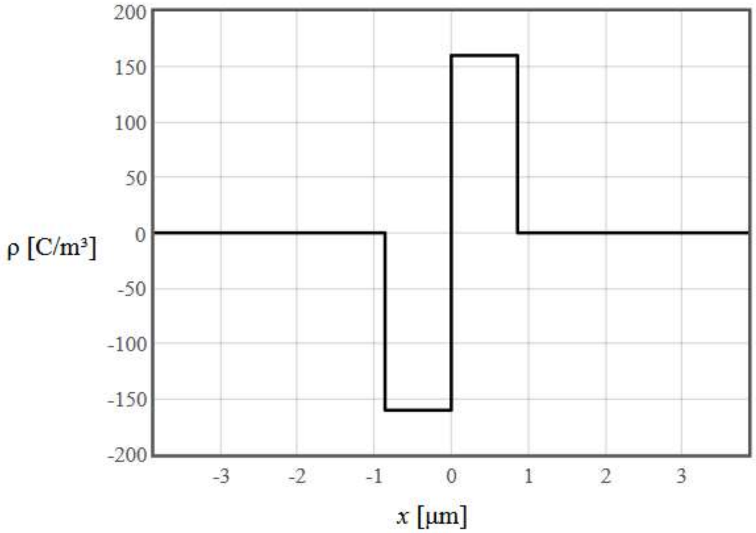
In an abrupt pn junction, the doping changes abruptly from p to n. It is common to solve for the band bending, the local electric field, the carrier concentration profiles, and the local conductivity in the depletion approximation. In this approximation it is assumed that there is a depletion width W around the transition from p to n where the charge carrier densities are negligible. Outside the depletion width the charge carrier densities are equal to the doping densities so that the semiconductor is electrically neutral outside the depletion width. Using this approximation it is possible to calculate the important properties of the pn junction.

$N_A =$ 1E15 1/cm^3
$N_D =$ 1E15 1/cm^3
$E_g =$ 1.166-4.73E-4*T*T/(T+636) eV
$N_v(300) =$ 9.84E18 1/cm^3
$N_c(300) =$ 2.78E19 1/cm^3
$\epsilon_r =$ 12
$T =$ 300 K
$\mu_p =$ 480 cm^2/V s
$\mu_n =$ 1350 cm^2/V s
$\tau_p =$ 1E-10 s
$\tau_n =$ 1E-10 s
$V =$ -0.5 V
V
Submit

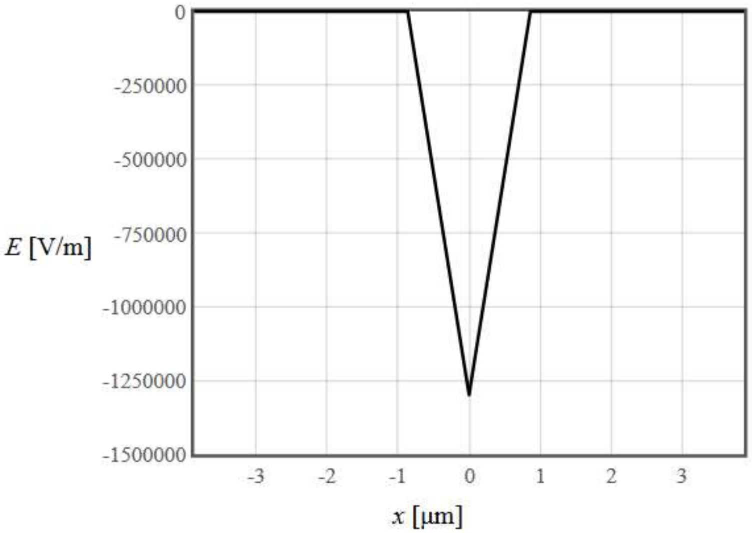
$E_g = 1.12$ eV $W = 1.72$ µm $x_p =$ -0.861 µm $x_n = 0.861$ µm $V_{bi} = 0.618$ V $C_j = 6.17$ nF/cm^2

$D_p = 12.4$ cm^2/s $D_n = 34.9$ cm^2/s $L_p = 0.352$ µm $L_n = 0.591$ µm

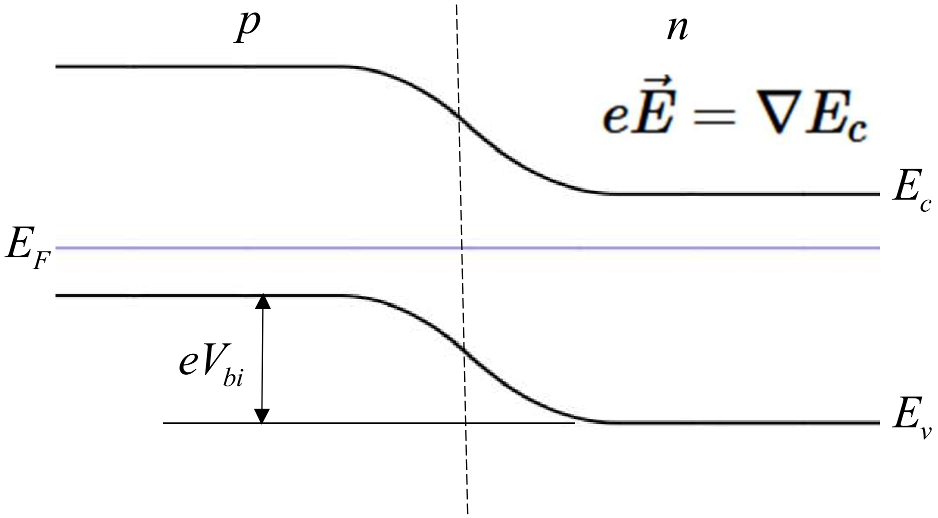
Charge density



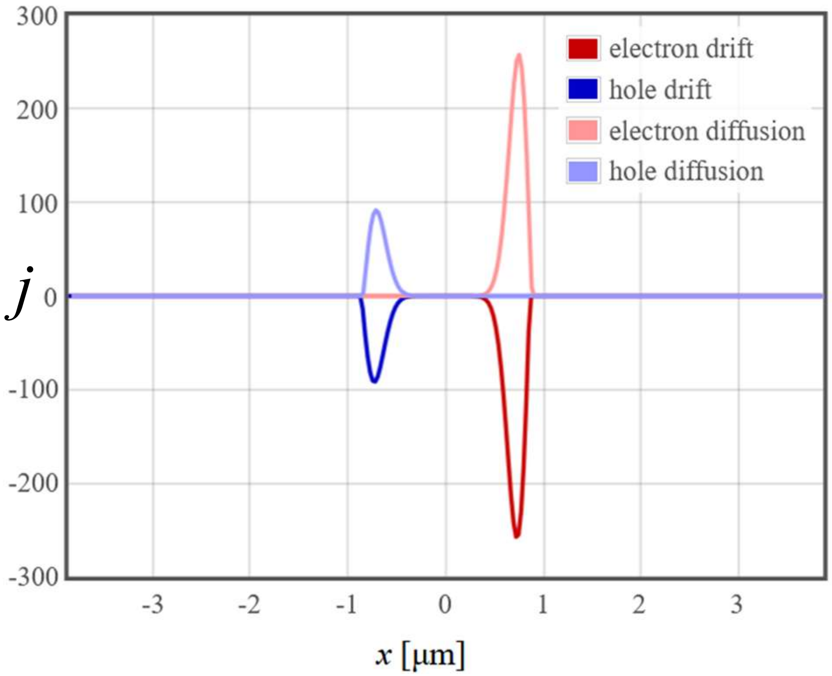
Electric field



Drift and Diffusion



$$e\vec{E} = \nabla E_c$$



$$\vec{j}_n = en\mu_n\vec{E} + eD_n\nabla n$$

$$n = N_c \exp\left(\frac{E_F - E_c}{k_B T}\right)$$

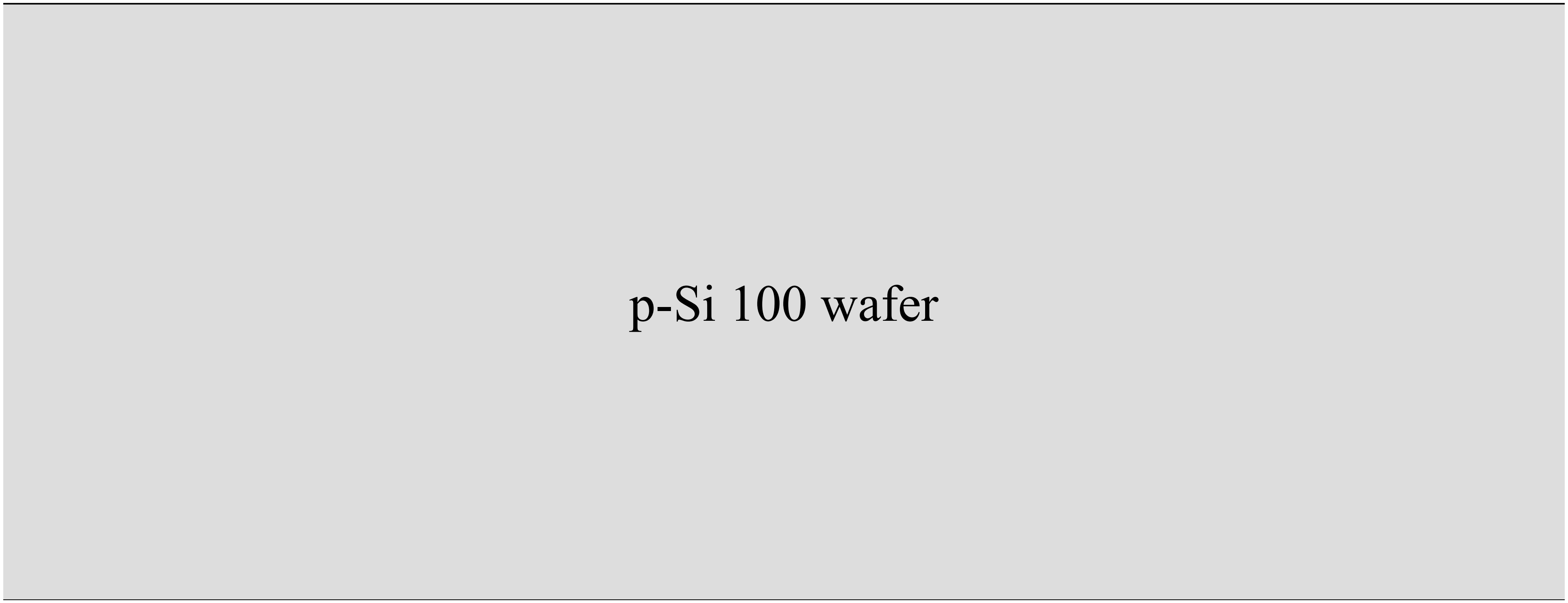
$$\nabla n = -\frac{\nabla E_c}{k_B T} N_c \exp\left(\frac{E_F - E_c}{k_B T}\right) = -\frac{\nabla E_c}{k_B T} n$$

$$\vec{j}_n = n\nabla E_c \left(\mu_n - \frac{eD_n}{k_B T}\right)$$

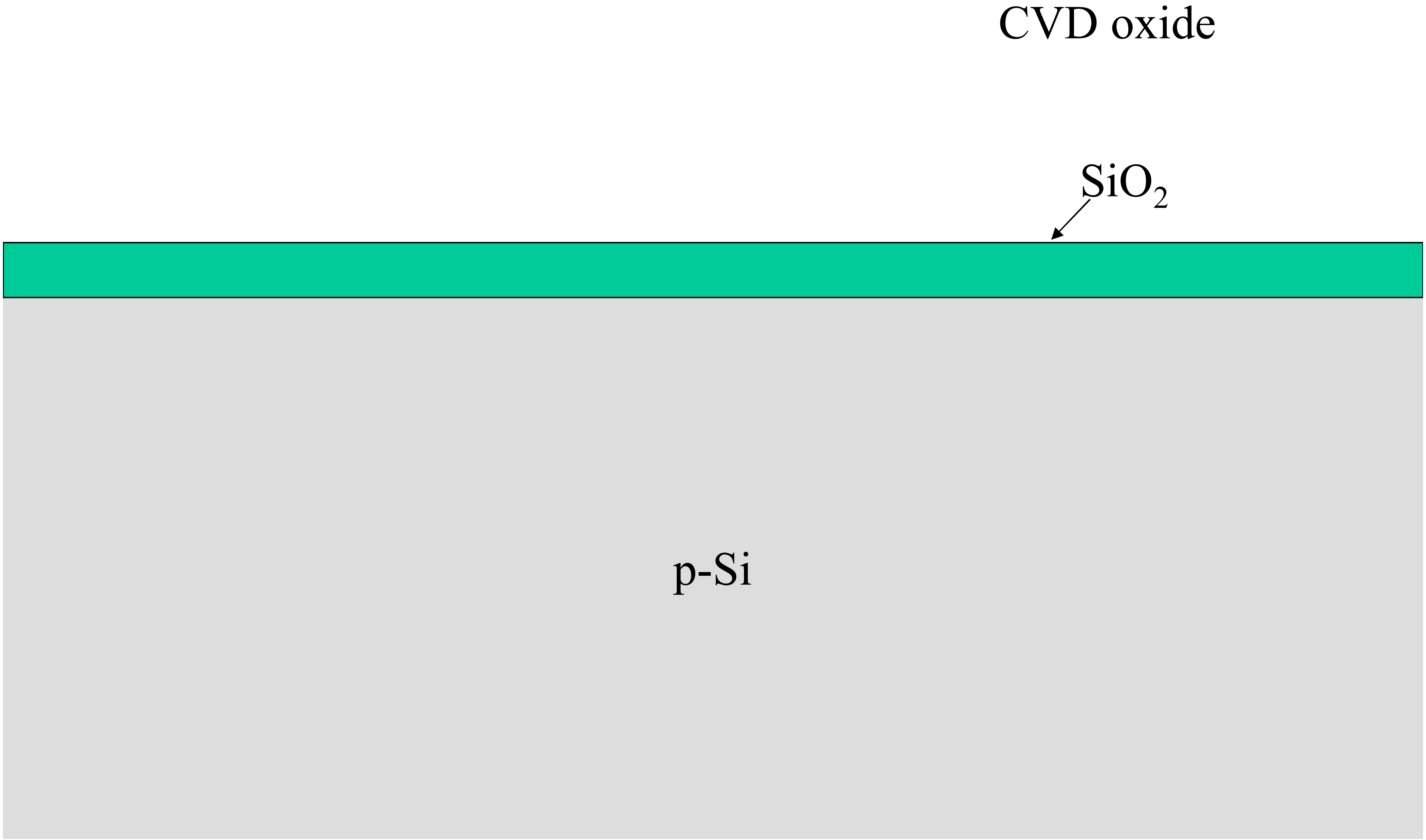
Einstein relation

If the E_F is constant, $j = 0$.

diode fabrication



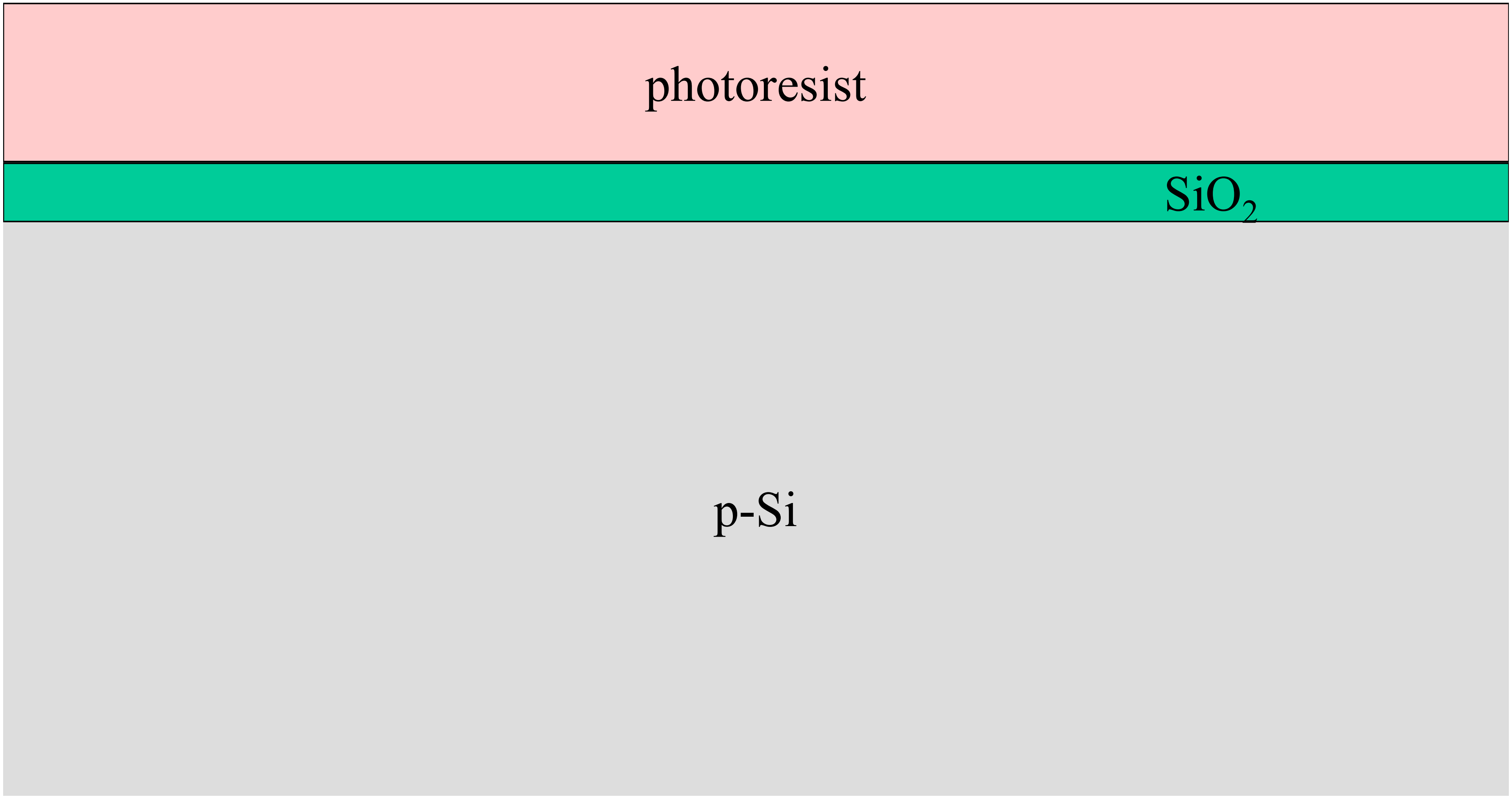
p-Si 100 wafer

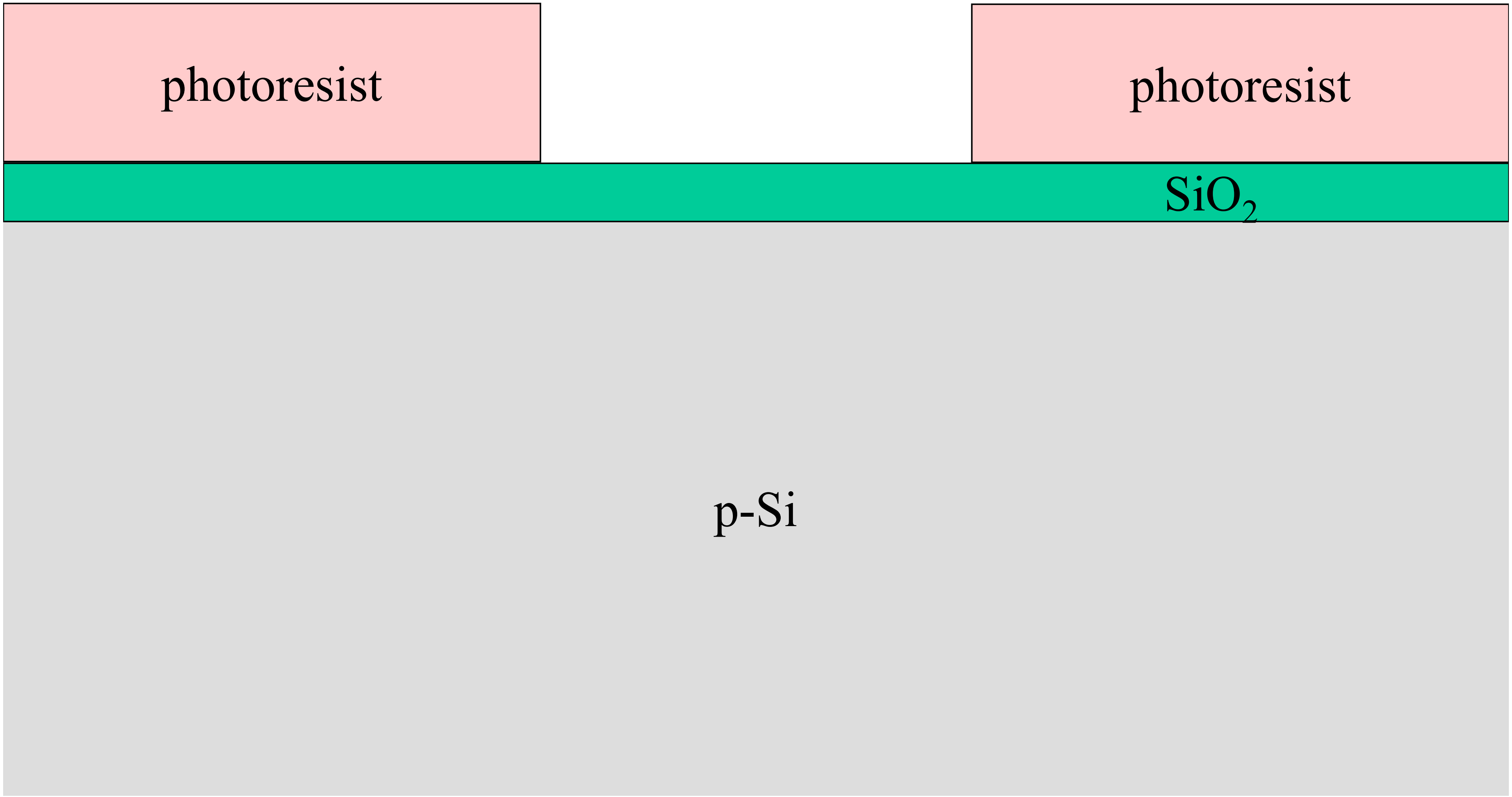


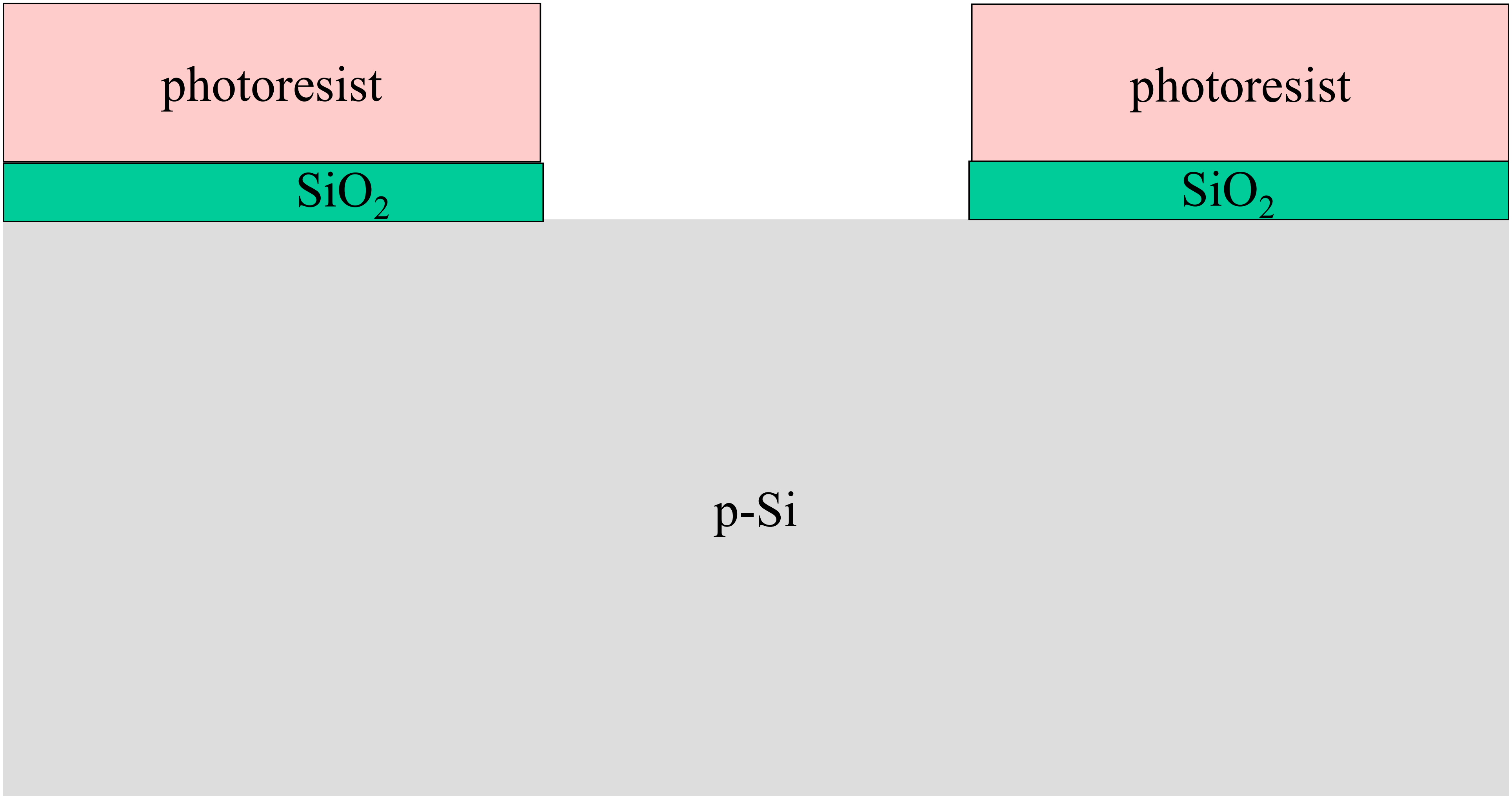
CVD oxide

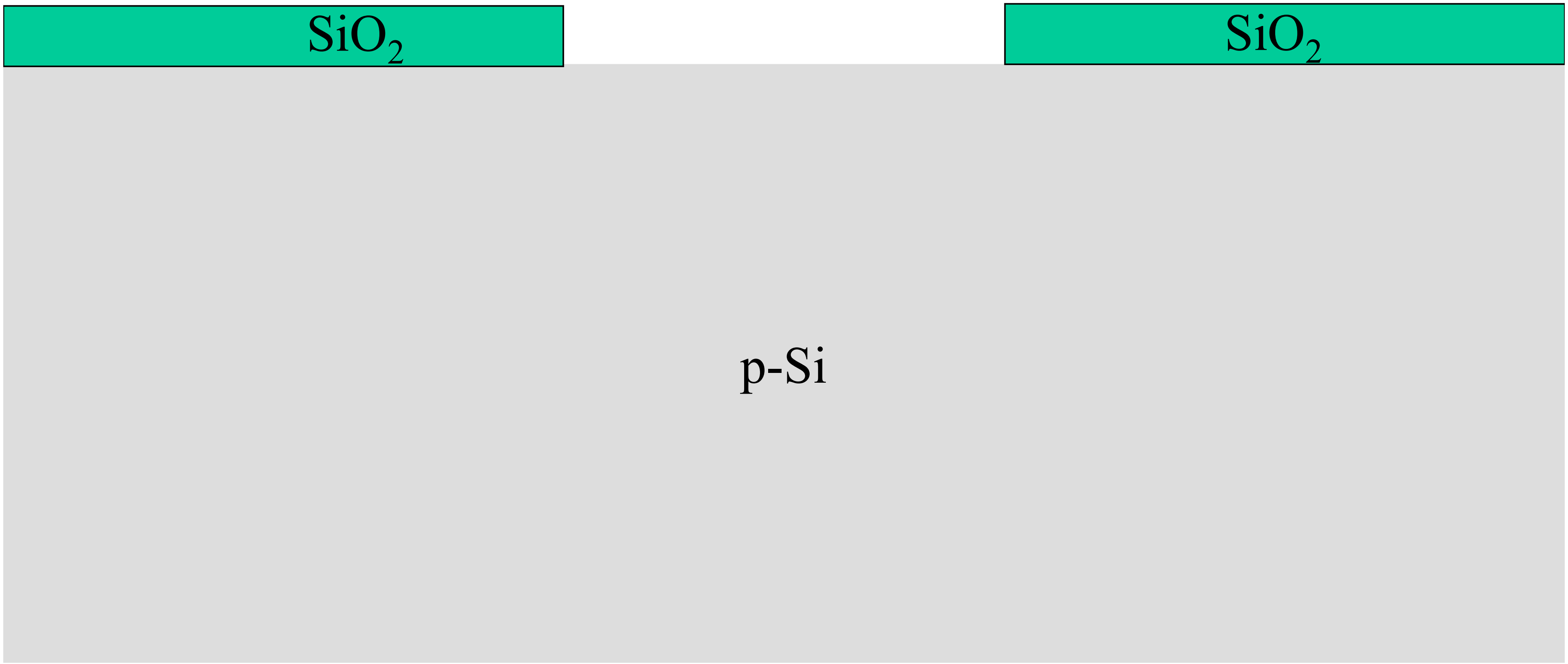
SiO_2

p-Si

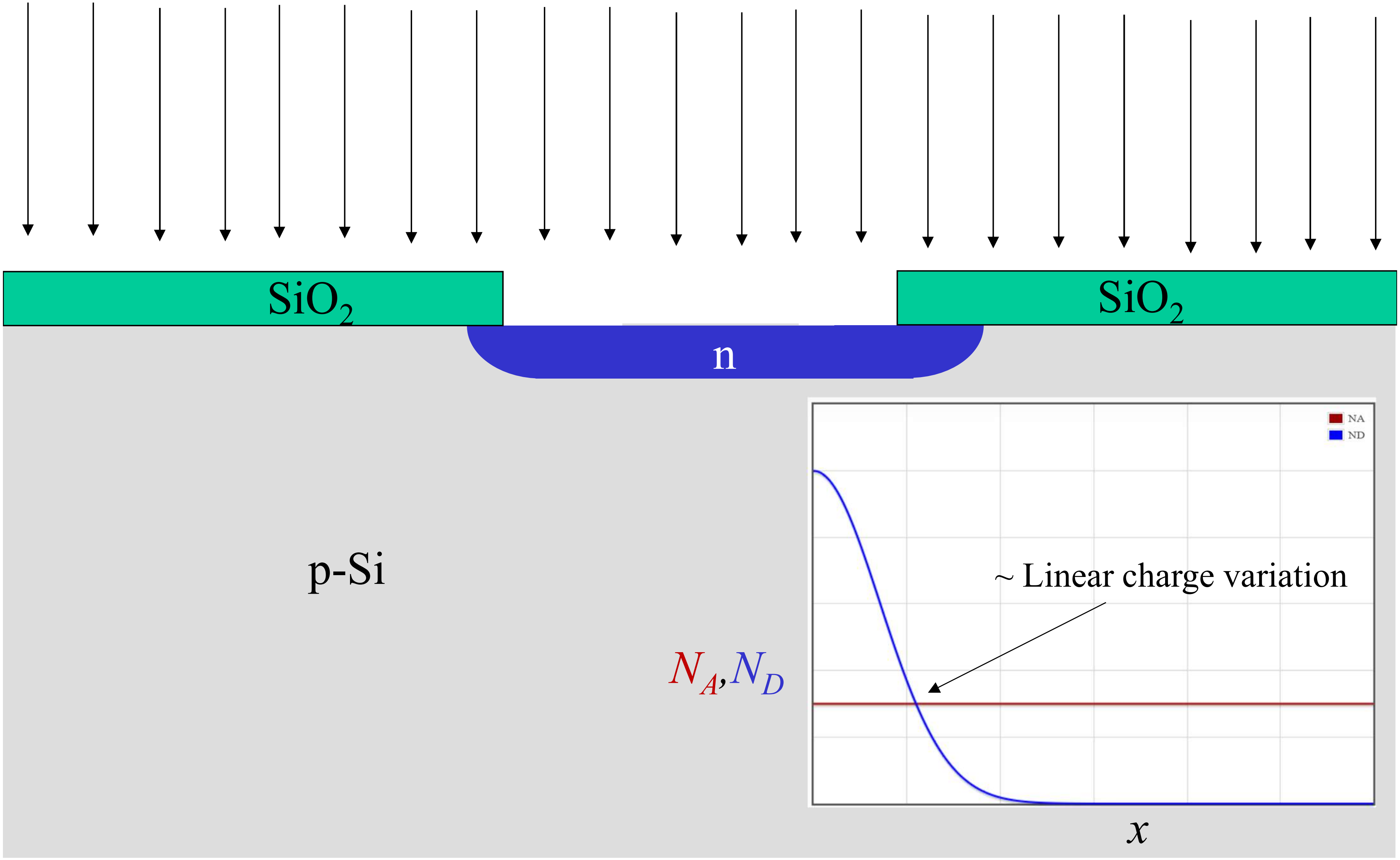

photoresist
SiO_2
p-Si


photoresist
photoresist
SiO_2
p-Si

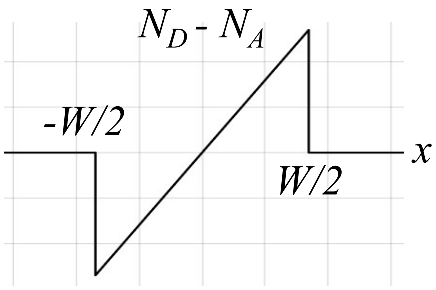

photoresist
photoresist
SiO_2
SiO_2
p-Si


SiO_2
SiO_2
p-Si

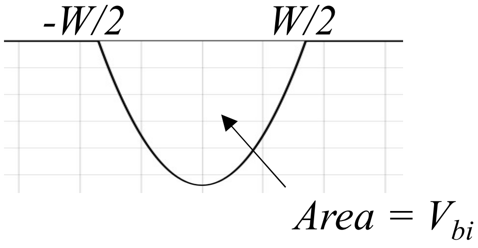
Diffuse or Implant



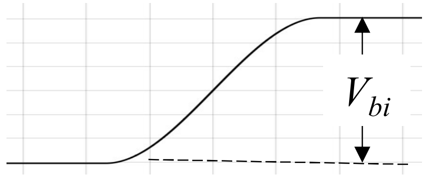
linearly graded junction



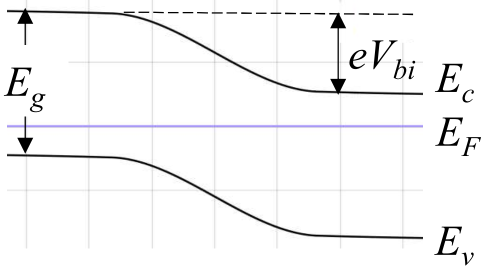
$$\rho = e\left(N_D(x) - N_A(x)\right) = eax$$



$$E = \int \frac{\rho}{\varepsilon} dx = \frac{-ea}{2\varepsilon}\left(\left(\frac{W}{2}\right)^2 - x^2\right) \qquad E_{\max} = \frac{-eaW^2}{8\varepsilon}$$



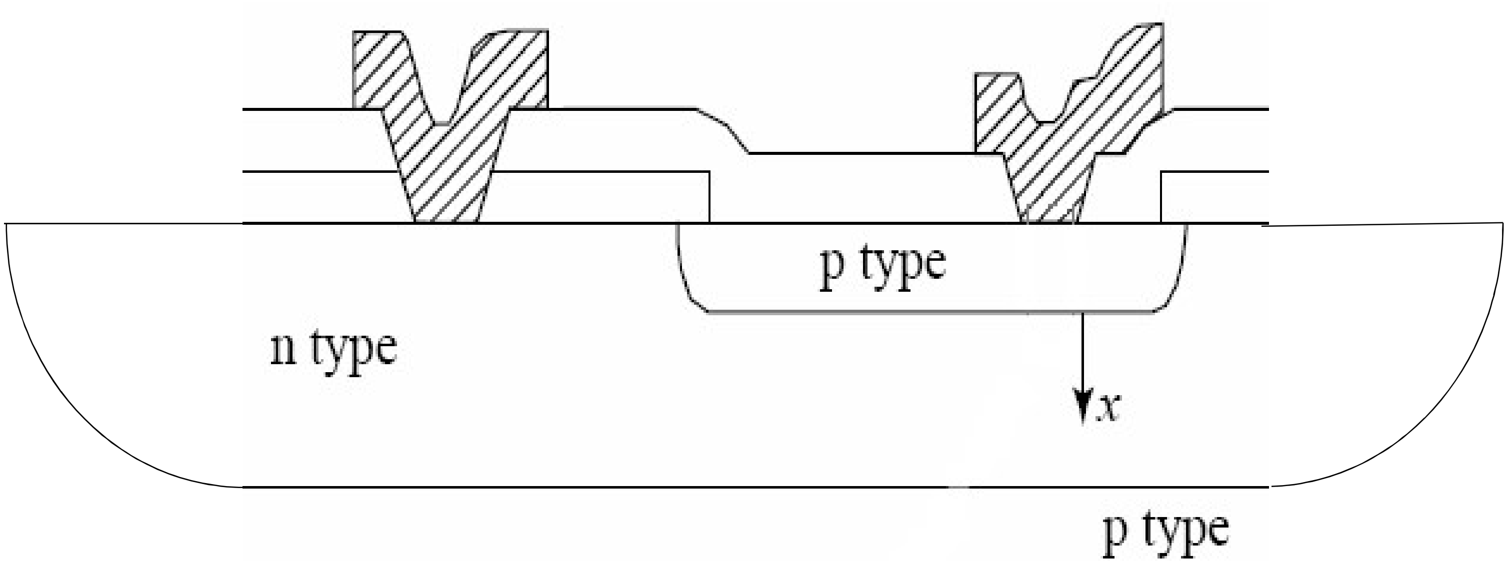
$$V = \int E dx = \frac{ea}{2\varepsilon}\left(\left(\frac{W}{2}\right)^2 x - \frac{x^3}{3}\right)$$



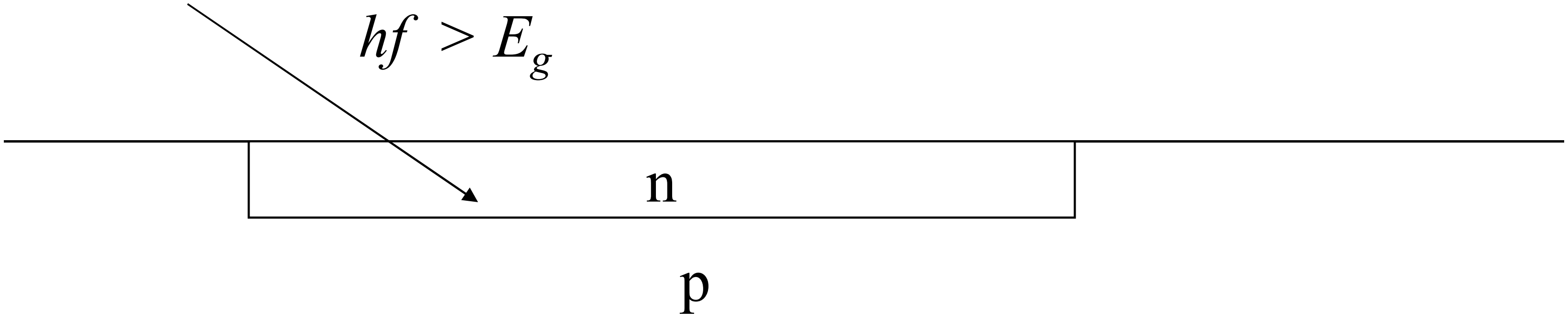
$$V_{bi} = \frac{eaW^3}{12\varepsilon}$$

http://lampx.tugraz.at/~hadley/psd/L6/linearly_graded.html

Isolation

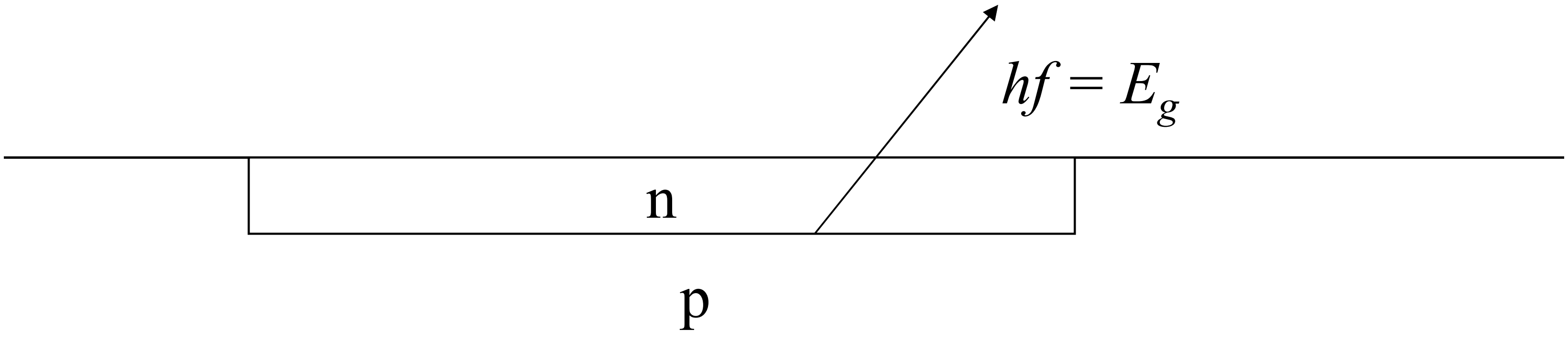


Solar cell



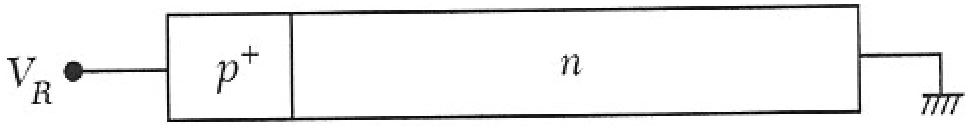
Light creates an electron-hole pair in the depletion region. The electric field sweeps the electrons towards the n-region and the holes towards the p-region.

Light emitting diode



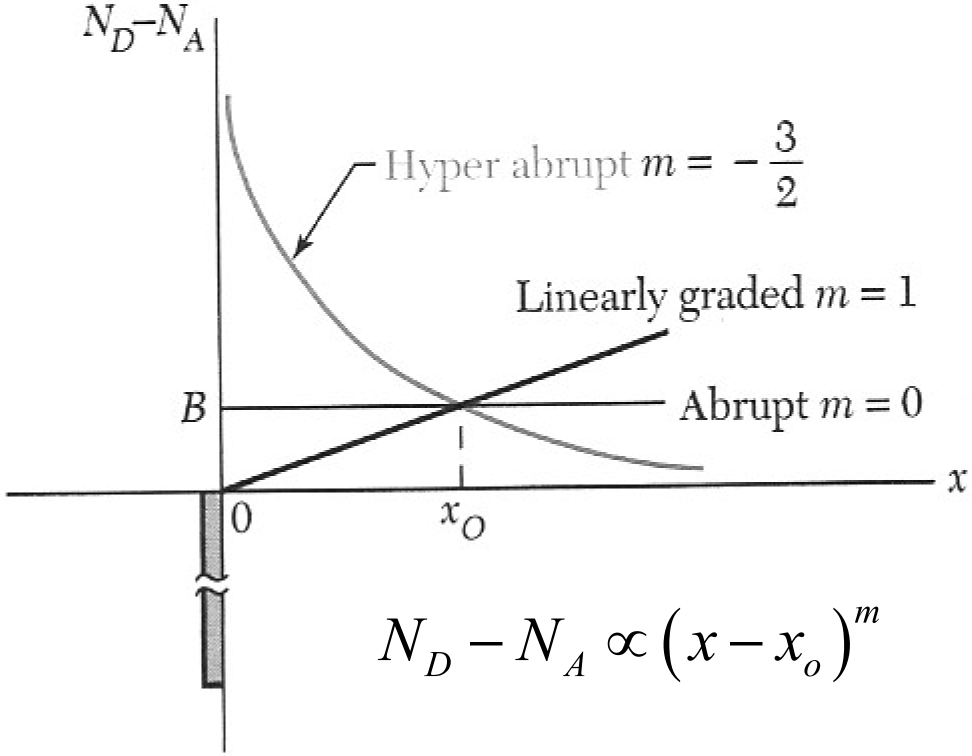
Electrons and holes are injected into the depletion region by forward biasing the junction. The electrons fall in the holes. For direct bandgap semiconductors, photons are emitted. For indirect bandgap semiconductors, phonons are emitted.

Varactor



$$C_j \propto \left(V_{bi} + V_R\right)^{-n}$$

abrupt: $n = 1/2$
linearly graded: $n = 1/3$



$$N_D - N_A \propto \left(x - x_o\right)^m$$

$n = 1/(m+2)$