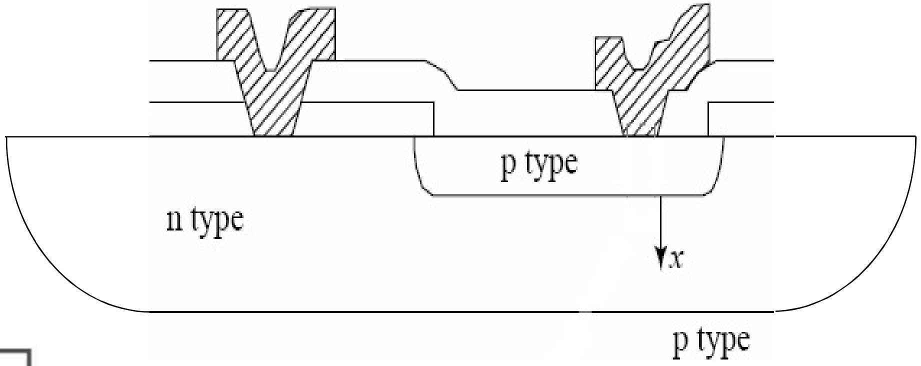
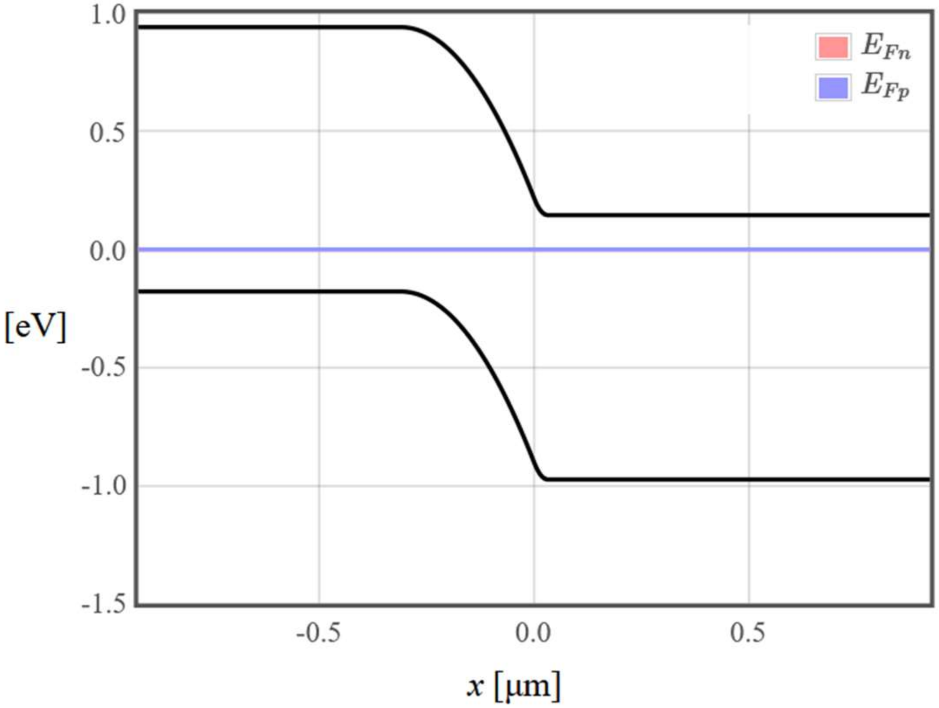
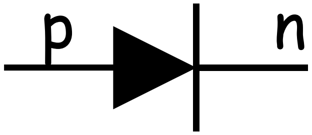
pn - Junctions

pn junctions





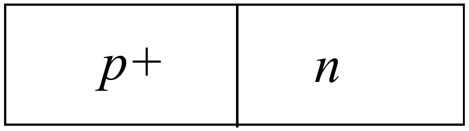


Capacitance-voltage characteristics

specific capacitance $$C_j = \frac{\varepsilon}{W} \quad \text{F m}^{-2}$$

abrupt junction: $$W = \frac{\varepsilon}{C_j} = \sqrt{\frac{2\varepsilon\left(N_D + N_A\right)\left(V_{bi} - V\right)}{eN_DN_A}}$$

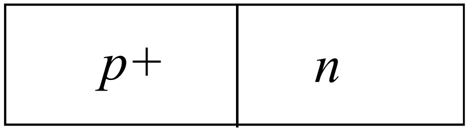
a one sided abrupt junction in reverse bias:



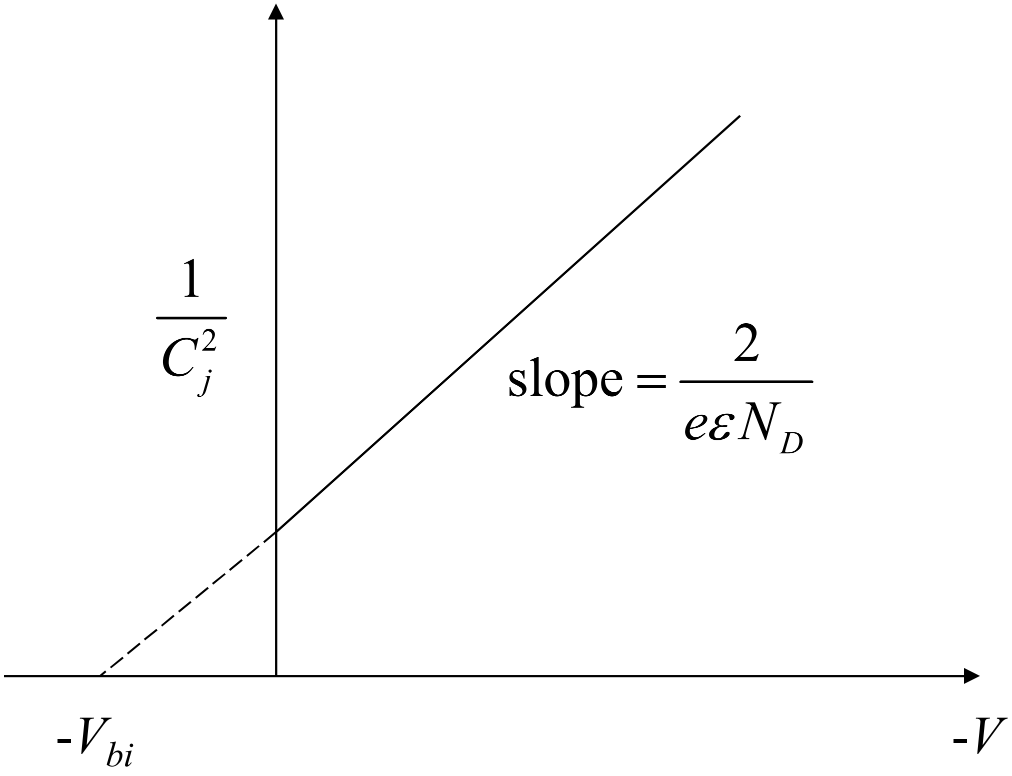
$$\frac{1}{C_j^2} = \frac{2\left(V_{bi} - V\right)}{e\varepsilon N_D}$$

Capacitance-voltage characteristics

a one sided abrupt junction in reverse bias:

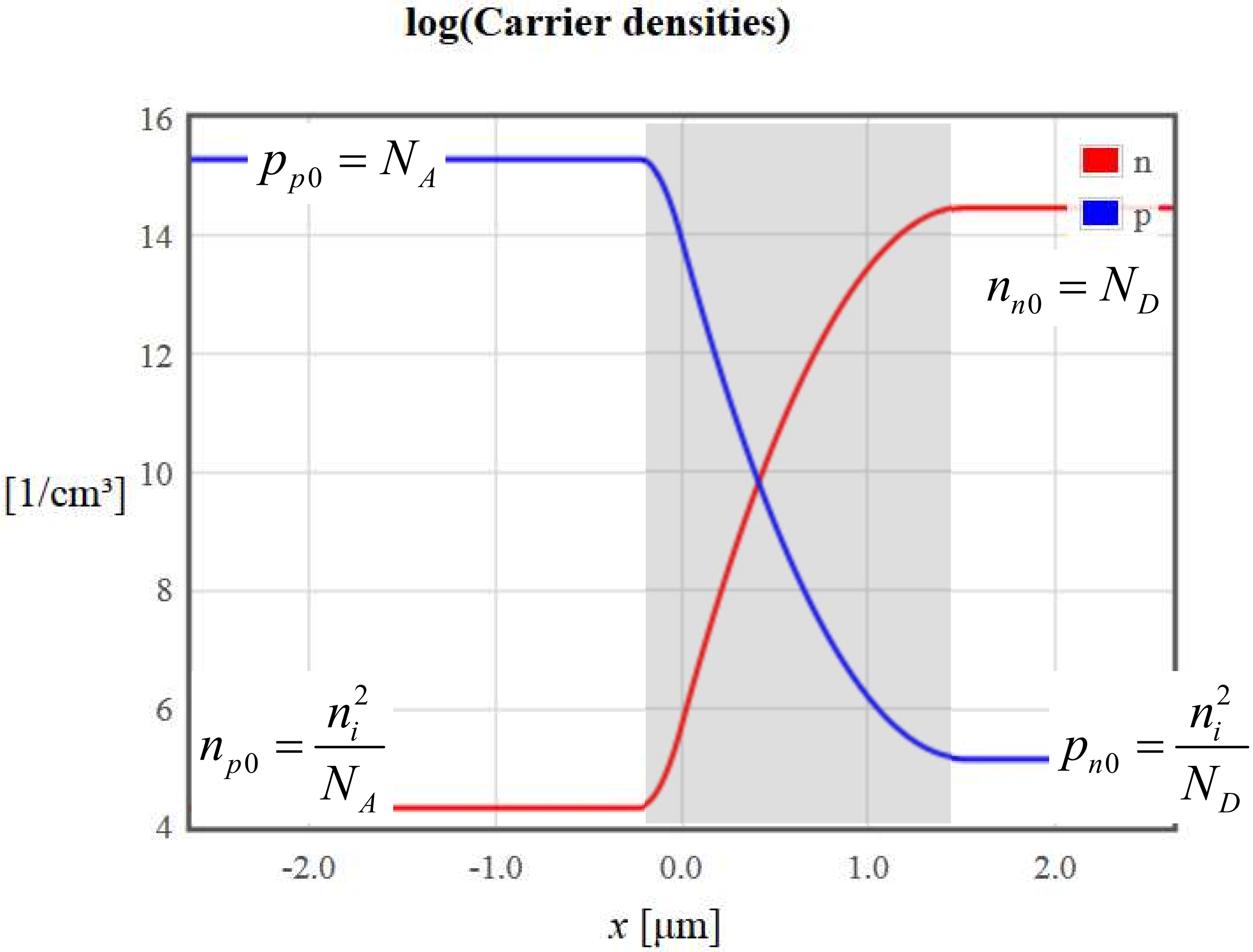


$$\frac{1}{C_j^2} = \frac{2\left(V_{bi} - V\right)}{e\varepsilon N_D}$$



slope gives impurity concentration and the intercept gives V_{bi}

Equilibrium concentrations, V = 0



$p_{p0} = N_A$

$n_{n0} = N_D$

$n_{p0} = \frac{n_i^2}{N_A}$

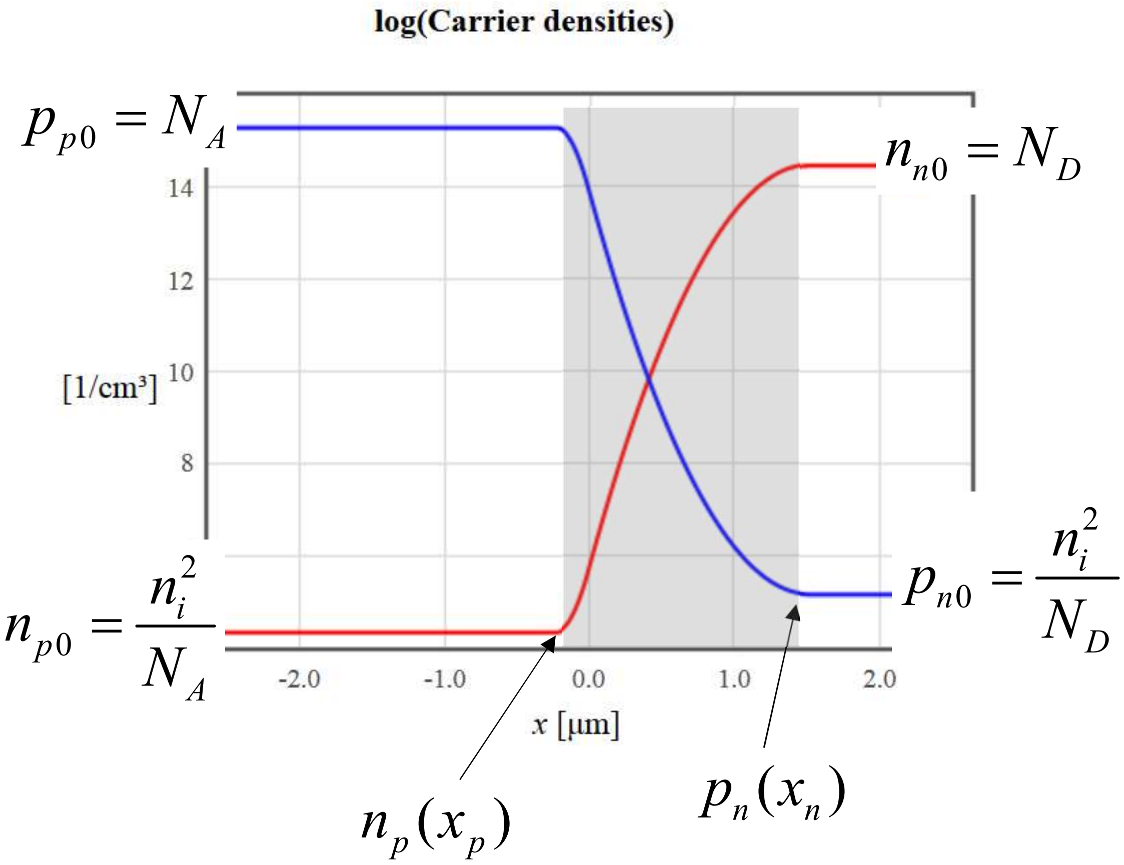
$p_{n0} = \frac{n_i^2}{N_D}$

$$n_{p0} p_{p0} = n_{n0} p_{n0} = n_i^2$$

Bias voltage, V = 0

$$eV_{bi} = k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right) = k_B T \ln\left(\frac{N_D}{n_{p0}}\right) = k_B T \ln\left(\frac{N_A}{p_{n0}}\right)$$

$$n_{p0} p_{p0} = n_{n0} p_{n0} = n_i^2$$



$$p_{p0} = N_A$$

$$n_{n0} = N_D$$

$$n_{p0} = \frac{n_i^2}{N_A}$$

$$p_{n0} = \frac{n_i^2}{N_D}$$

$$n_p(x_p)$$

$$p_n(x_n)$$

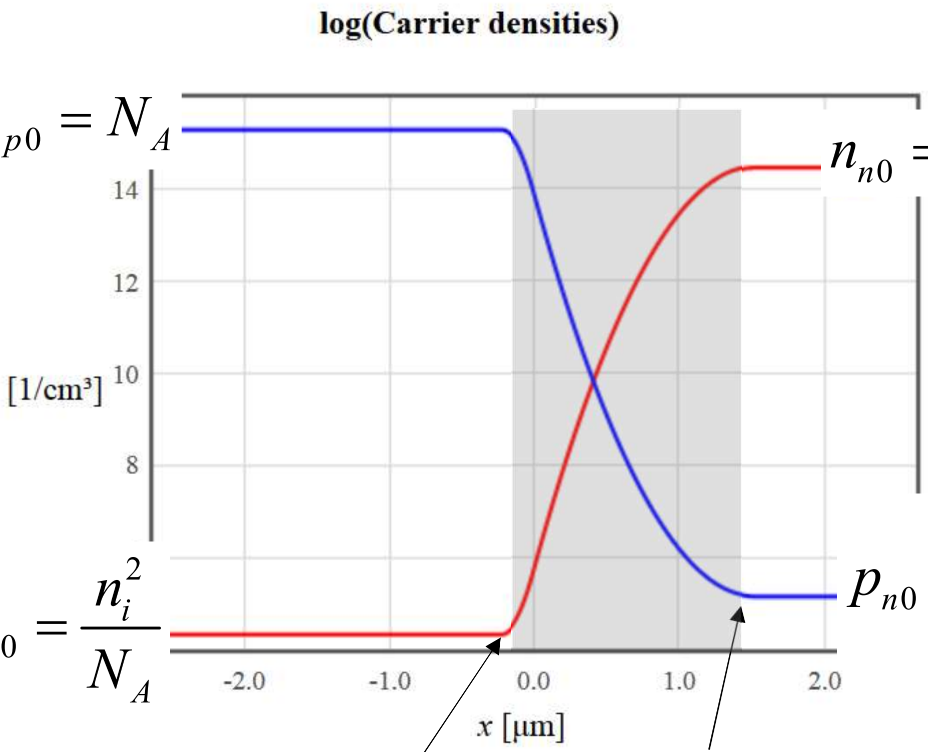
V = 0

$$n_{p0} = N_D \exp\left(\frac{-eV_{bi}}{k_B T}\right)$$

$$p_{n0} = N_A \exp\left(\frac{-eV_{bi}}{k_B T}\right)$$

Bias voltage, $V \neq 0$

$$eV_{bi} = k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right) = k_B T \ln\left(\frac{N_D}{n_{p0}}\right) = k_B T \ln\left(\frac{N_A}{p_{n0}}\right)$$



$$p_{p0} = N_A$$

$$n_{n0} = N_D$$

$$p_{n0} = \frac{n_i^2}{N_D}$$

$$n_{p0} = \frac{n_i^2}{N_A}$$

$$n_p(x_p)$$

$$p_n(x_n)$$

$$n_{p0} p_{p0} = n_{n0} p_{n0} = n_i^2$$

$V = 0$

$$n_{p0} = N_D \exp\left(\frac{-eV_{bi}}{k_B T}\right)$$

$$p_{n0} = N_A \exp\left(\frac{-eV_{bi}}{k_B T}\right)$$

$V \neq 0$

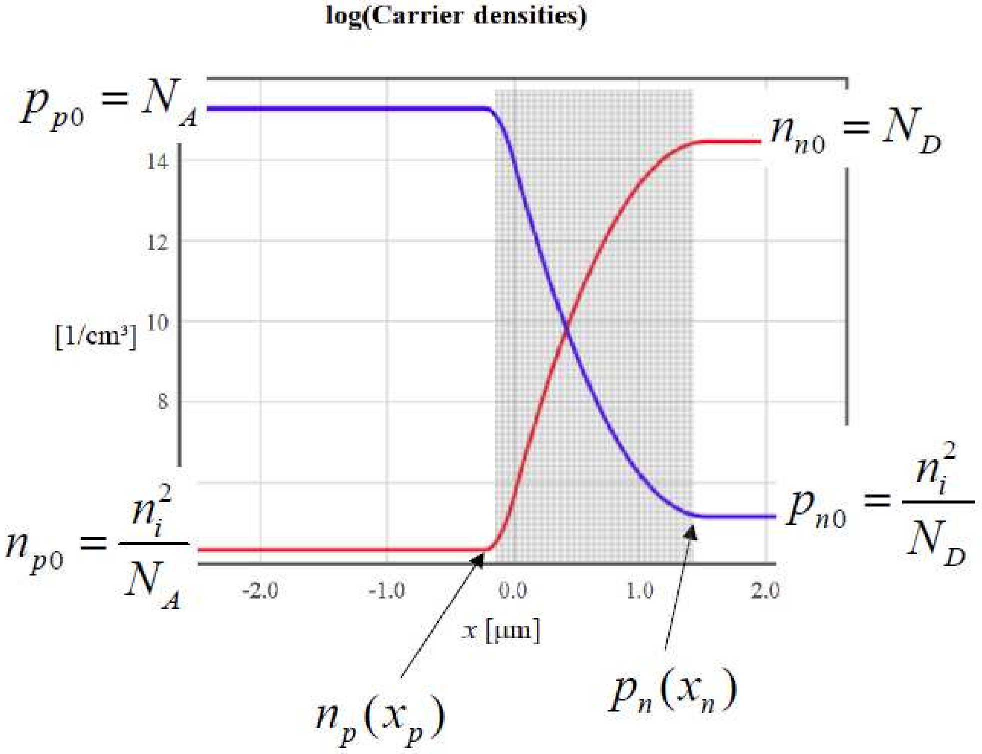
$$n_p(x_p) = N_D \exp\left(\frac{-e(V_{bi} - V)}{k_B T}\right)$$

$$p_n(x_n) = N_A \exp\left(\frac{-e(V_{bi} - V)}{k_B T}\right)$$

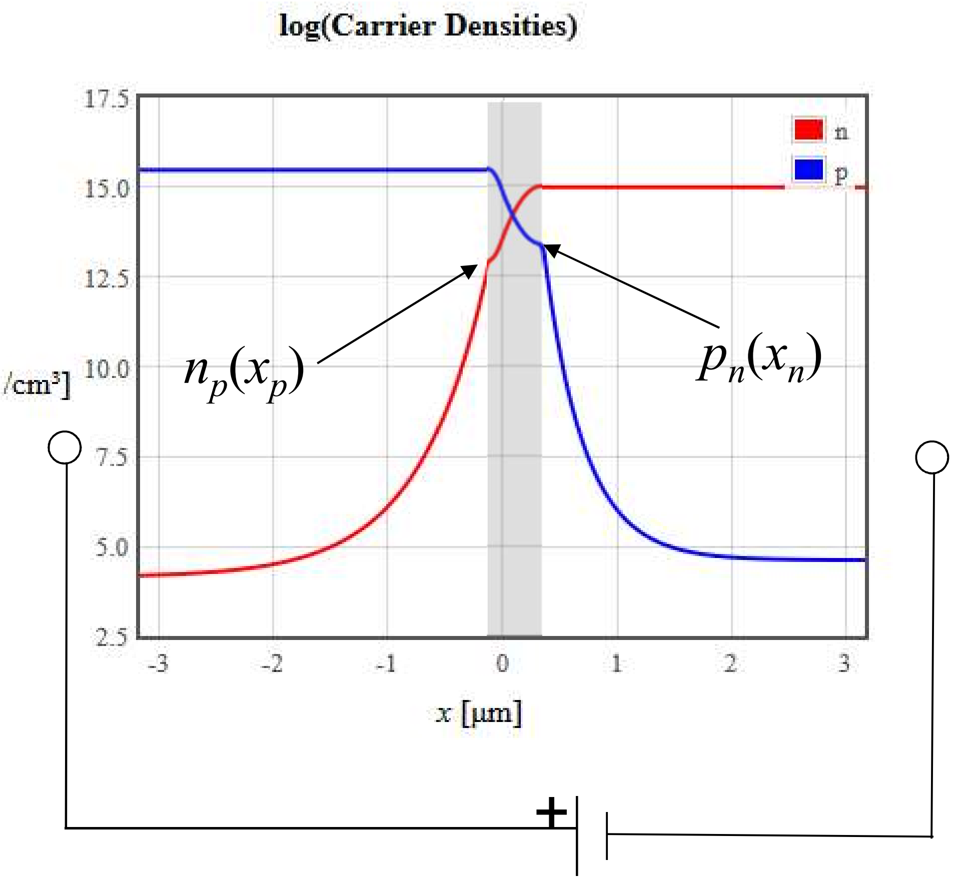
Bias voltage, $V \neq 0$

$$n_p(x_p) = N_D \exp\left(\frac{-e\left(V_{bi} - V\right)}{k_B T}\right) = n_{p0} \exp\left(\frac{eV}{k_B T}\right)$$

$$p_n(x_n) = N_A \exp\left(\frac{-e\left(V_{bi} - V\right)}{k_B T}\right) = p_{n0} \exp\left(\frac{eV}{k_B T}\right)$$



Forward bias, V > 0



Electrons and holes are driven towards the junction.
The depletion region becomes narrower

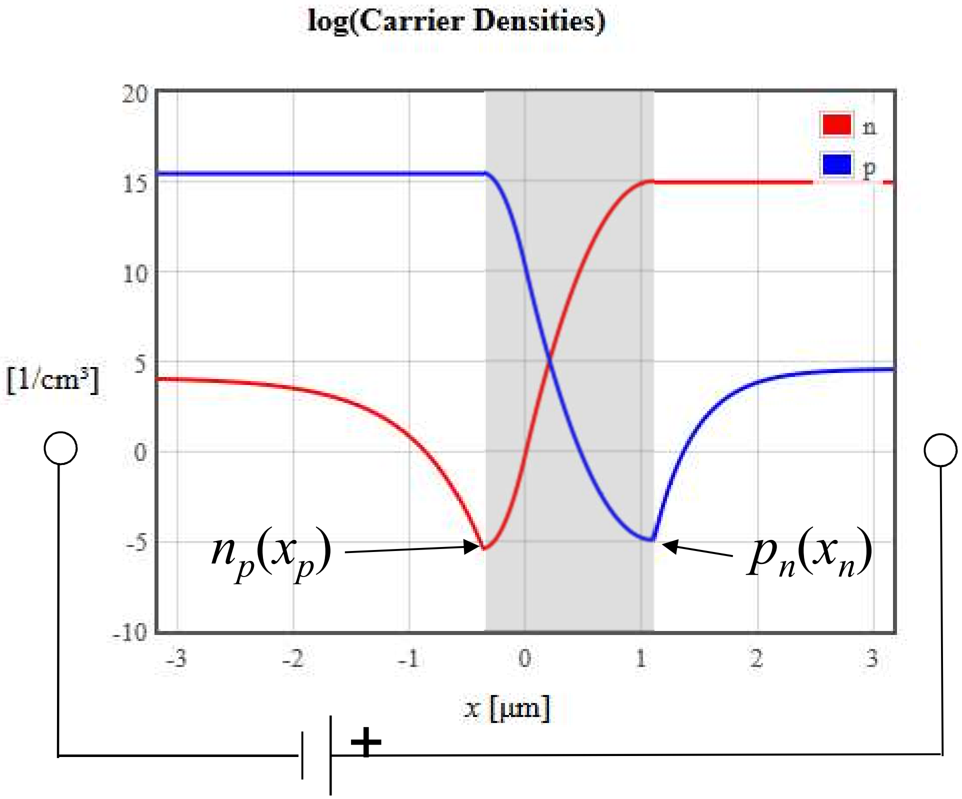
$$n_p(x_p) = n_{p0} \exp\left(\frac{eV}{k_B T}\right)$$

$$p_n(x_n) = p_{n0} \exp\left(\frac{eV}{k_B T}\right)$$

Minority electrons are injected into the p-region
Minority holes are injected into the n-region

Reverse bias, V < 0

Electrons and holes are driven away from the junction.
The depletion region becomes wider



$$n_p(x_p) = n_{p0}\exp\left(\frac{eV}{k_BT}\right)$$

$$p_n(x_n) = p_{n0}\exp\left(\frac{eV}{k_BT}\right)$$

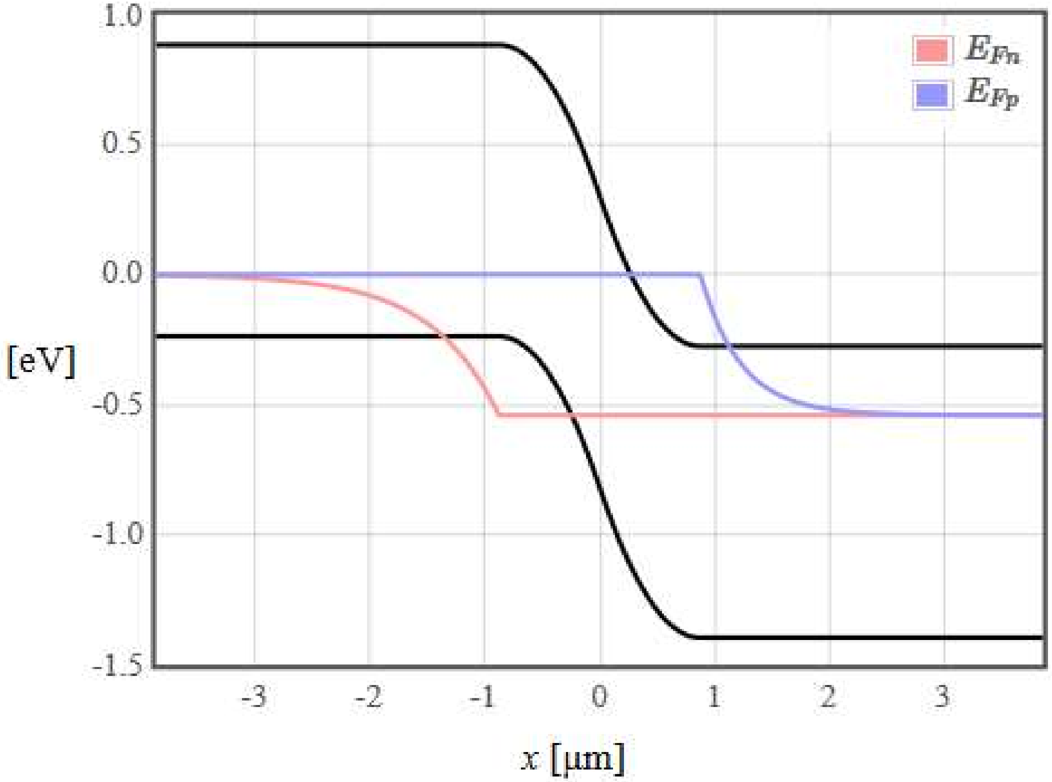
Minority electrons are extracted from the p-region by the electric field
Minority holes are extracted from the n-region by the electric field

Quasi Fermi level

When the charge carriers are not in equilibrium the Fermi energy can be different for electrons and holes.

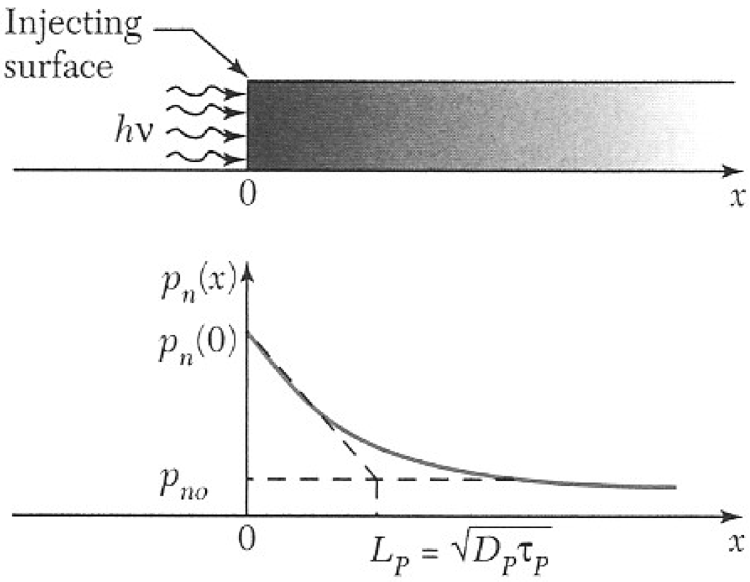
$$n = N_c \exp\left(\frac{E_{Fn} - E_c}{k_B T}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_{Fp}}{k_B T}\right)$$



Review of Diffusion

n-type



$$D_p \frac{\partial^2 p_n}{\partial x^2} = \frac{p_n - p_{n0}}{\tau_p}$$

recombination time

$$p_n(x) = p_{n0} + \left(p_n(0) - p_{n0}\right)\exp\left(\frac{-x}{L_p}\right)$$

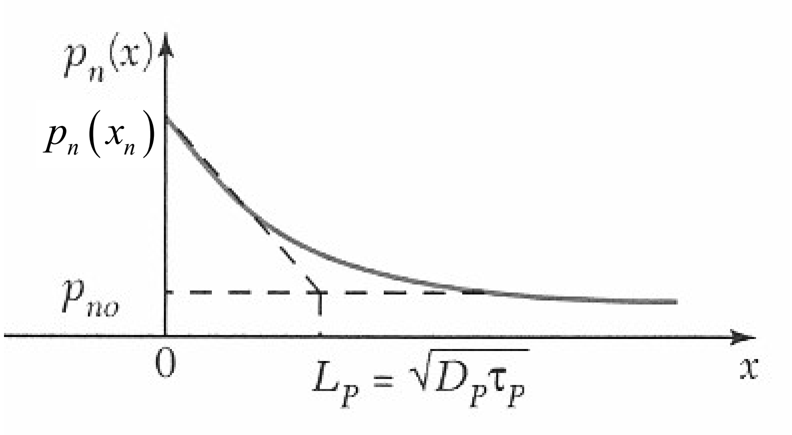
$$L_p = \sqrt{D_p \tau_p}$$

diffusion length

Injection only occurs at the surface. There the minority carrier density is $p_n(0)$.

Diffusion current

n-type



$$p_n(x) = p_{n0} + \left(p_n(x_n) - p_{n0}\right)\exp\left(\frac{-x}{L_p}\right)$$

$$J_{diff,p} = -eD_p \frac{dp}{dx}$$

$$J_{diff,p} = -eD_p \frac{dp}{dx} = \left(p_n(x_n) - p_{n0}\right)\frac{eD_p}{L_p}\exp\left(\frac{-x}{L_p}\right)$$

At the edge of the depletion region:

$$J_{diff,p} = -eD_p \frac{dp}{dx} = \left(p_n(x_n) - p_{n0}\right)\frac{eD_p}{L_p}$$

Diffusion current

$$J_{diff,p} = \left(p_n(x_n) - p_{n0}\right)\frac{eD_p}{L_p}$$

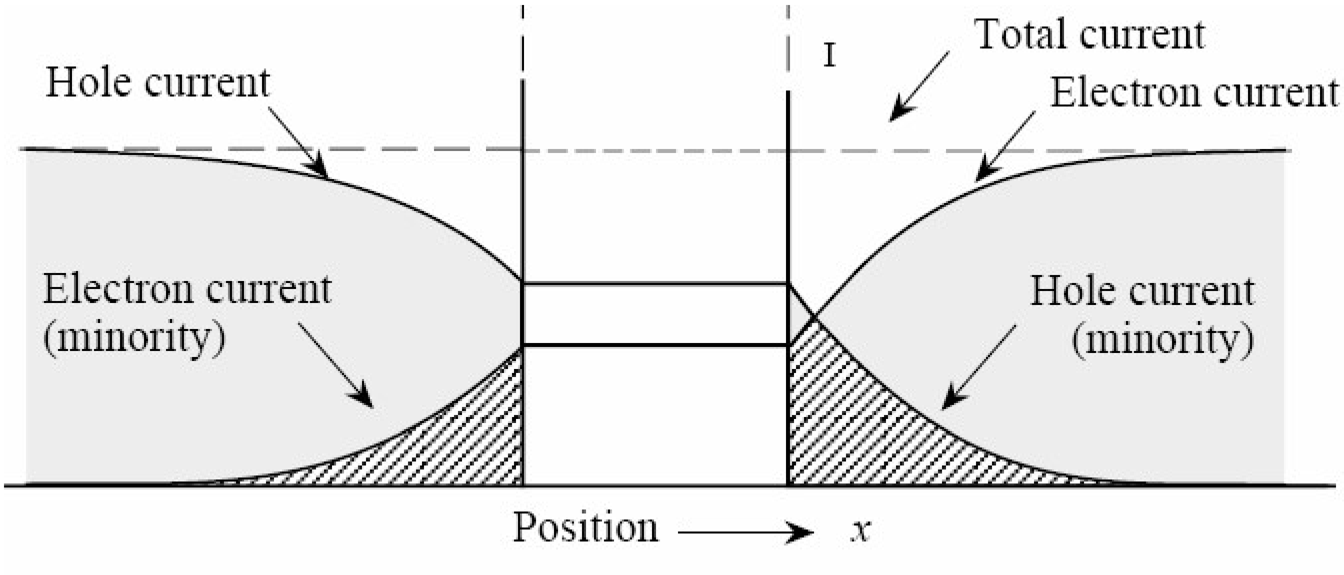
$$p_n(x_n) = p_{n0}\exp\left(\frac{eV}{k_BT}\right)$$

$$J_{diff,p} = p_{n0}\frac{eD_p}{L_p}\left(\exp\left(\frac{eV}{k_BT}\right) - 1\right)$$

Diffusion current

$$J_{diff,p} = \frac{p_{n0} e D_p}{L_p}\left(\exp\left(\frac{eV}{k_B T}\right) - 1\right)$$

$$J_{diff,n} = \frac{n_{p0} e D_n}{L_n}\left(\exp\left(\frac{eV}{k_B T}\right) - 1\right)$$

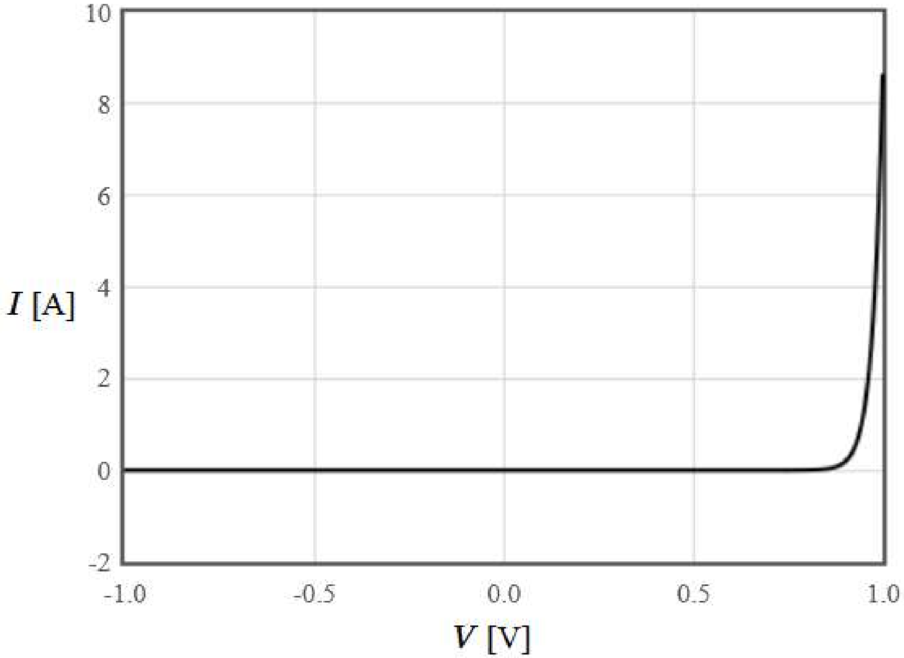


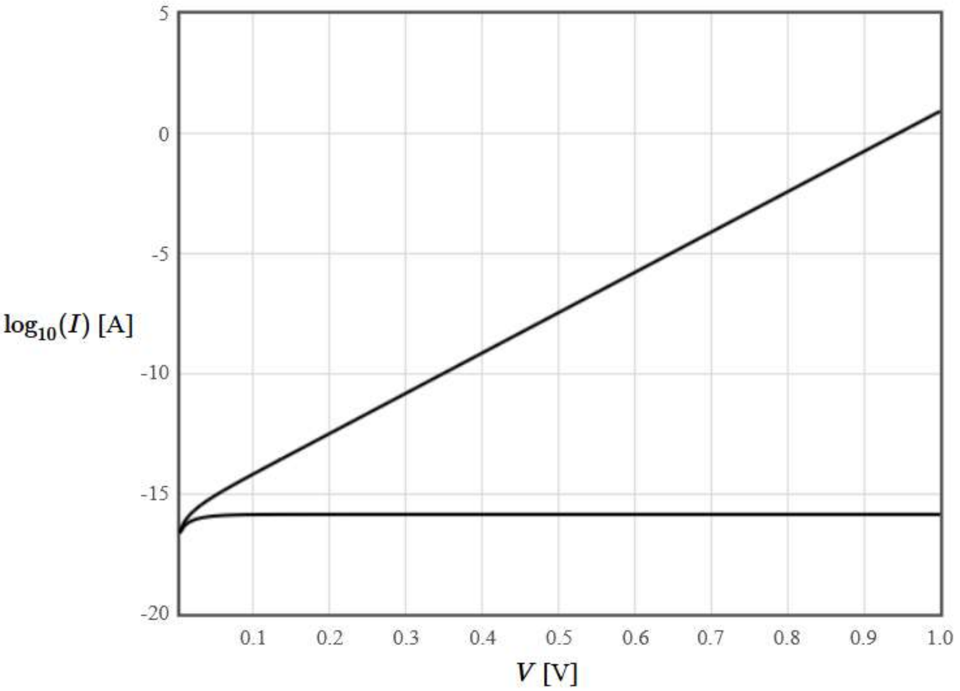
Diode current

$$I = eA\left(\frac{p_{n0}D_p}{L_p} + \frac{n_{p0}D_n}{L_n}\right)\left(\exp\left(\frac{eV}{k_BT}\right) - 1\right) = I_s\left(\exp\left(\frac{eV}{k_BT}\right) - 1\right)$$

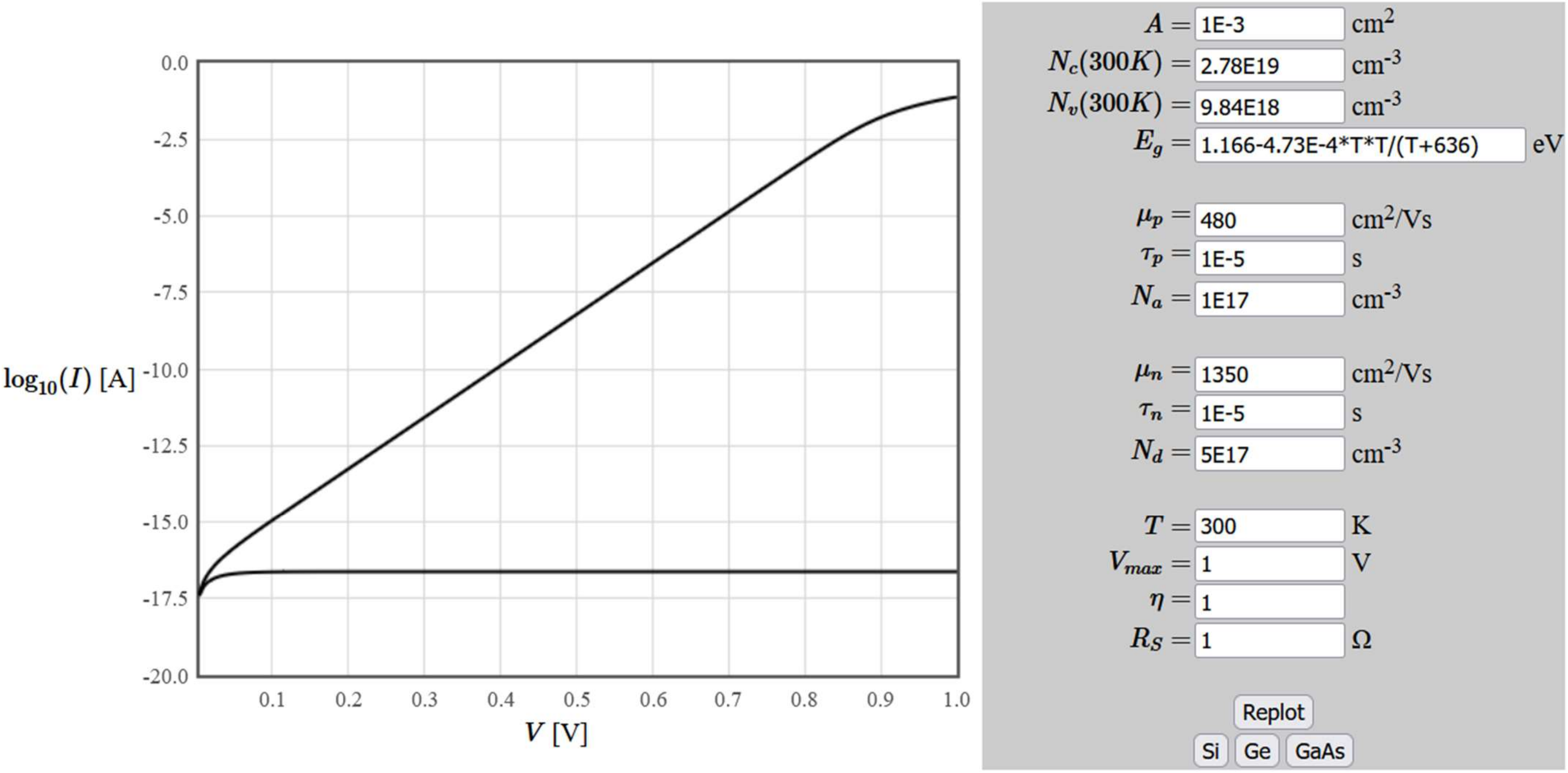
Area

Saturation current





Diode I-V charateristics



$A =$ 1E-3 cm^2

$N_c(300K) =$ 2.78E19 cm^{-3}

$N_v(300K) =$ 9.84E18 cm^{-3}

$E_g =$ 1.166-4.73E-4*T*T/(T+636) eV

$\mu_p =$ 480 cm^2/Vs

$\tau_p =$ 1E-5 s

$N_a =$ 1E17 cm^{-3}

$\mu_n =$ 1350 cm^2/Vs

$\tau_n =$ 1E-5 s

$N_d =$ 5E17 cm^{-3}

$T =$ 300 K

$V_{max} =$ 1 V

$\eta =$ 1

$R_S =$ 1 Ω

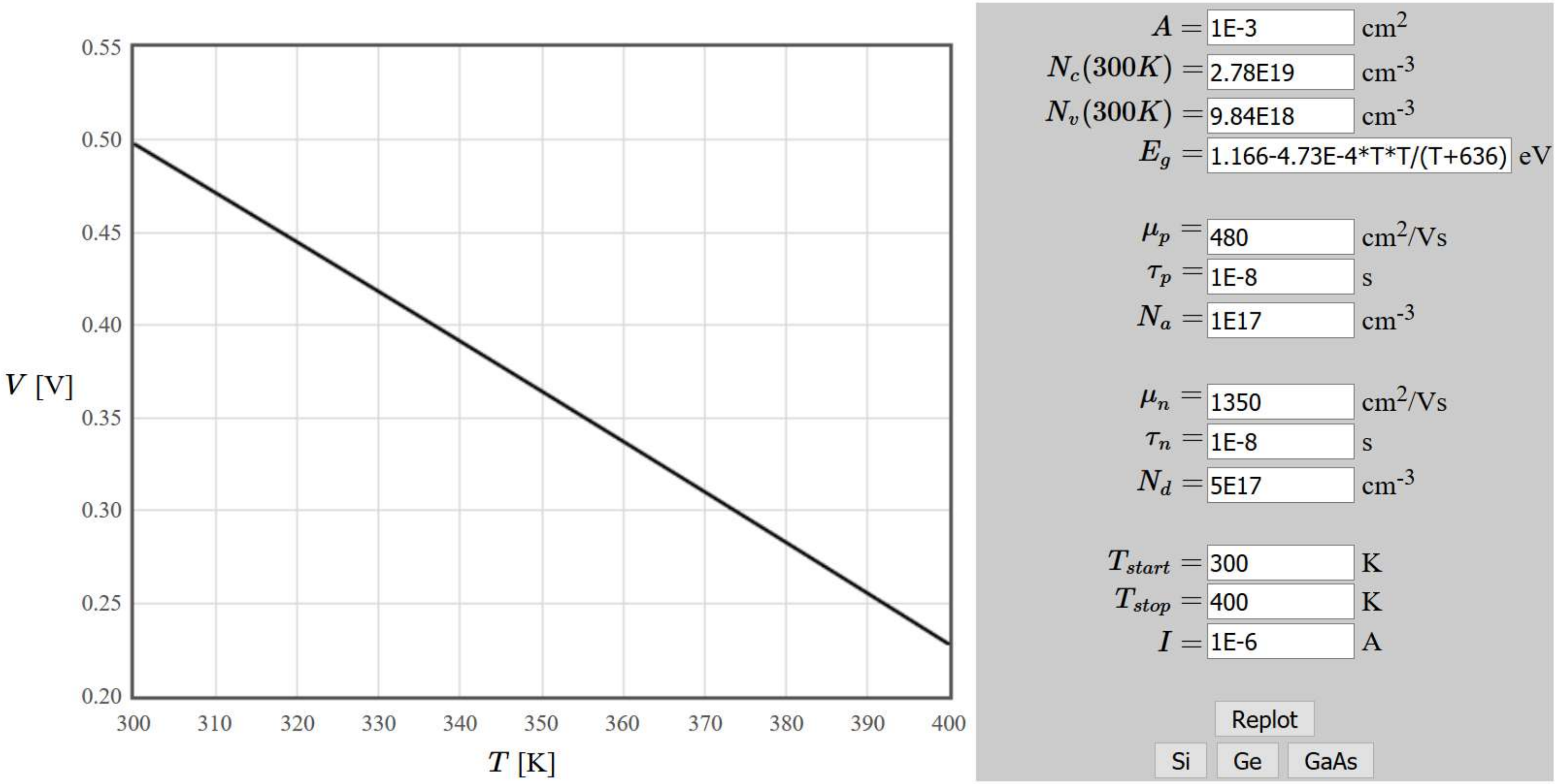
http://lamp.tu-graz.ac.at/~hadley/psd/L6/pnIV.php

Thermometer

$$I_S = Aen_i^2\left(\frac{D_p}{L_pN_d} + \frac{D_n}{L_nN_a}\right)$$

$$n_i = \sqrt{N_c\left(\frac{T}{300}\right)^{3/2} N_v\left(\frac{T}{300}\right)^{3/2}} \exp\left(\frac{-E_g}{2k_BT}\right)$$

$$D_n = \frac{\mu_n k_B T}{e}$$



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$E_g =$ 1.166-4.73E-4*T*T/(T+636) eV

$\mu_p =$ 480 cm^2/Vs

$\tau_p =$ 1E-8 s

$N_a =$ 1E17 cm^{-3}

$\mu_n =$ 1350 cm^2/Vs

$\tau_n =$ 1E-8 s

$N_d =$ 5E17 cm^{-3}

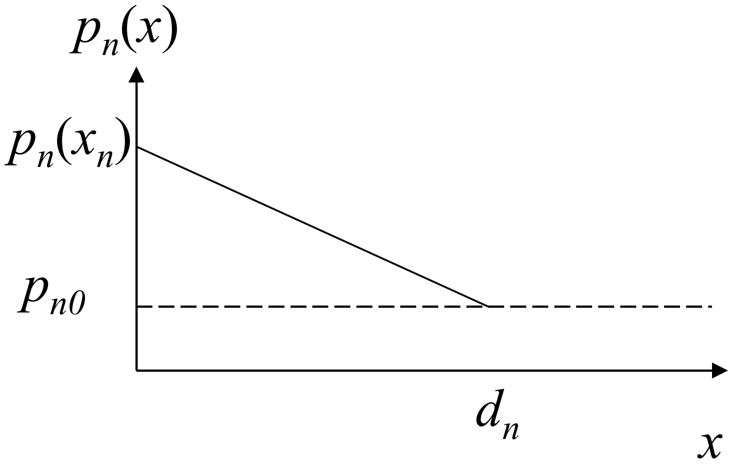
$T_{start} =$ 300 K

$T_{stop} =$ 400 K

$I =$ 1E-6 A

Short diode

n-type



$d_n << L_p$

Metal contact is much closer to the depletion region than the diffusion length

$$J_{diff,p} = -eD_p \frac{dp}{dx}$$

$$J_{diff,p} = -eD_p \frac{dp}{dx} = \frac{eD_p}{d_n}\left(p_n(x_n) - p_{n0}\right)$$

Diffusion current

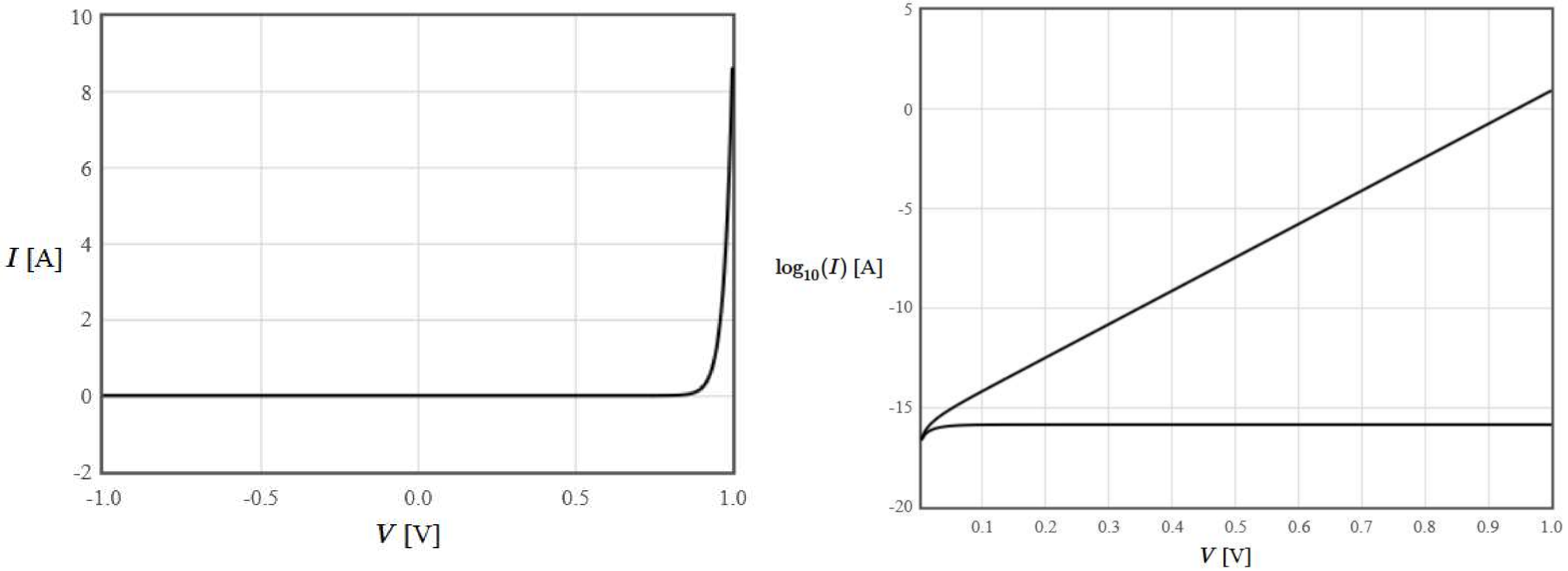
$$J_{diff,p} = \left(p_n(x_n) - p_{n0}\right)\frac{eD_p}{d_n}$$

$$J_{diff,p} = \left(p_{n0}\exp\left(\frac{e(V)}{k_B T}\right) - p_{n0}\right)\frac{eD_p}{d_n}$$

$$J_{diff,p} = \frac{p_{n0}eD_p}{d_n}\left(\exp\left(\frac{eV}{k_B T}\right) - 1\right)$$

Short diode current

$$I = e\underset{\substack{\nearrow \\ \text{Area}}}{A}\left(\frac{p_{n0}D_p}{d_n}+\frac{n_{p0}D_n}{d_p}\right)\left(\exp\left(\frac{eV}{k_BT}\right)-1\right)=I_s\left(\exp\left(\frac{eV}{k_BT}\right)-1\right)$$

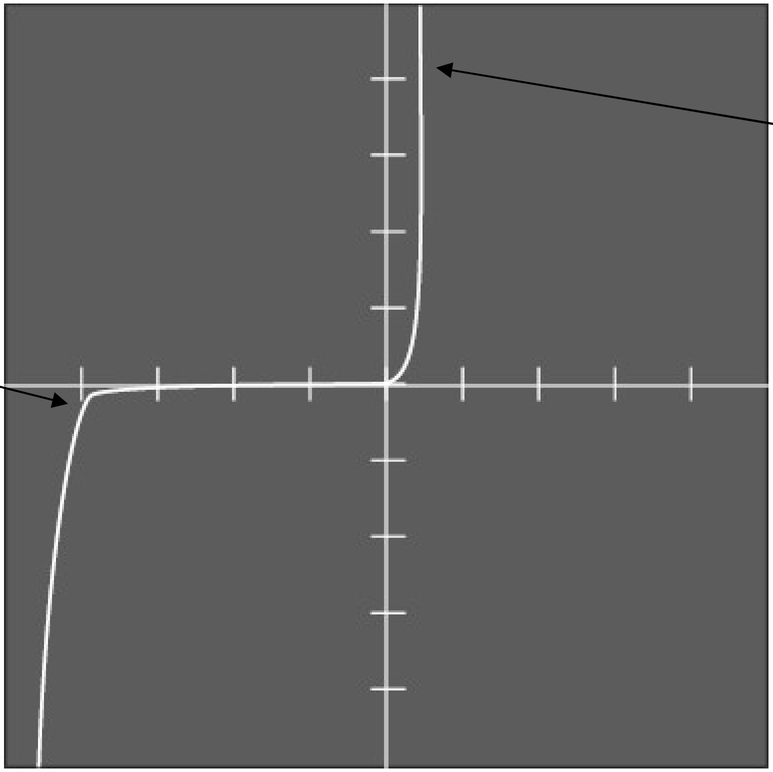


Real diodes

$$I = I_s\left(\exp\left(\frac{eV}{nk_BT}\right)-1\right)$$

n = nonideality factor

$n = 1$ for an ideal diode



series resistance

reverse breakdown voltage

Real diodes

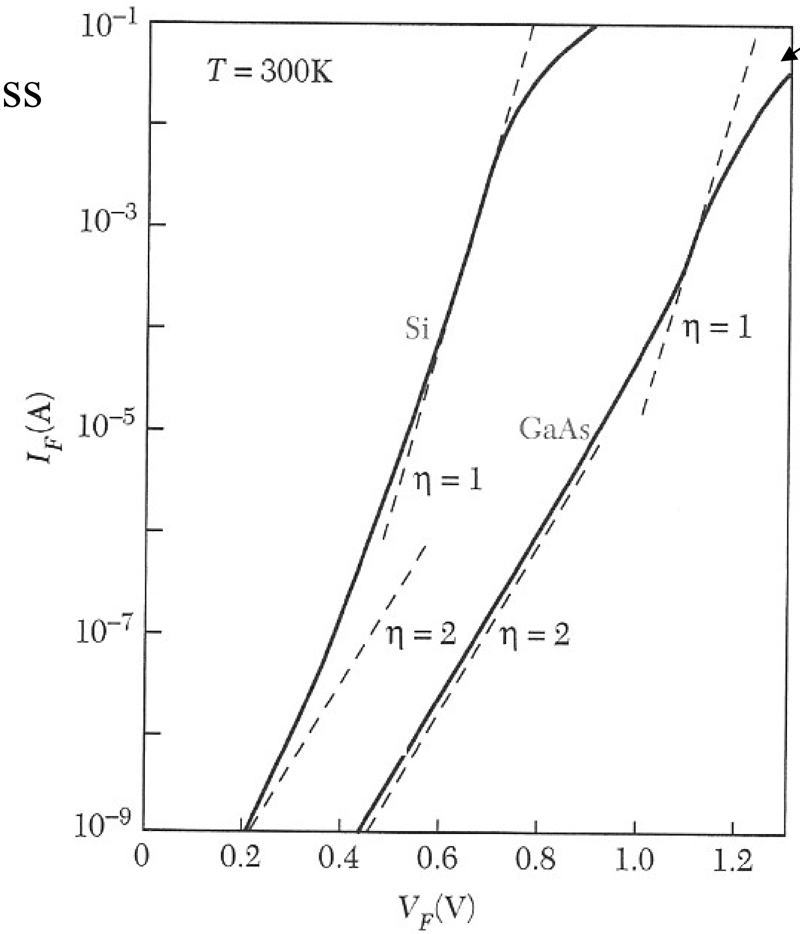
$$I = I_s\left(\exp\left(\frac{eV}{nk_BT}\right)-1\right)$$

There is constant generation/recombination of electron hole pairs.

In forward bias there is less current due to recombination.

In reverse bias there is an extra current from generation.

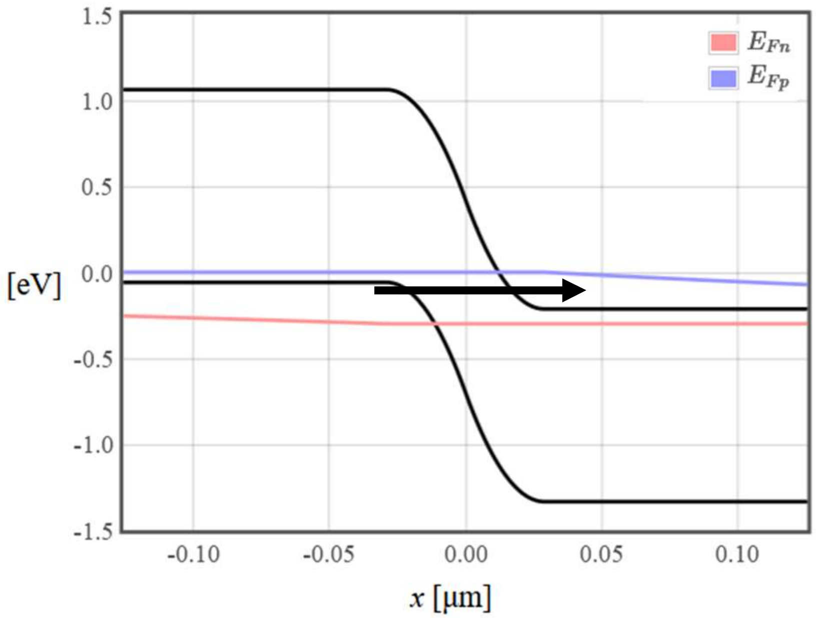
Low bias: recombination dominates, $n = 2$

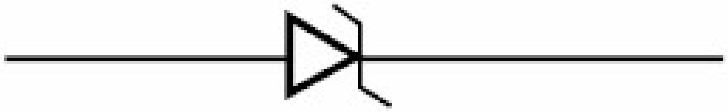


Very high bias: series resistance

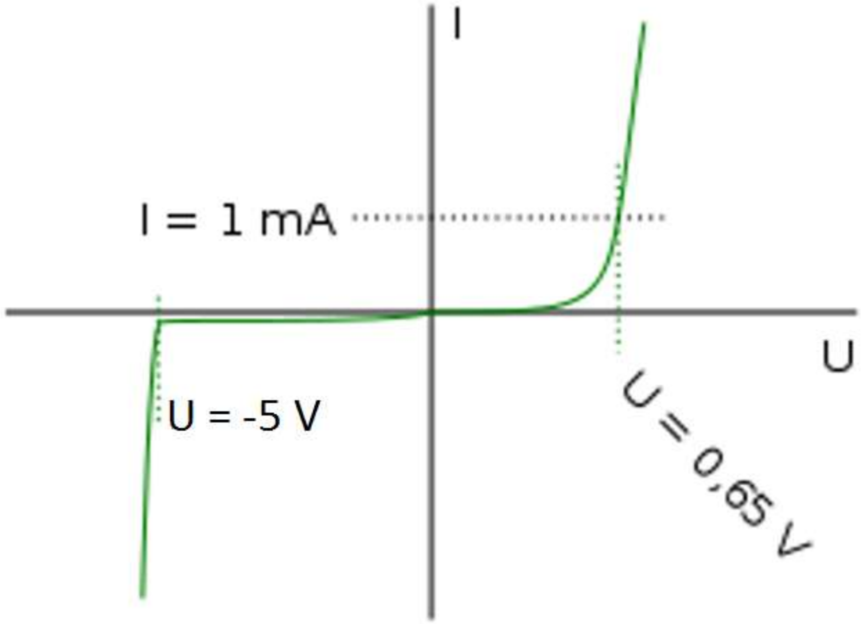
High bias: ideal behavior, $n = 1$

Zener tunneling





(Zener diode)

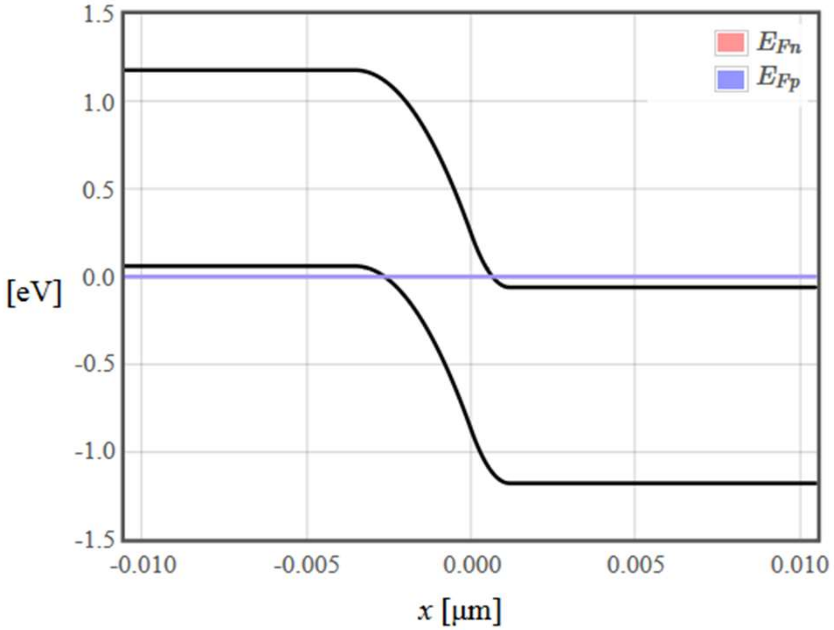


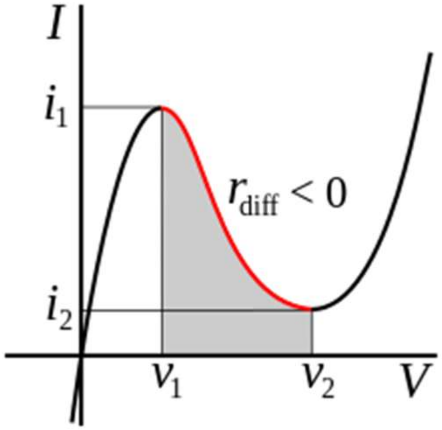
Electrons tunnel from valence band to conduction band

Occurs at high doping

$|V_{\text{zener}}| < 5.6$ V

Tunnel diodes / Esaki diodes

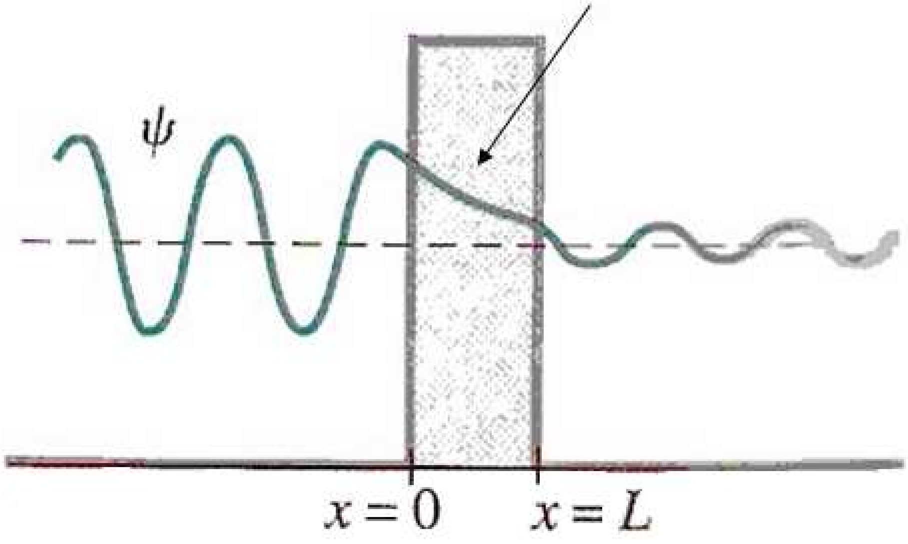


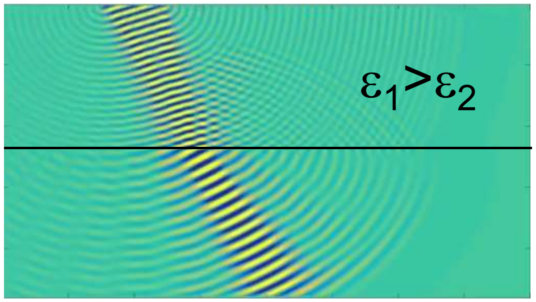


Both sides of the diode are degenerately doped

Tunneling

wave decays exponentially in the classically forbidden region





Tunneling is a wave phenomena. Tunneling and total internal reflection are used in a beam splitter.

Zener tunneling

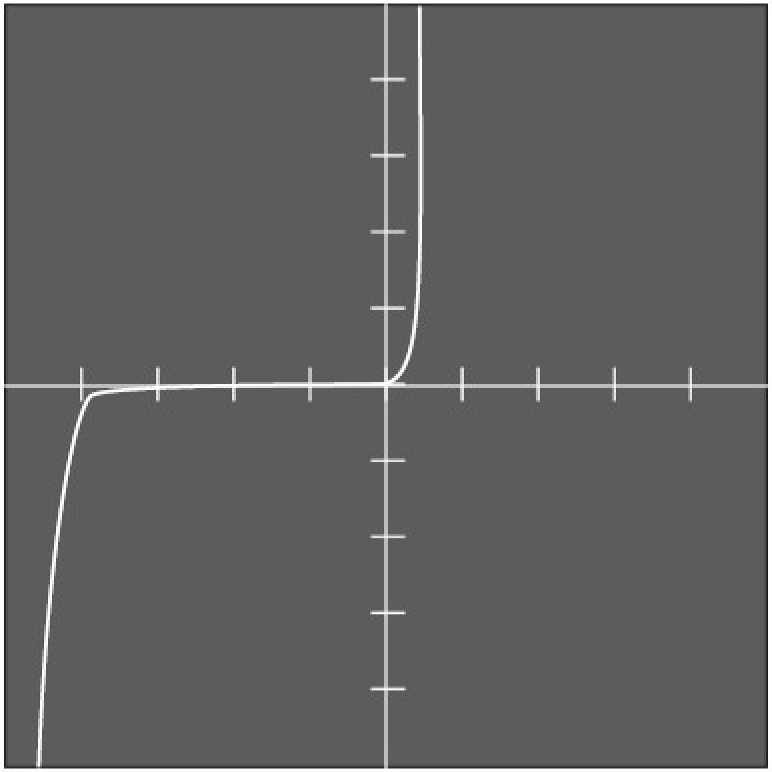
Breakdown voltage is typically much lower than the breakdown voltage of an avalanche diode and can be tuned by adjusting the width of the depletion layer.

Used to provide a reference voltage.

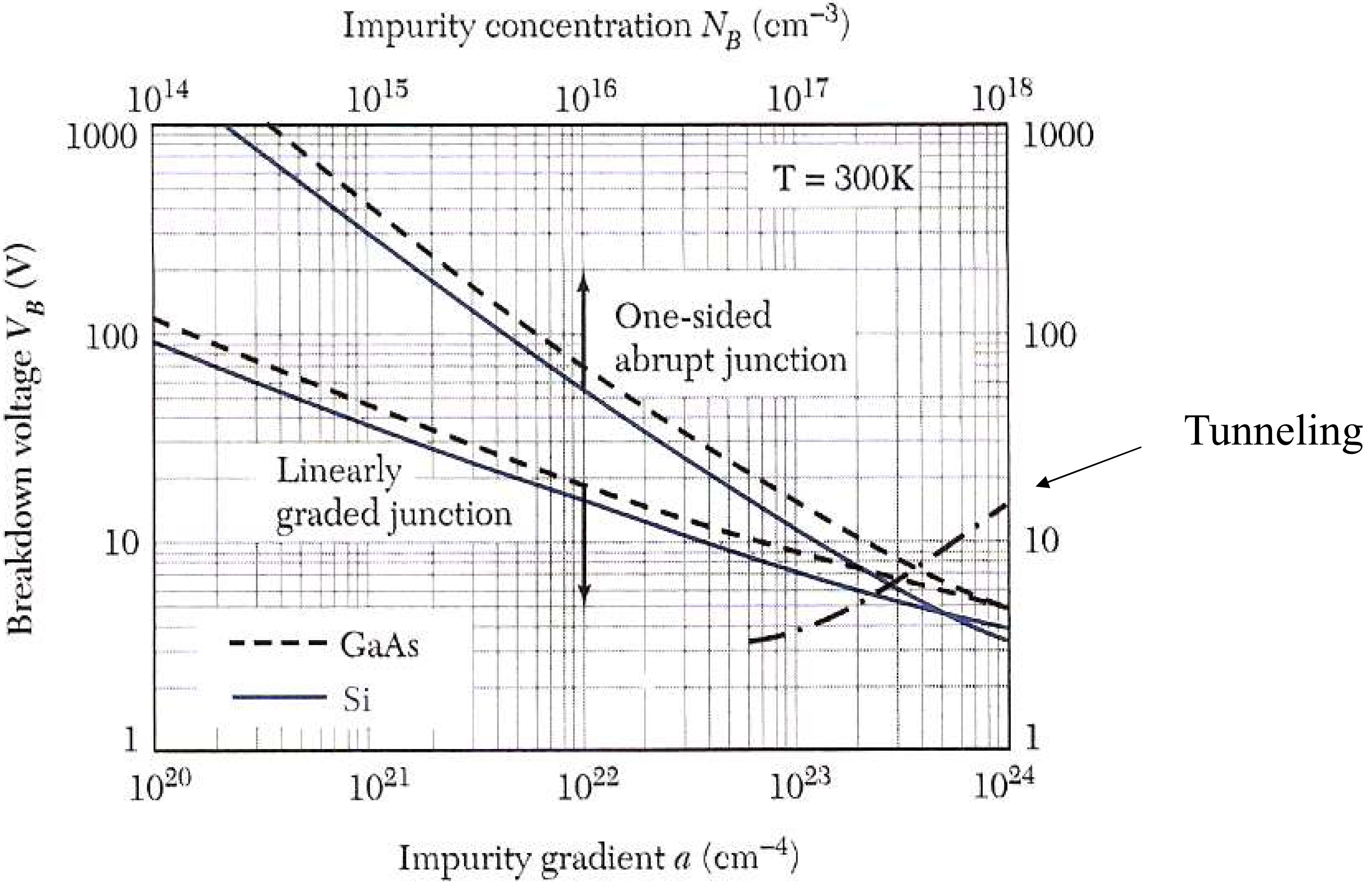
Avalanche breakdown

Impact ionization causes an avalanche of current

Occurs at low doping


Vertical: 5 mA/div
Horizontal: 5 V/div

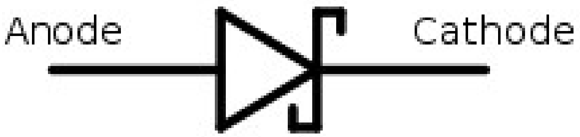
Avalanche breakdown

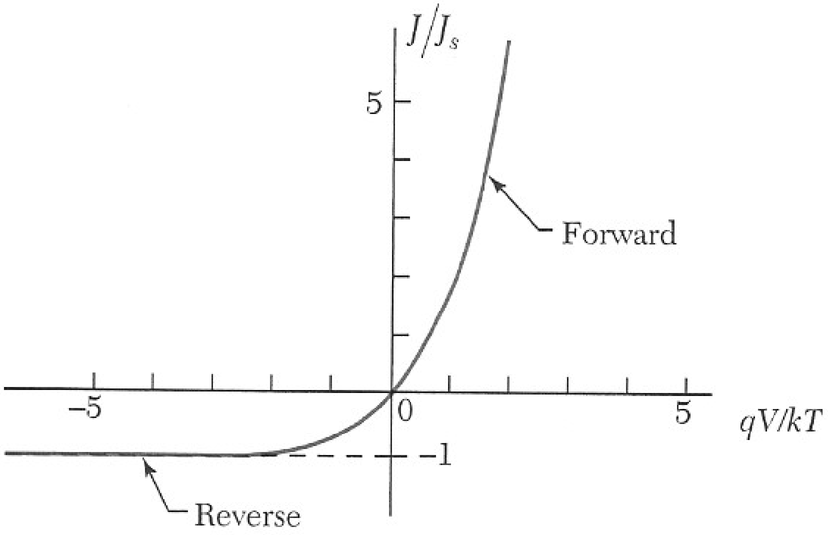


Metal-Semiconductor Contacts

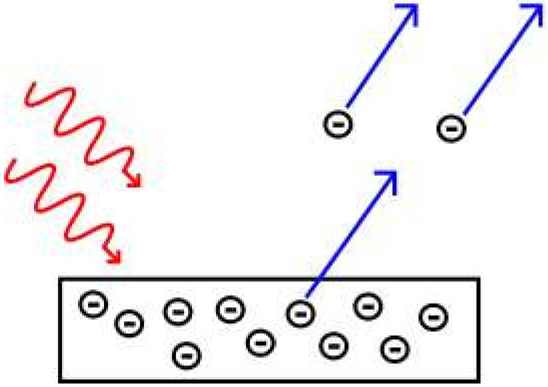
metal - semiconductor contacts

Photoelectric effect
Schottky barriers
Schottky diodes
Ohmic contacts
Thermionic emission
Tunnel contacts



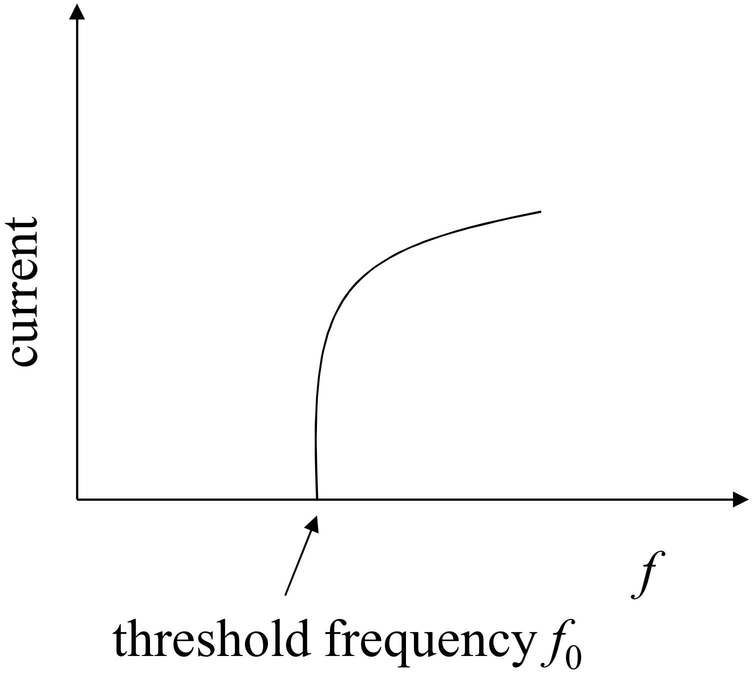


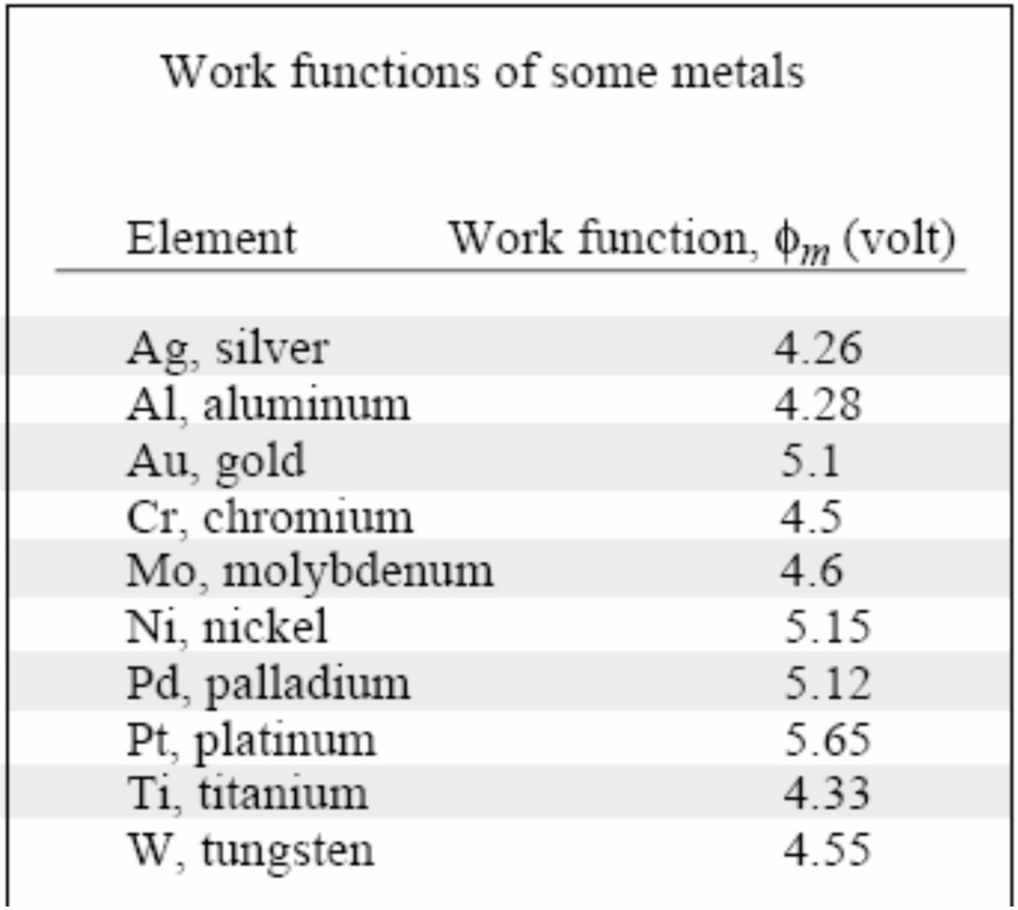
Photoelectric effect



$hf_0 = e\phi$ at threshold

workfunction





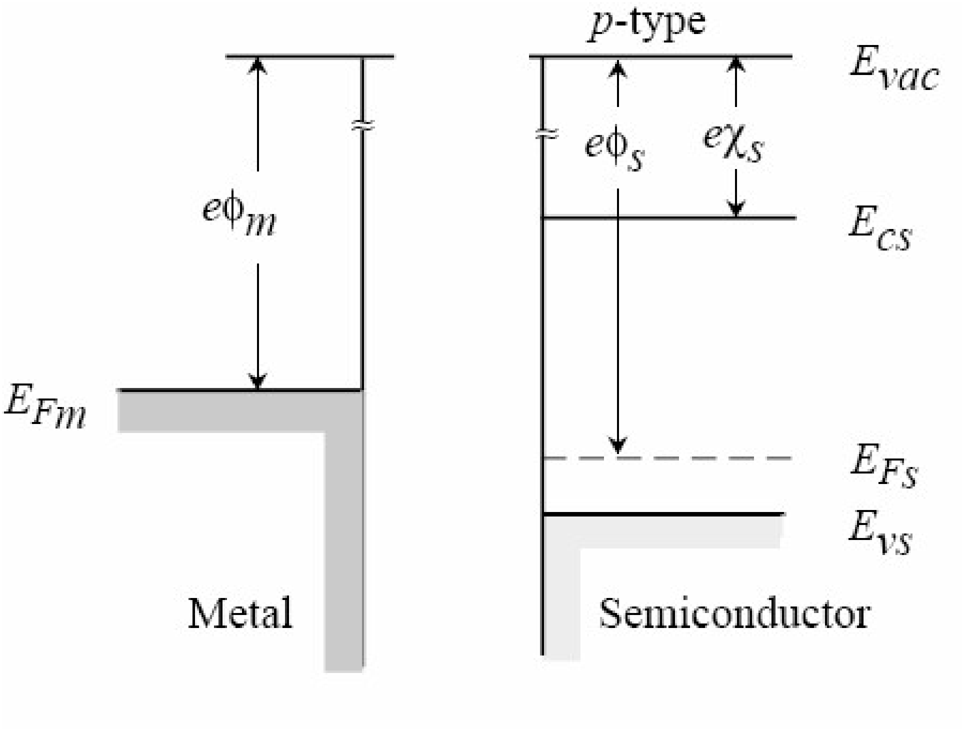
Work functions of some metals

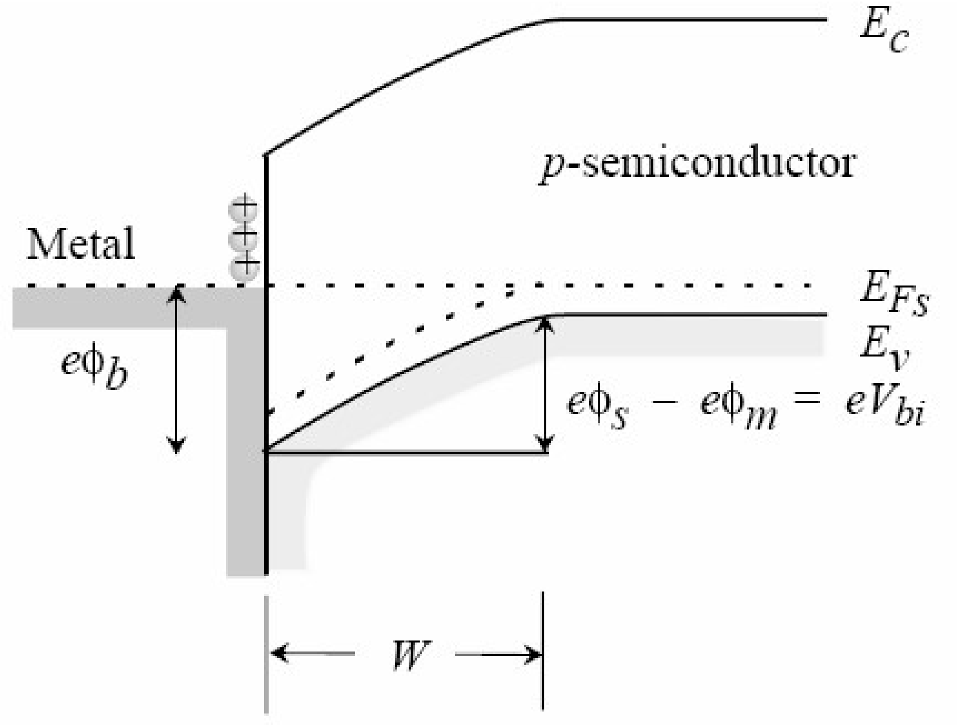
Element	Work function, ϕ_m (volt)
Ag, silver	4.26
Al, aluminum	4.28
Au, gold	5.1
Cr, chromium	4.5
Mo, molybdenum	4.6
Ni, nickel	5.15
Pd, palladium	5.12
Pt, platinum	5.65
Ti, titanium	4.33
W, tungsten	4.55

There is a dipole field at the surface of a metal. This electric field must be overcome for an electron to escape.

Singh

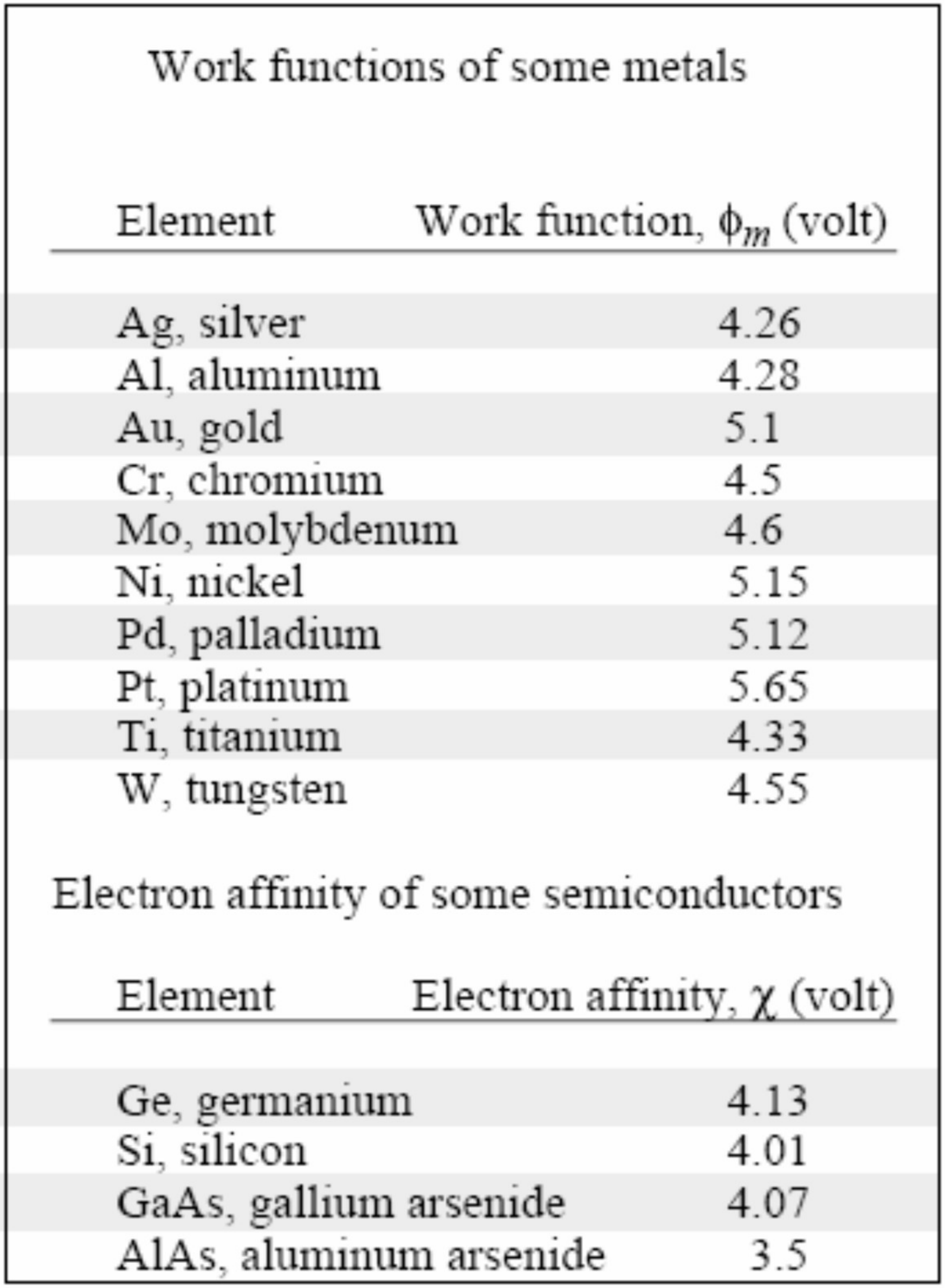
work function - electron affinity





If $\phi_s < \phi_m$, the semiconductor bands bend down.

If $\phi_s > \phi_m$, the semiconductor bands bend up.



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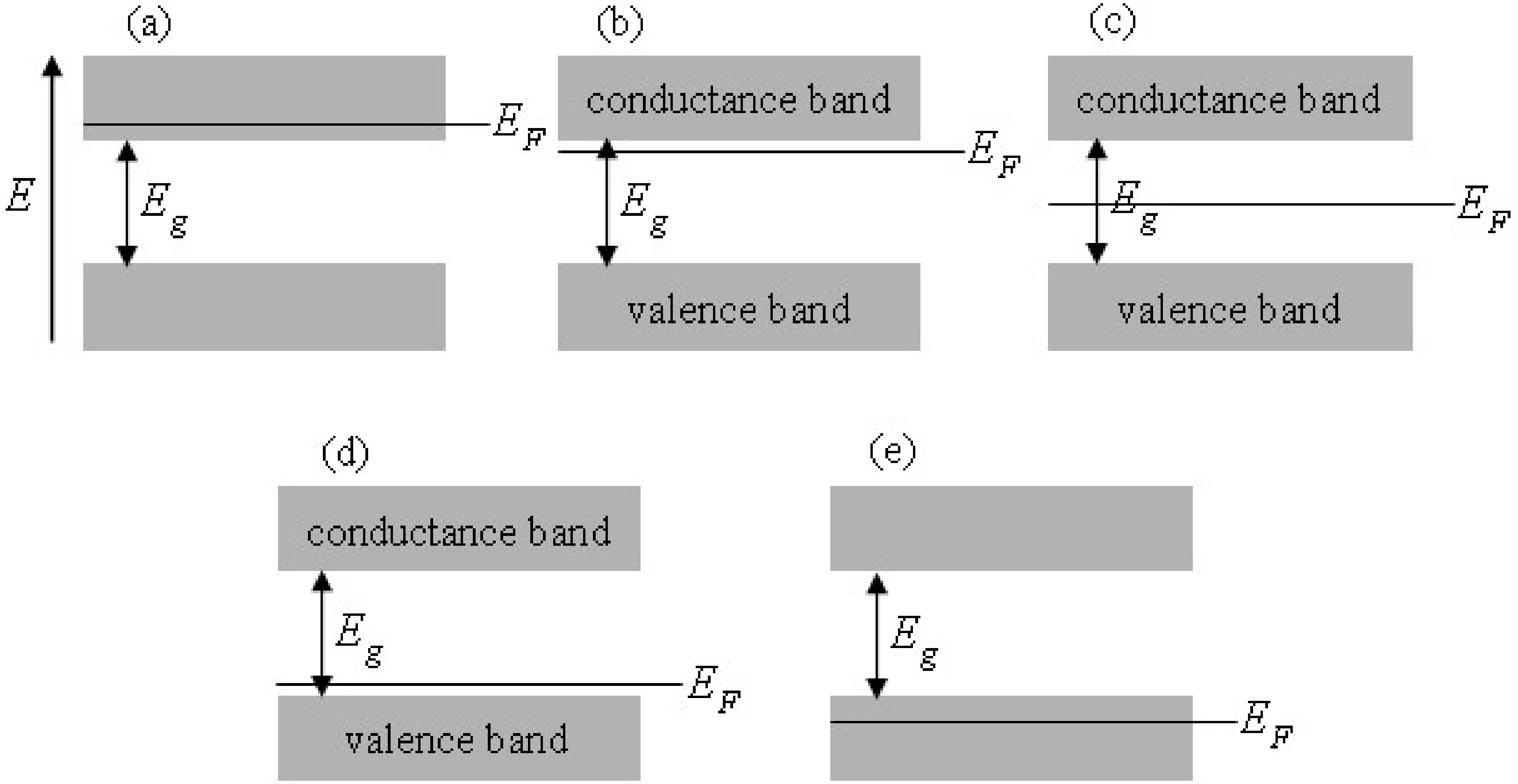
Ti, titanium	4.33
W, tungsten	4.55

Electron affinity of some semiconductors

Element	Electron affinity, χ (volt)
Ge, germanium	4.13
Si, silicon	4.01
GaAs, gallium arsenide	4.07
AlAs, aluminum arsenide	3.5

Singh

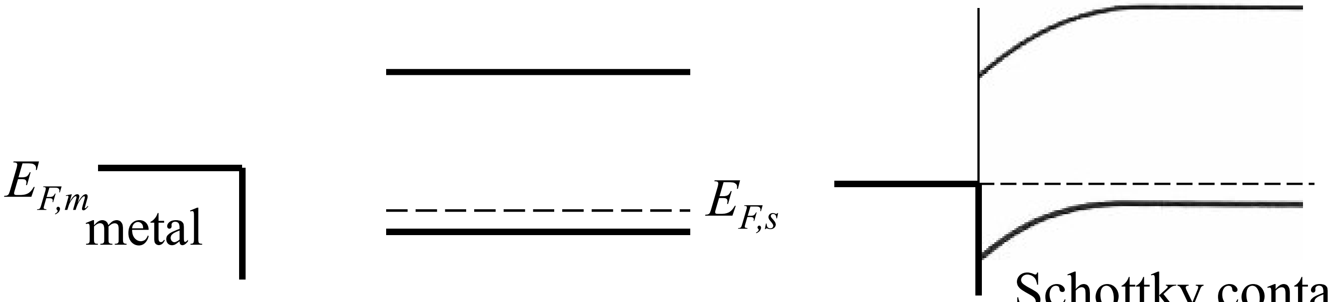


(a) A metal with a small workfunction. (b) An n-type semiconductor. (c) An insulator.
(d) A p-type semiconductor. (e) A metal with a large workfunction.

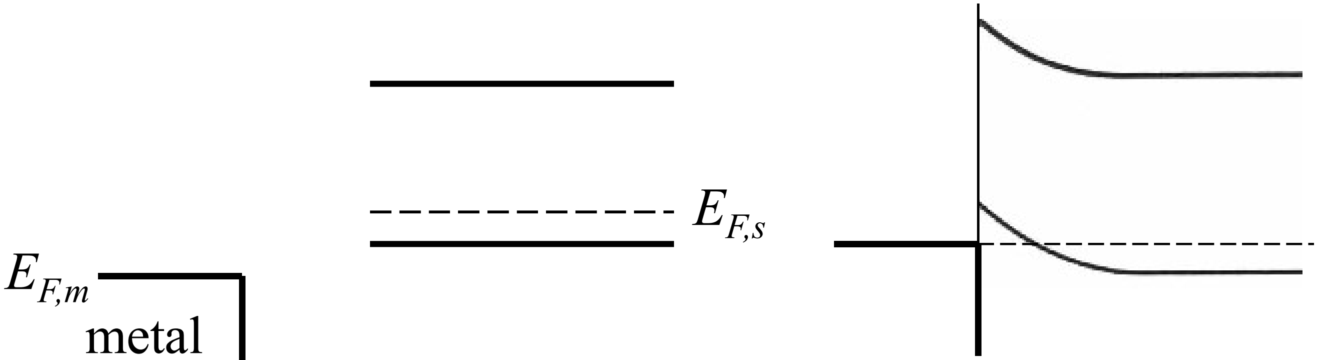
p-type

Schottky contact / ohmic contact


Walter Schottky



Schottky contact

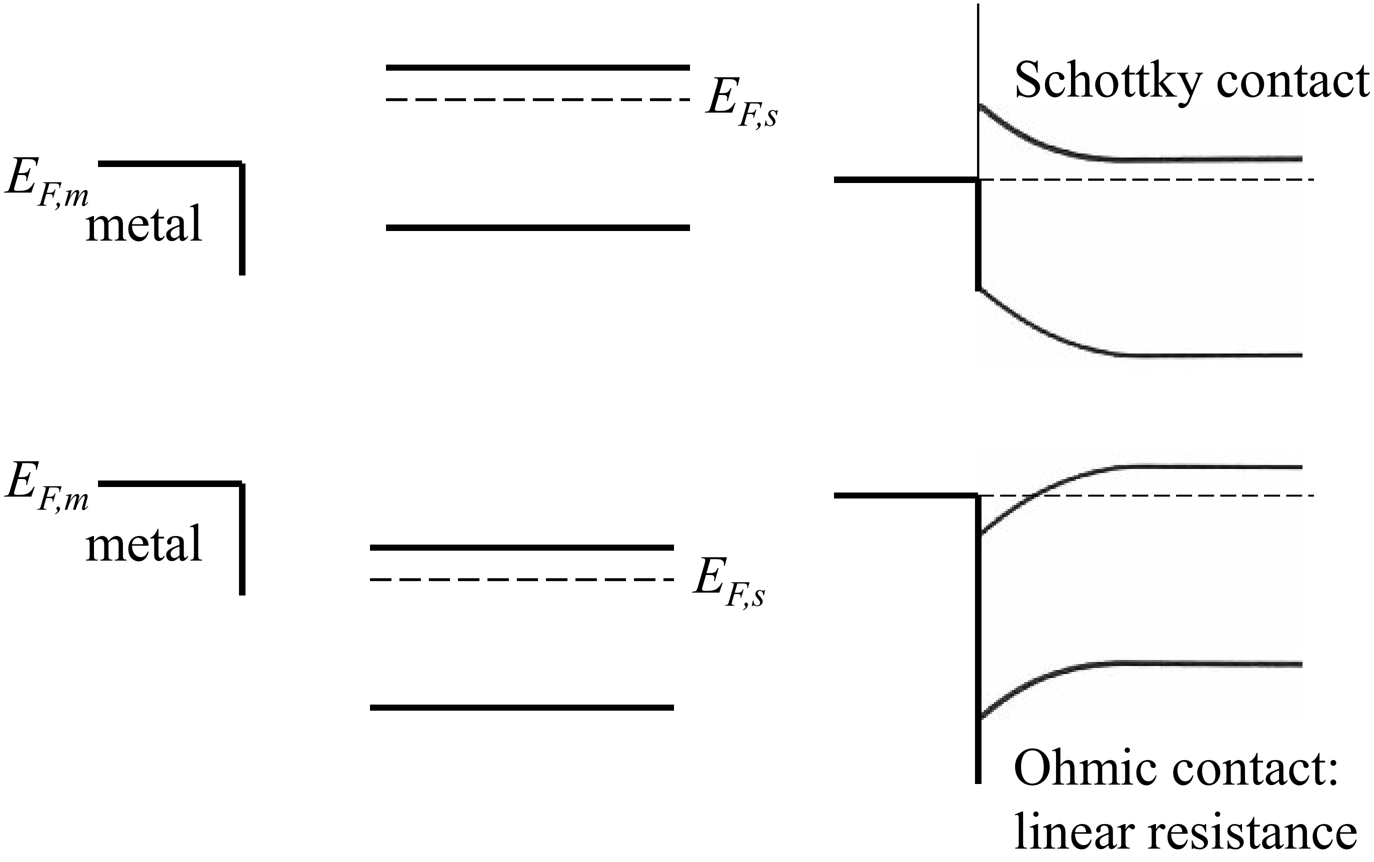


Ohmic contact:
linear resistance

specific contact resistance: $R_c = \left(\frac{\partial J}{\partial V}\right)^{-1}$ Ω-cm^2

n-type

Schottky contact / ohmic contact

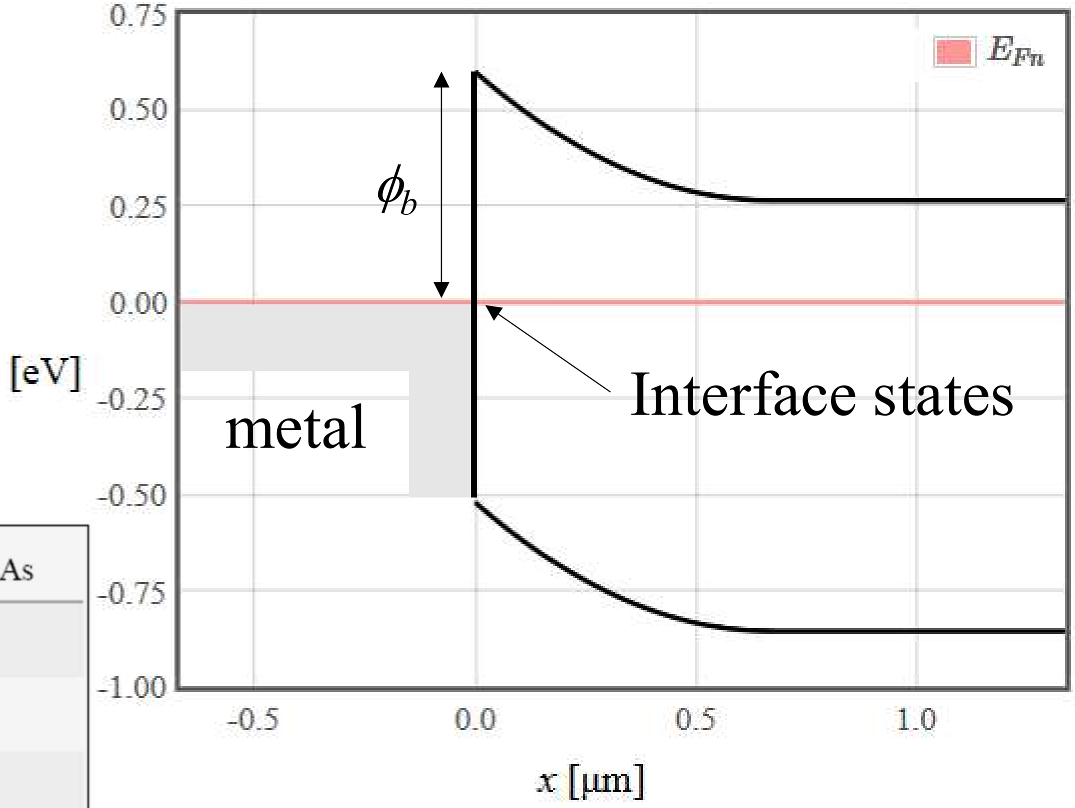


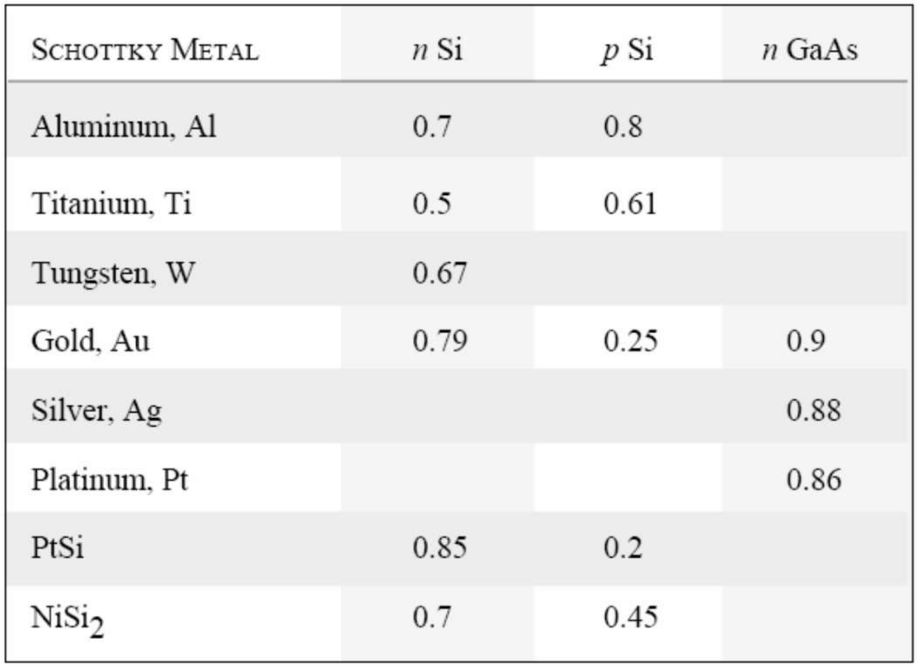
Schottky contact

Ohmic contact:
linear resistance

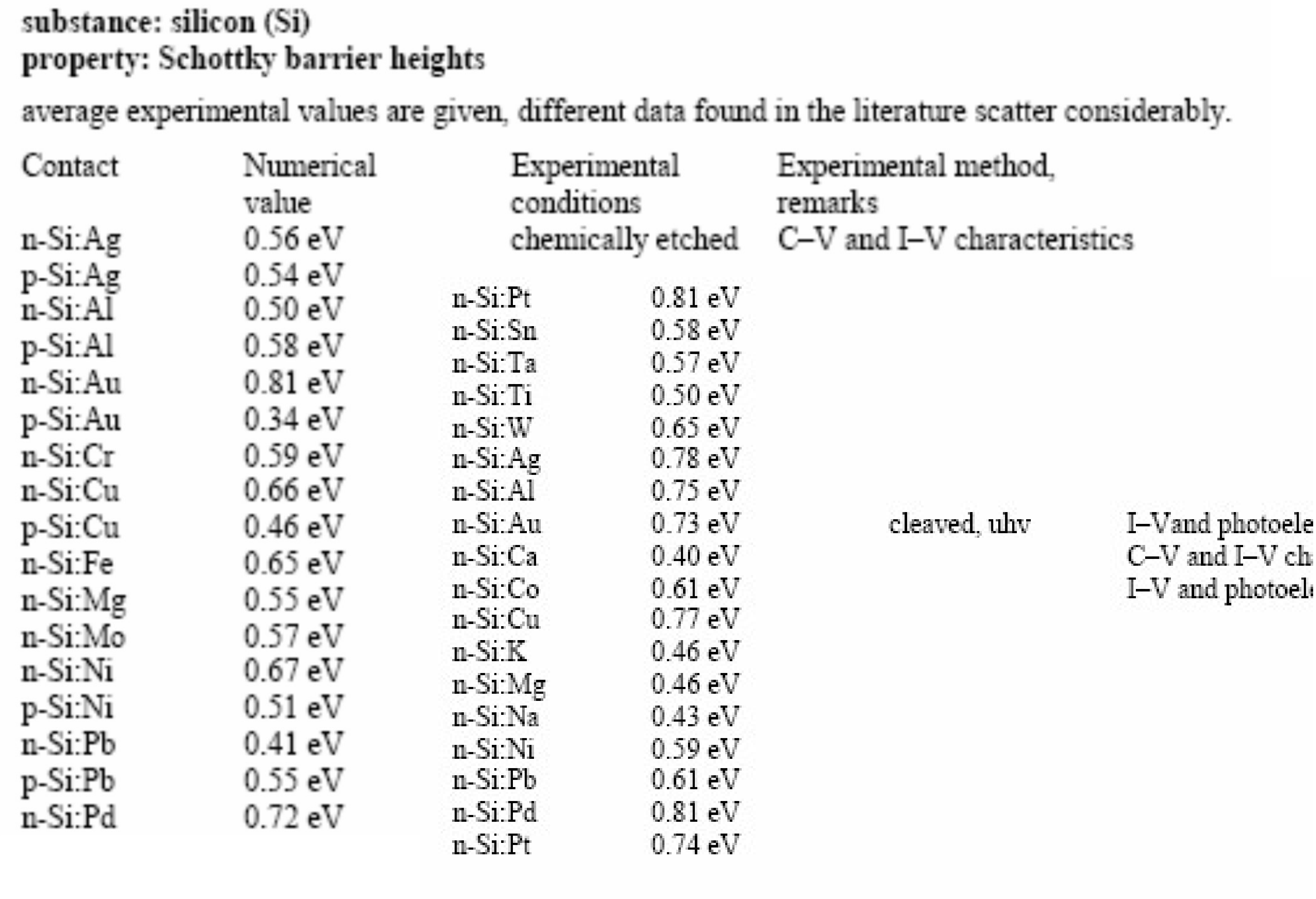
specific contact resistance: $R_c = \left(\frac{\partial J}{\partial V}\right)^{-1}$ Ω-cm^2

Interface states





SCHOTTKY METAL	*n* Si	*p* Si	*n* GaAs
Aluminum, Al	0.7	0.8	
Titanium, Ti	0.5	0.61	
Tungsten, W	0.67		
Gold, Au	0.79	0.25	0.9
Silver, Ag			0.88
Platinum, Pt			0.86
PtSi	0.85	0.2	
$NiSi_2$	0.7	0.45	



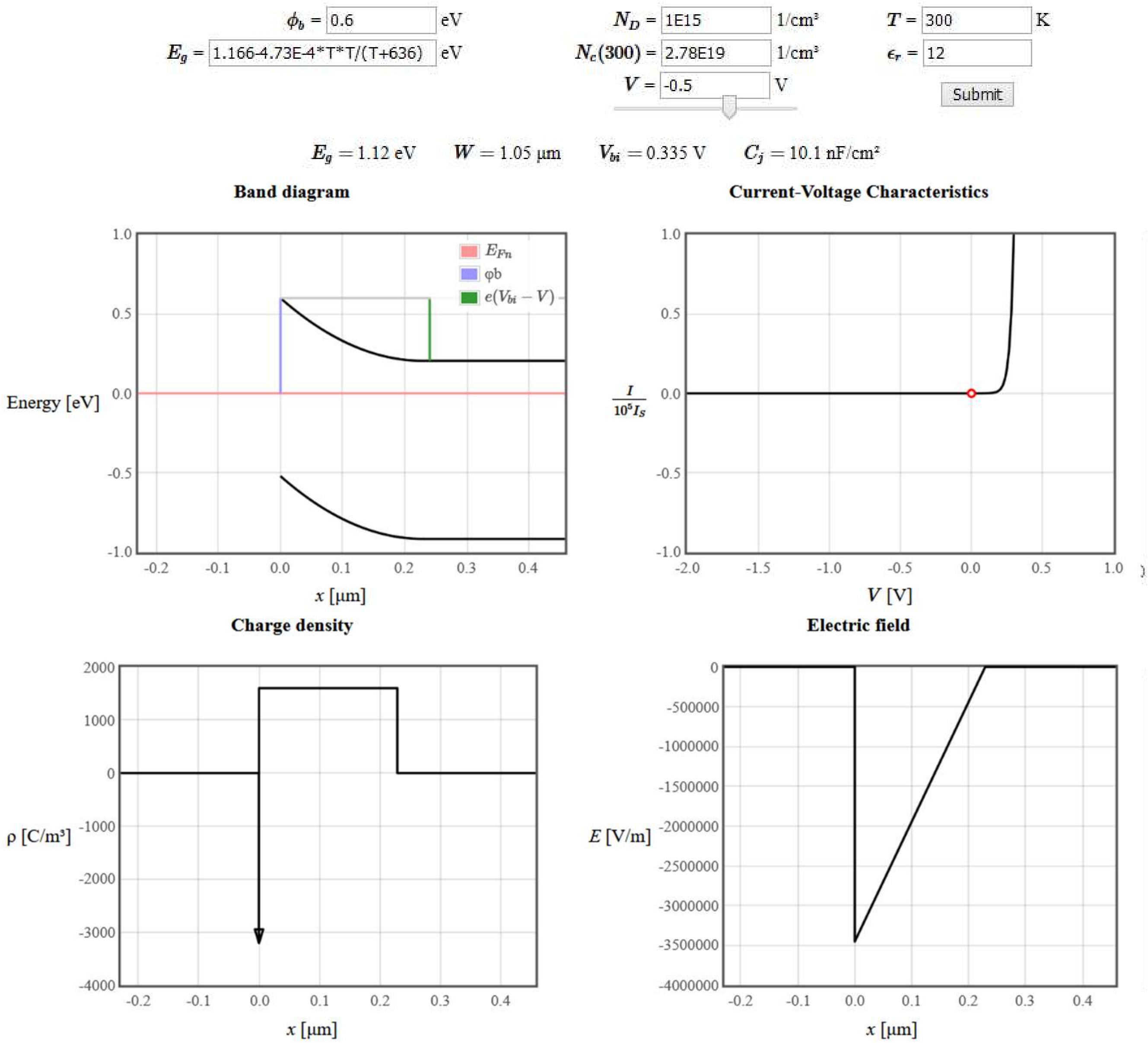
substance: silicon (Si)
property: Schottky barrier heights

average experimental values are given, different data found in the literature scatter considerably.

Contact	Numerical value	Experimental conditions	Experimental method, remarks
n-Si:Ag	0.56 eV	chemically etched	C–V and I–V characteristics
p-Si:Ag	0.54 eV		
n-Si:Al	0.50 eV		
p-Si:Al	0.58 eV		
n-Si:Au	0.81 eV		
p-Si:Au	0.34 eV		
n-Si:Cr	0.59 eV		
n-Si:Cu	0.66 eV		
p-Si:Cu	0.46 eV		
n-Si:Fe	0.65 eV		
n-Si:Mg	0.55 eV		
n-Si:Mo	0.57 eV		
n-Si:Ni	0.67 eV		
p-Si:Ni	0.51 eV		
n-Si:Pb	0.41 eV		
p-Si:Pb	0.55 eV		
n-Si:Pd	0.72 eV		
n-Si:Pt	0.81 eV		
n-Si:Sn	0.58 eV		
n-Si:Ta	0.57 eV		
n-Si:Ti	0.50 eV		
n-Si:W	0.65 eV		
n-Si:Ag	0.78 eV		
n-Si:Al	0.75 eV		
n-Si:Au	0.73 eV	cleaved, uhv	I–Vand photoele
n-Si:Ca	0.40 eV		C–V and I–V ch
n-Si:Co	0.61 eV		I–V and photoel
n-Si:Cu	0.77 eV		
n-Si:K	0.46 eV		
n-Si:Mg	0.46 eV		
n-Si:Na	0.43 eV		
n-Si:Ni	0.59 eV		
n-Si:Pb	0.61 eV		
n-Si:Pd	0.81 eV		
n-Si:Pt	0.74 eV		

http://www.springermaterials.com/navigation/#n_240905_Silicon+%2528Si%2529

Schottky barrier



ϕ_b = 0.6 eV

E_g = 1.166-4.73E-4*T*T/(T+636) eV

N_D = 1E15 1/cm³

$N_c(300)$ = 2.78E19 1/cm³

V = -0.5 V

T = 300 K

ϵ_r = 12

$E_g = 1.12$ eV $W = 1.05$ µm $V_{bi} = 0.335$ V $C_j = 10.1$ nF/cm²

$$E = \frac{eN_D}{\varepsilon_r \varepsilon_0}\left(x - x_n\right)$$

$$W \approx x_n = \sqrt{\frac{2\varepsilon\left(V_{bi} - V\right)}{eN_D}}$$

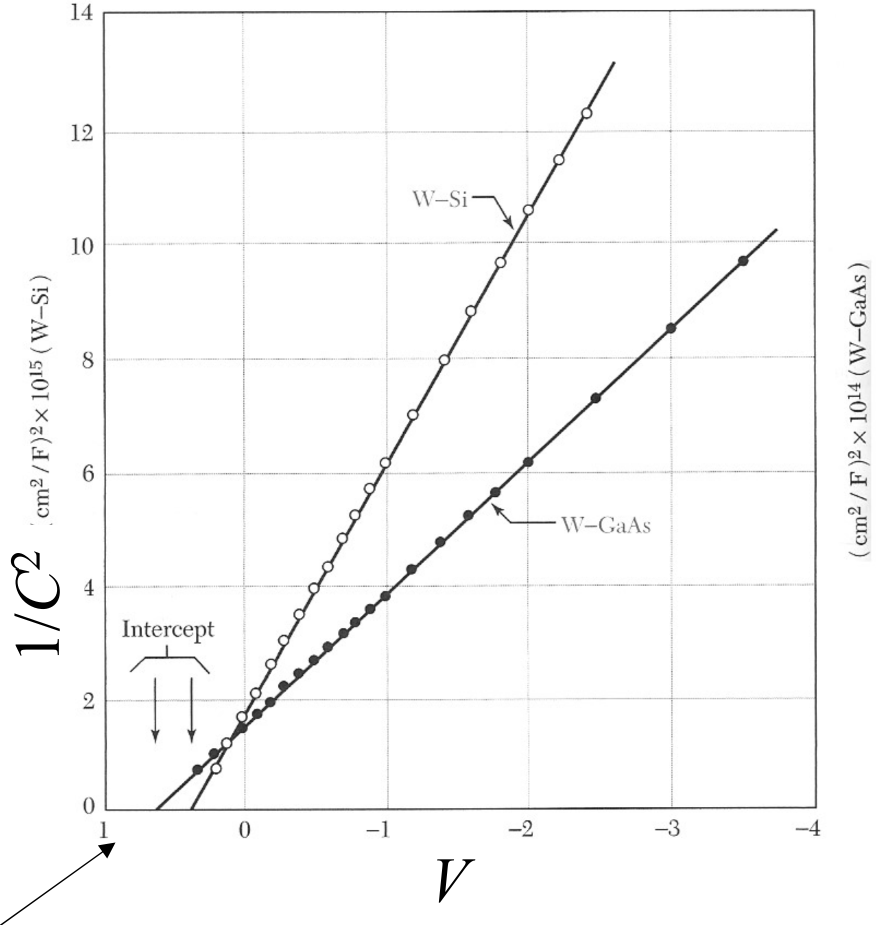
$$V = \frac{-eN_D}{\varepsilon}\left(\frac{x^2}{2} - x x_n\right)$$

CV measurements

$$x_p = \sqrt{\frac{2\varepsilon\left(V_{bi} - V\right)}{eN_A}}$$

$$C = \frac{\varepsilon}{x_p} = \sqrt{\frac{e\varepsilon N_A}{2\left(V_{bi} - V\right)}} \quad \text{F m}^{-2}$$

$$\frac{1}{C^2} = \frac{2\left(V_{bi} - V\right)}{e\varepsilon N_A}$$



GaAs has larger E_g and V_{bi}

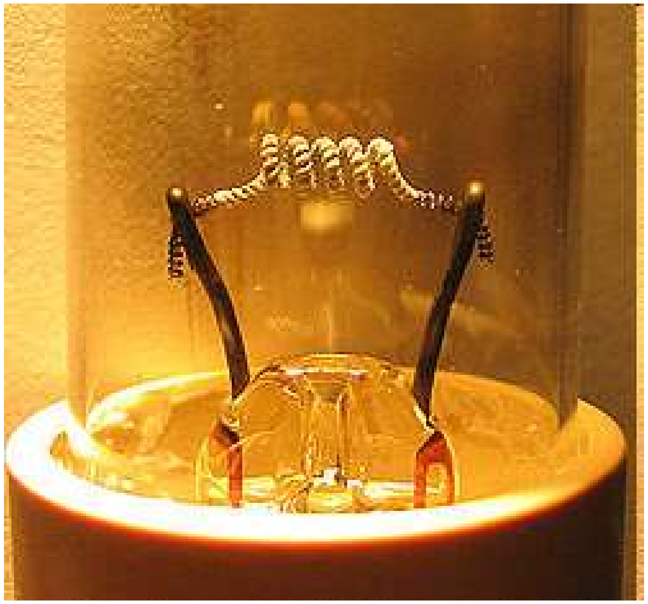
$$eV_{bi} = \phi_b - k_B T \ln\left(\frac{N_v(T)}{N_A}\right)$$

Thermionic emission

1901 Richardson

Owen Willans Richardson

Current from a heated wire is:

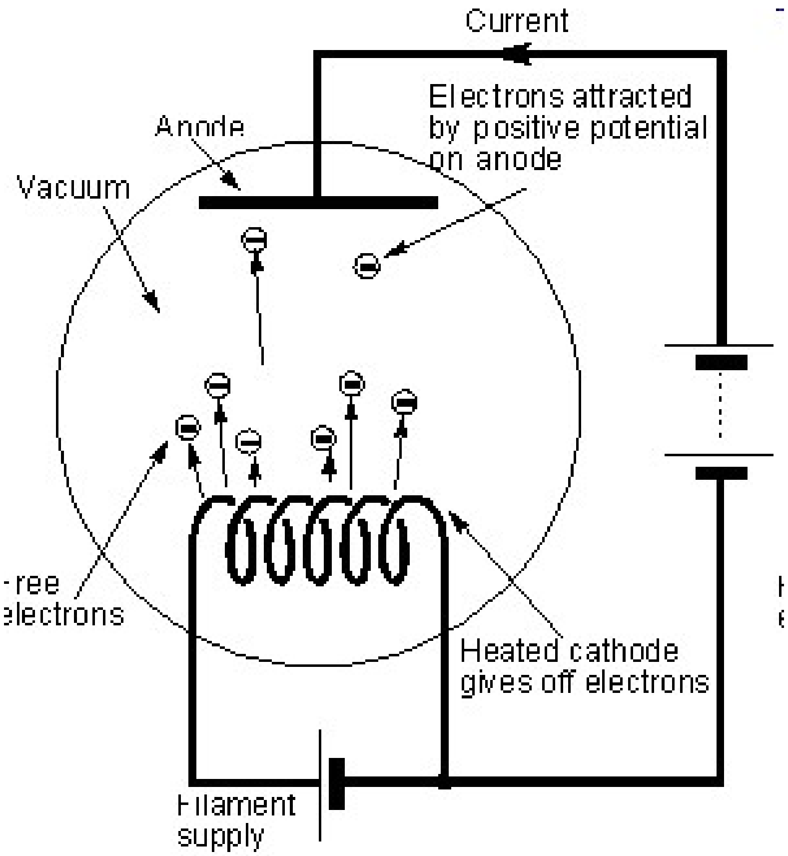


$$I = \frac{Aem^*k_B^2}{2\pi^2\hbar^3}T^2\exp\left(-\frac{\phi_b}{k_BT}\right)\left(\exp\left(\frac{eV}{k_BT}\right)-1\right)$$

Some electrons have a thermal energy that exceeds the work function and escape from the wire.

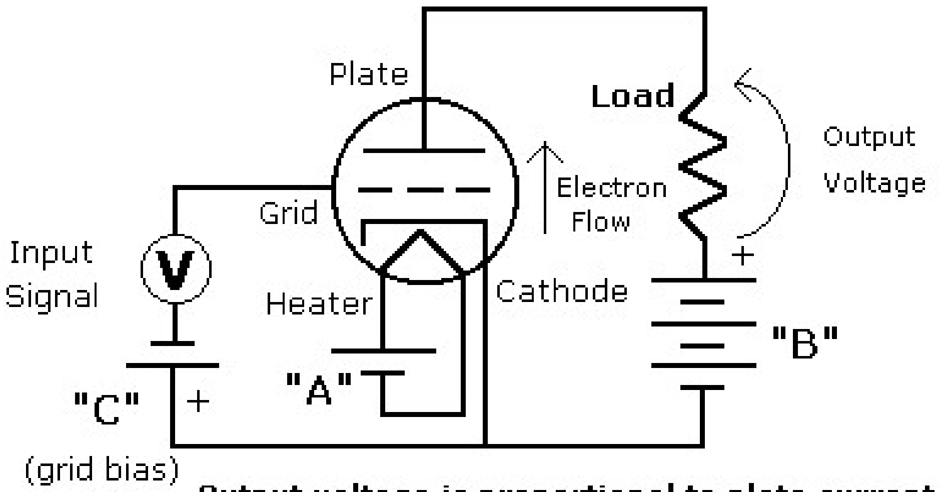
http://lampz.tugraz.at/~hadley/psd/L6/richardson.html

Vacuum diodes



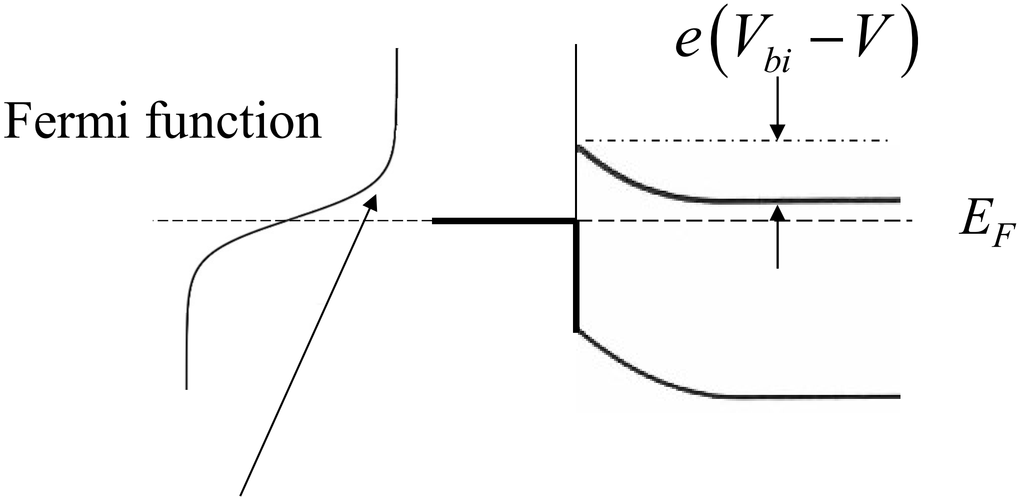
diode

The Common-cathode Triode Amplifier



Output voltage is proportional to plate current, which is controlled by grid voltage.

Thermionic emission



$$f(E) \approx \exp\left(\frac{E_F - E}{k_B T}\right) = \exp\left(\frac{E_F}{k_B T}\right)\exp\left(\frac{-E}{k_B T}\right) \propto \exp\left(\frac{-E}{k_B T}\right)$$

The density of electrons with enough energy to go over the barriers $\propto \exp\left(\frac{-E}{k_B T}\right)$

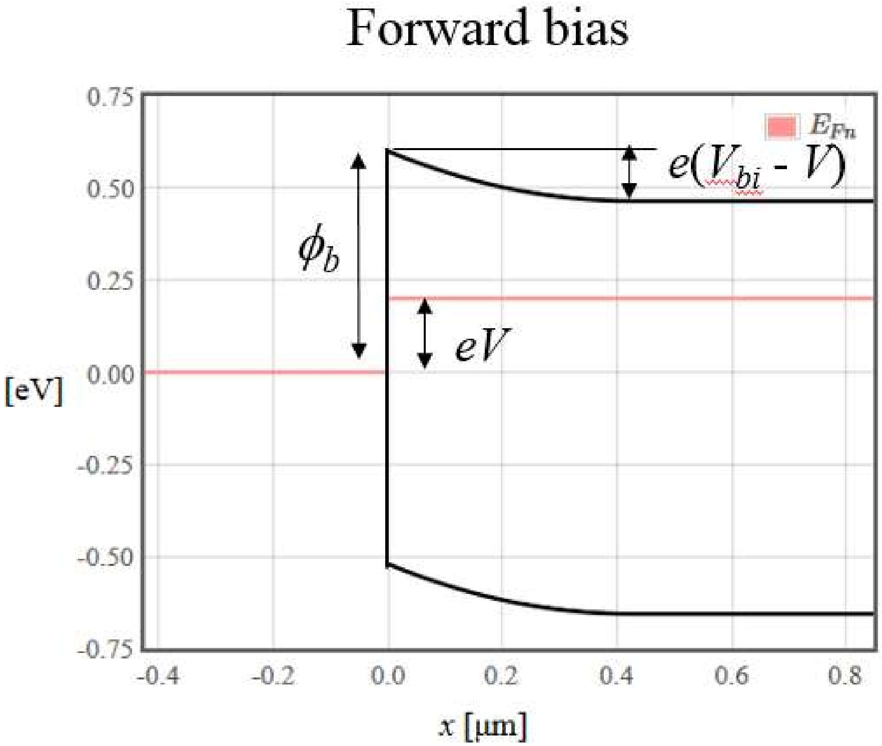
$$n_{th} \propto \exp\left(\frac{-E}{k_B T}\right) = \exp\left(\frac{-e\left(V_{bi}-V\right)}{k_B T}\right) = \exp\left(\frac{-eV_{bi}}{k_B T}\right)\exp\left(\frac{eV}{k_B T}\right)$$

Thermionic emission

$$n_{th} \propto \exp\left(\frac{eV}{k_B T}\right)$$

$$I_{sm} \propto n_{th} \propto \exp\left(\frac{eV}{k_B T}\right)$$

$$I_{ms} = I_{sm}(V=0)$$



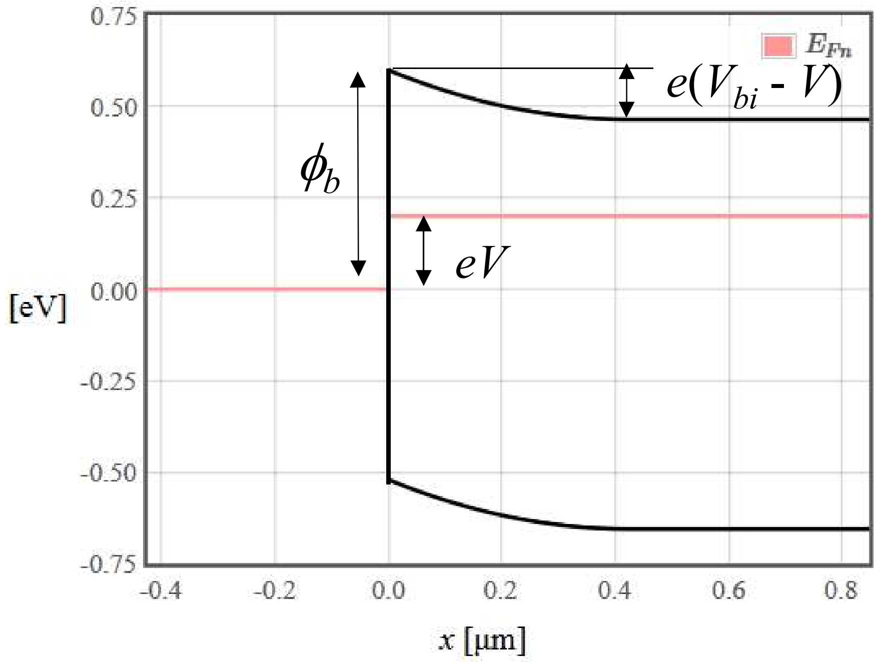
Forward bias

$$I = I_{sm} + I_{ms} = I_{ms}(T)\left(exp(\frac{eV}{k_B T}) - 1\right)$$

$$I = \frac{Aem^* k_B^2}{2\pi^2\hbar^3} T^2 \exp\left(-\frac{\phi_b}{k_B T}\right)\left(\exp\left(\frac{eV}{k_B T}\right) - 1\right)$$

Schottky barrier

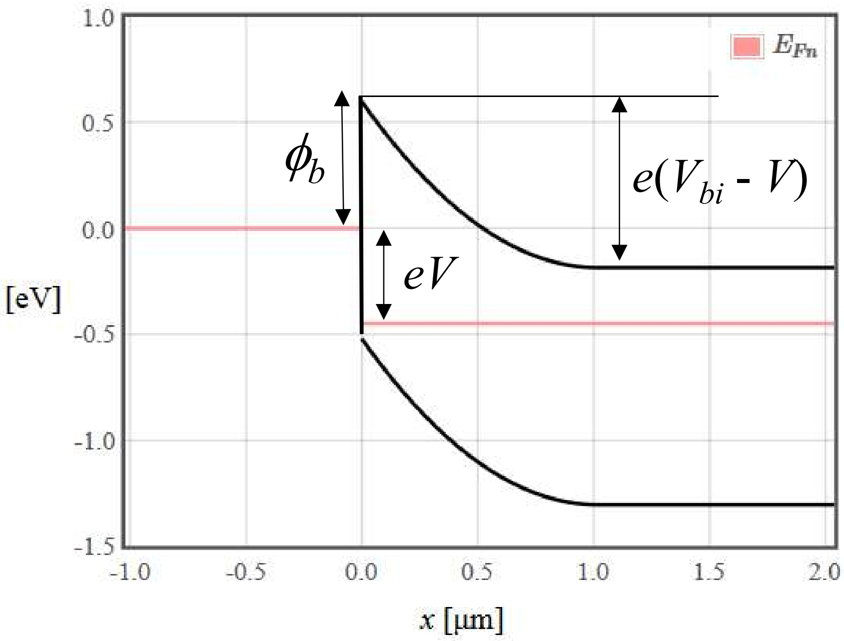
Forward bias



$I_{\text{sm}} \sim \exp(eV/k_BT)$

I_{ms} constant

Reverse bias



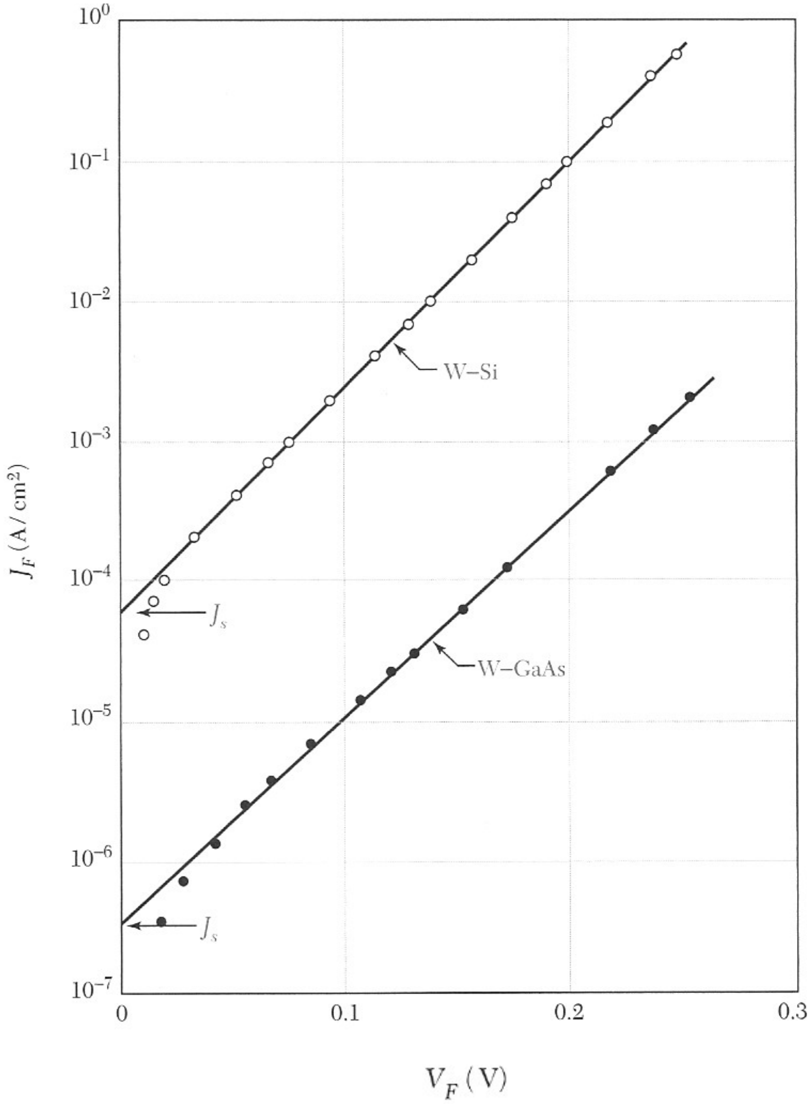
$I_{\text{sm}} \sim 0$

I_{ms} constant

Thermionic emission

$$I = I_{sm} + I_{ms} = I_s\left(e^{\frac{eV}{k_B T}} - 1\right)$$

Nonideality factor = 1



Schottky diodes

Majority carrier current dominates.

nonideality factor ~ 1.

Fast response, no recombination of electron-hole pairs required.

Used as rf mixers.

Low turn on voltage - high reverse bias current

$$I = I_s\left(e^{\frac{eV}{k_B T}} - 1 \right)$$