Superconductivity

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1 What is a type I and a type II superconductor?

The difference between type I and type II superconductors can be found in their magnetic behaviour.
A type I superconductor keeps out the whole magnetic field until a critical applied field H_c reached. Above that field a type I superconductor is no longer in its superconductiong state.
A type II superconductor will only keep the whole magnetic field out until a first critical field H_{c1} is reached. Then vortices start to appear. A vortex is a magnetic flux quantum that penetrates the superconductor. Where the vortex appears the superconducting order parameter drops to zero. In this region the metal is no longer a superconductor. Around the vortex a current starts to circulate. Even though the vortices have formed, the rest of the metal stays superconducting. If the field is increased to the second critical field H_{c2} the metal stops to be superconducting. H_{c2} is usually a lot bigger than H_c that's why type II superconductors are typically used for superconducting magnets.

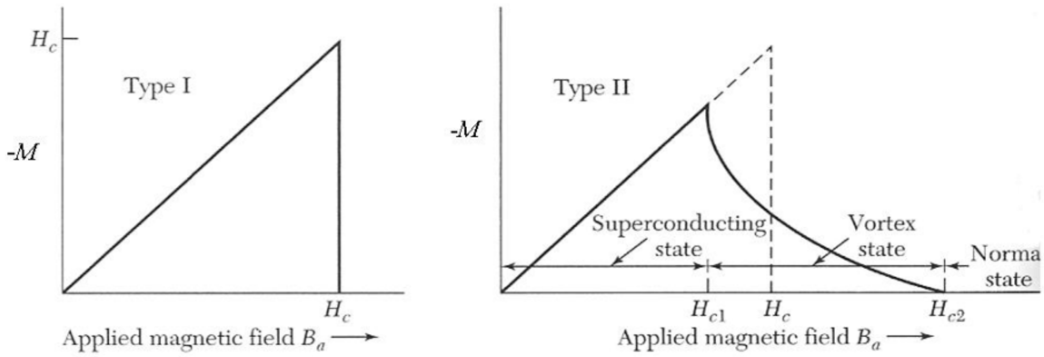

Abbildung 1: Magnetization of type I (left) and type II (right) superconductors depending on the applied field

Figure 1 shows the difference in the magnetic behaviour of type I and type II superconductors. As

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \tag{1}$$

$-\vec{M} = \vec{H}$ means that the whole field is kept out. If $-\vec{M} < \vec{H}$ not the whole field is kept out any more.

In a type I superconductor the coherence length ξ (length over which superconductivity changes) is bigger than the penetration depth λ ($\vec{B} = \vec{B}_0 * exp(x/\lambda)\vec{e}_z$).

In a type II superconductor the coherence length is shorter than the penetration depth. Then it is energetically favourable for vortices to form.

2 Draw the magnetic field and current density around a vortex

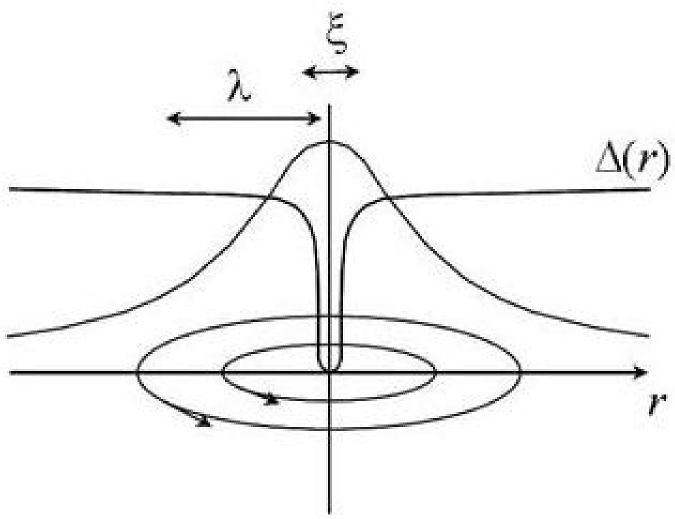

Abbildung 2: superconducting order parameter, amplitude of magnetic field and current circulating around flux quantum

Figure 2 shows what happens in the region of a vortex. At the vortex there is one magnetic flux quantum ($\phi_0 = \frac{h}{2e} \approx 2 * 10^{-15} Tm^2$) that enters the superconductor. Around the vortex superconducting currents are trying to keep the field out. The magnetic field decreases exponentially from the center of the vortex. In the center of the vortex the superconducting order parameter Δ goes to zero. This means that in this region the metal is no longer a superconductor.
You can also see that the coherence length ξ is shorter than the penetration depth λ. As explained above, this defines a type II superconductor and makes the formation of vortices favourable.

3 Explain flux quantization

The equation for a supercurrent can be calculated from the imaginary part of the Schrödinger equation for a charged particle in an electric and magnetic field.

This is the equation for the current density.

$$\vec{j} = \frac{-e\hbar n_{cp}}{m_e}(\nabla\Theta + \frac{2e}{\hbar}\vec{A}) \tag{2}$$

n_{cp} is the number of cooper pairs, so half of the number of electrons and Θ is the phase.

We now take a superconducting ring, which is thicker than the penetration depth. If you choose a path through the middle of the ring (the distance to any edge should be bigger than the penetration depth) there should be no field and no current along the path. So $\vec{j} = 0$ and therfore $\nabla\Theta + \frac{2e}{\hbar}\vec{A} = 0$ and $\nabla\Theta = -\frac{2e}{\hbar}\vec{A}$.

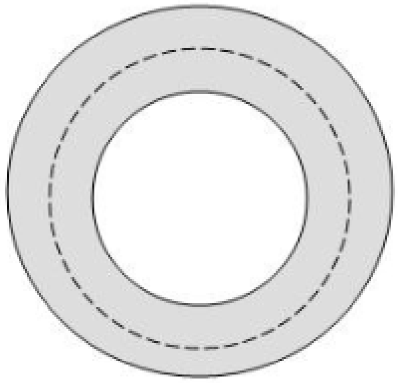

Abbildung 3: path through the superconducting ring

We now integrate both sides of the equation along the dotted path:

$$\oint \nabla\Theta d\vec{l} = -\frac{2e}{\hbar}\oint \vec{A}d\vec{l} \tag{3}$$

We can then use Stokes' theorem to get

$$\oint \nabla\Theta d\vec{l} = -\frac{2e}{\hbar}\int_S \nabla \times \vec{A}d\vec{s} = -\frac{2e}{\hbar}\int_S \vec{B}d\vec{s} = -\frac{2e}{\hbar}\Phi \tag{4}$$

where S is the area enclosed by the dotted path and Φ is the magnetic flux.
On the left you can see a circular integral over a phase. When you integrate over a ring like this the phase should be the same after one lap as when you started (except for a factor of 2π, or a multiple of 2π). So you get

$$\oint \nabla\Theta d\vec{l} = 2\pi n = -\frac{2e}{\hbar}\Phi \tag{5}$$

where n is a whole number positve or negative. We can transfer the negative sign into the n and get

$$2\pi n = \frac{2e}{\hbar}\Phi = \frac{\Phi}{\Phi_0} \tag{6}$$

So the magnetic flux in a superconductor can only appear in whole number portions of the superconducting flux quantum Φ_0 and therefore is quantized:

$$\Phi = n\Phi_0 \tag{7}$$

And the flux quantum equals:

$$\Phi_0 = \frac{2\pi\hbar}{2e} = \frac{h}{2e} = 2.0679 * 10^{-15} Tm^2 \tag{8}$$