Magnon dispersion

The dispersion relation for ferromagnetic magnons in one dimension is,

\[ \hbar \omega = 4J |S| \left(1 - \cos(k_0)\right) \]

Here \( J \) is the exchange energy and \( S \) is the magnetic moment of the spins.

a) Draw the dispersion relation and the density of states.

b) Draw (approximately) the specific heat as a function of temperature. Is the specific heat more like the specific heat for photons or the specific heat for phonons? Why?

c) How could you observe the magnons experimentally?
Solution

a) Draw the dispersion relation and the density of states:

The dispersion relation for magnons is very similar to the dispersion relation for electrons in the tight binding model.

Magnon dispersion relation: \( E = 4J|S|(1 - \cos(ka)) \)

Tight binding dispersion relation: \( E = \epsilon - 2t \cos(ka) \)

Comparing these two equations, one finds \( \epsilon = 4J|S| \) and \( t = 2J|S| \). The dispersion relation and the density of states of magnons have the same form as for electrons in the tight binding model:

![Figure 1: Dispersion relation and DOS for magnons](image)

b) Draw (approximately) the specific heat as a function of temperature. Is the specific heat more like the specific heat for photons or the specific heat for phonons? Why?

Electrons: \( c_v \propto T \)

Phonons: \( c_v \propto T^2 \)

Magnons: \( c_v \propto T^{3/2} \)

![Figure 2: Specific heat for electrons, phonons and magnons](image)
The specific heat of magnons is more phonon like although the dispersion relation and the DOS is similar to electrons in the tight binding model.

The dispersion relation in the low temperature limit has a parabolic form $E \approx 2JSk^2a^2$, this results in a density of states $D(\omega) \propto \sqrt{\omega}$, which is the same as for free electrons in 3 dimensions. To calculate the specific heat, the total internal energy is needed:

$$u = \int_0^\infty \hbar\omega D(\omega)(\langle n(\omega)\rangle) d\omega$$

$$\langle n(E)\rangle = \frac{1}{e^{\frac{E}{k_BT}} - 1}$$

Magnons are bosons. The occupation probability of non-interacting bosons is given by the bose-einstein factor $\langle n(E)\rangle$. This leads to:

$$u = \int_0^\infty \hbar\omega D(\omega) \frac{1}{\hbar\omega e^{\frac{\hbar\omega}{k_BT}} - 1} d\omega \propto T^{5/2} \rightarrow \frac{du}{dT} \propto T^{3/2}$$

The reason that the specific heat of magnons is more phonon like is, that magnons are bosons and the occupation probability which is needed to get the thermodynamic properties is given by the bose-einstein factor. This results in a different internal energy and consequently in a more phonon like specific heat.

c) How could you observe the magnons experimentally?

Magnons can experimentally be observed via neutron scattering. Neutrons have a spin, so they can interact with magnons and create or annihilate them, when they scatter inelastically from a magnetic material, by loosing or gaining some energy. The scattering process has to fullfill the conservation of momentum:

$$\vec{k}_n = \vec{k}'_n + \vec{k}_{magnon} + \vec{G}$$

The wavevector of the incoming neutron $\vec{k}_n$ must be equal to the wavevector of the scattered neutron $\vec{k}'_n$ plus the wavevector of the magnon $\vec{k}_{magnon}$ plus some reciprocal lattice vector $\vec{G}$. If this condition is fullfilled a peak can be measured, which is near a stronger peak. The stronger peak comes from elastic scattering at which the laue condition ($\vec{k}_n = \vec{k}'_n + \vec{G}$) is fulfiled. The position of the weaker peak gives the k-vector of the magnon. To get the dispersion relation the energy of the magnon has to be analyzed. At first glance one doesn’t know if the measured peak comes from a phonon or a magnon, because the used technique is in both cases the same. It is possible to find out which contribution comes from a phonon and which from a magnon, by comparing the data points to a model. Near $k=0$ for phonons one would expect a dispersion relation $E(k) \propto k$ and for magnons $E(k) \propto k^2$. The datapoints usually are far enough apart, so it is possible to select datapoints and check with which model they fit and thus assign the contributions to the phonons and magnons.