Superconductivity

Superconductivity

Primary characteristic: zero resistance at dc

There is a critical temperature T_c above which superconductivity disappears

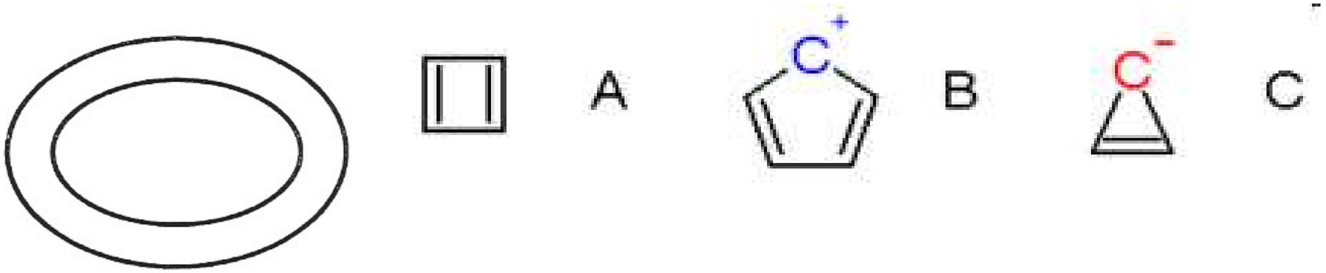
About 1/3 of all metals are superconductors

Metals are usually superconductors OR magnetic, not both

Good conductors are bad superconductors

Kittel chapter 10

Superconductivity



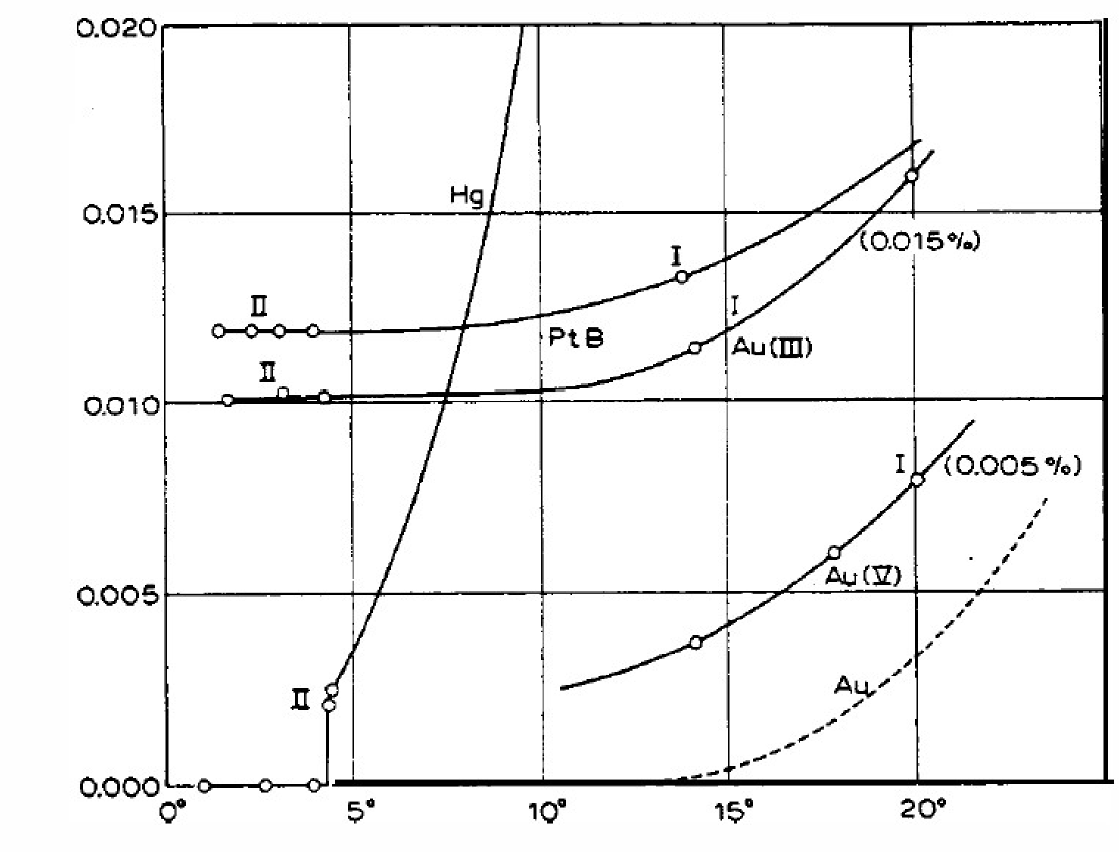
Superconducting ring

Molecule with magnetic moment

Antiaromatic molecules are unstable and highly reactive

No measurable decay in current after 2.5 years. $\rho < 10^{-25}$ Ωm.

Superconductivity

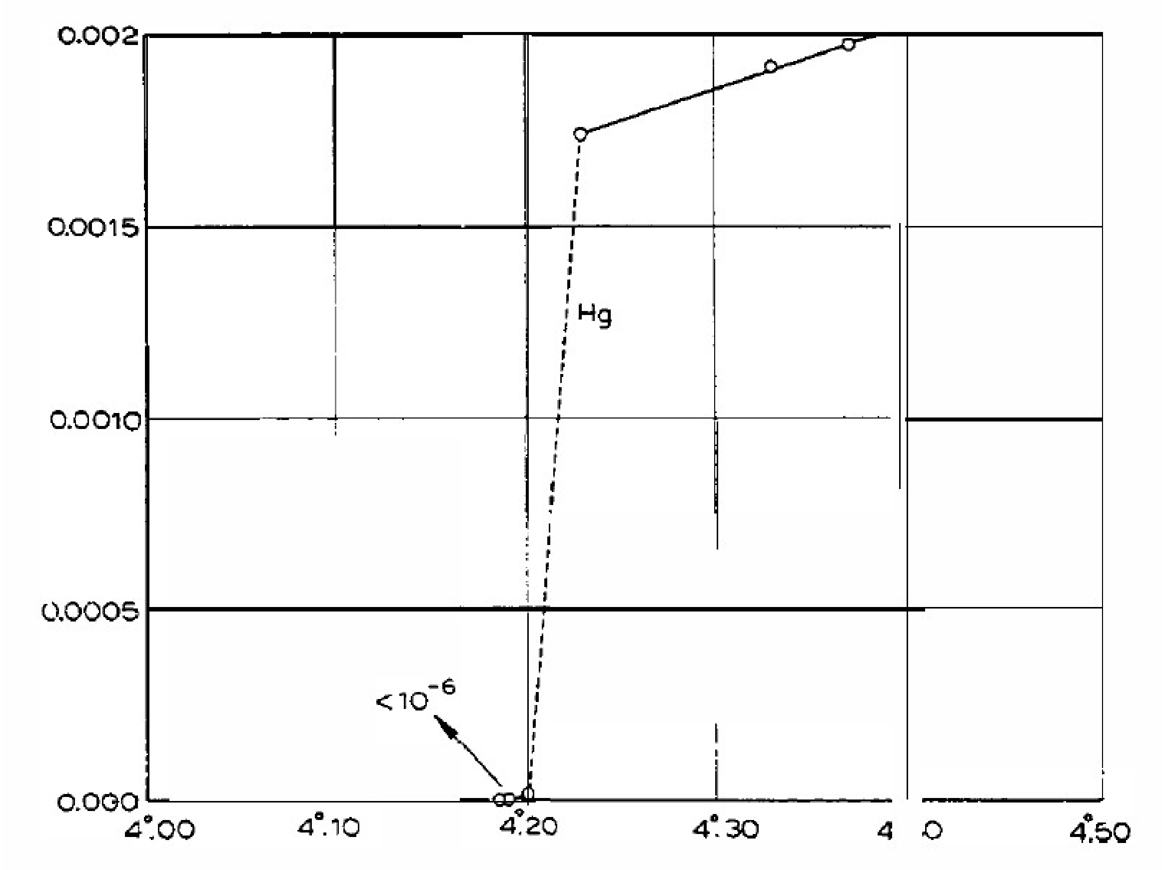


Nobel Lecture 1913

Heike Kammerling-Onnes

Superconductivity was discovered in 1911

Superconductivity

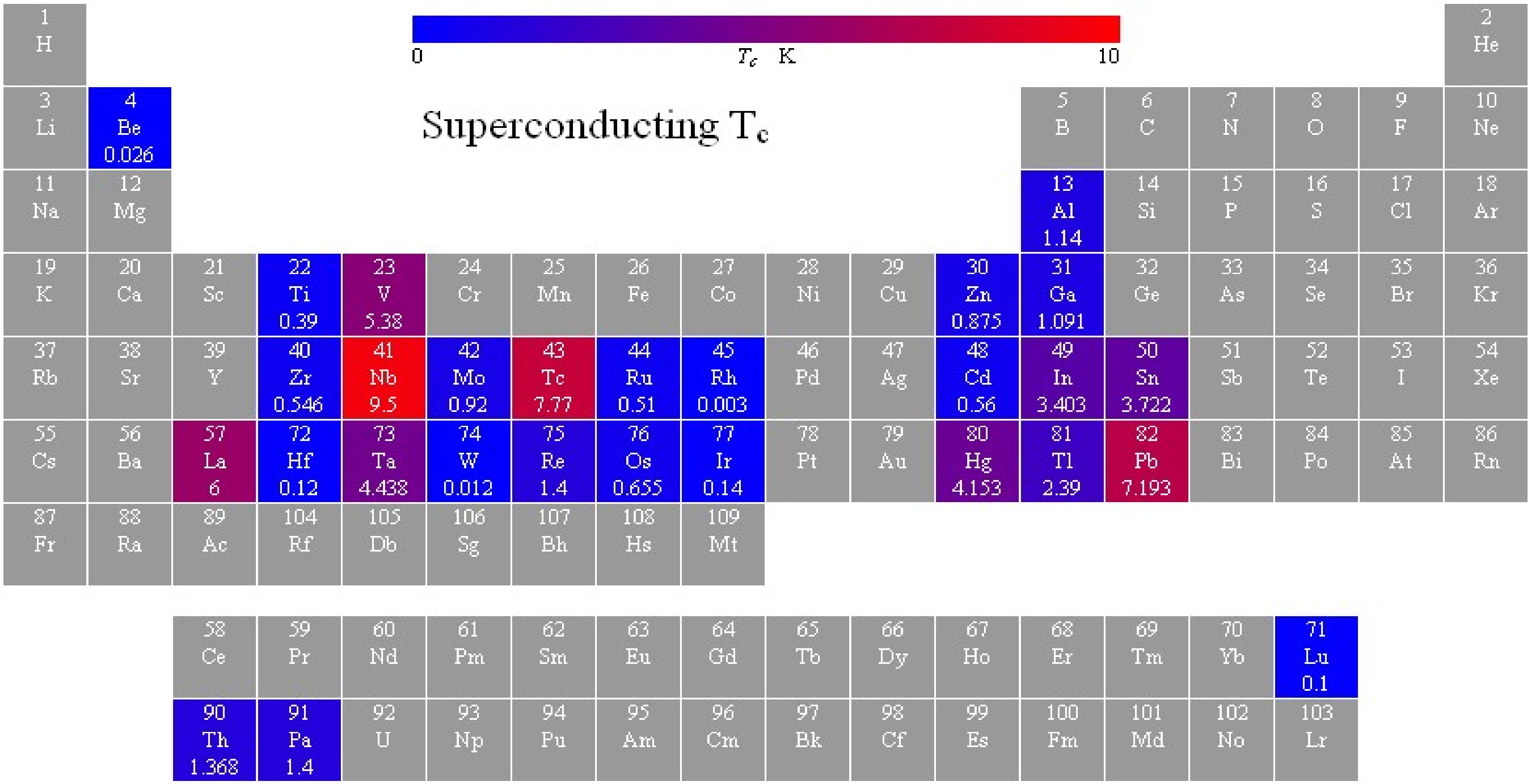


Nobel Lecture 1913

Heike Kammerling-Onnes

Superconductivity was discovered in 1911

Critical temperature

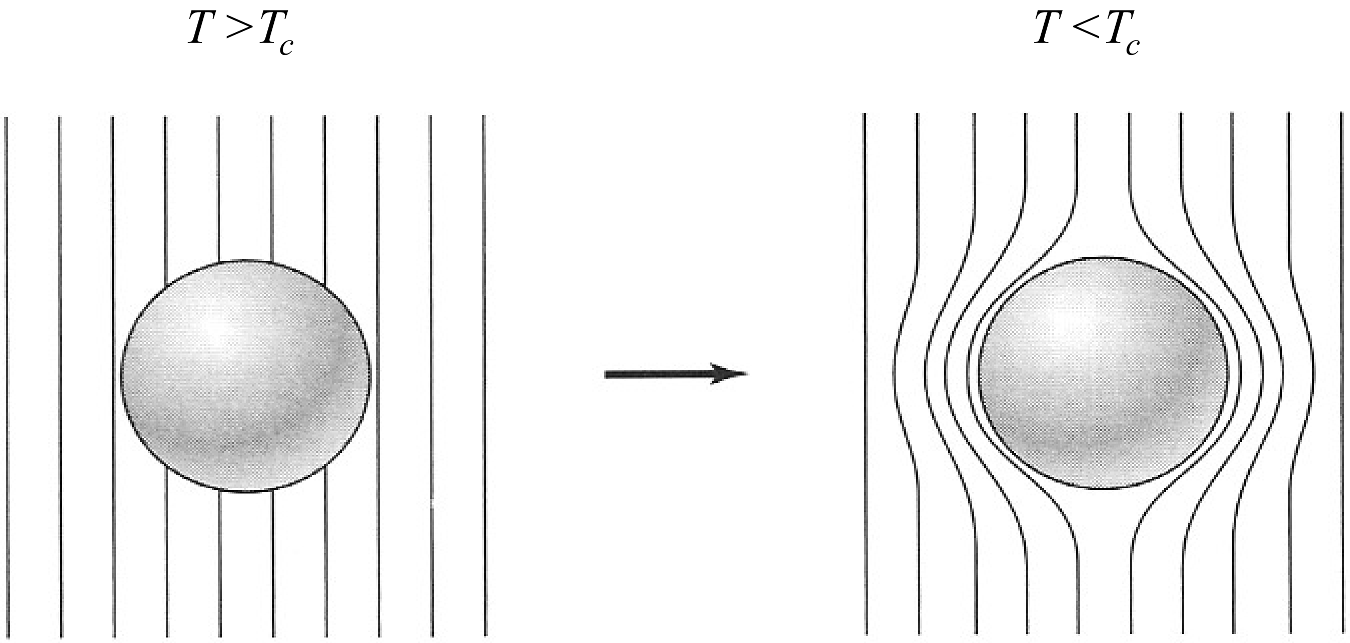


0 T_c K 10

Superconducting T_c

1 H																	2 He
3 Li	4 Be 0.026											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al 1.14	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti 0.39	23 V 5.38	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn 0.875	31 Ga 1.091	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr 0.546	41 Nb 9.5	42 Mo 0.92	43 Tc 7.77	44 Ru 0.51	45 Rh 0.003	46 Pd	47 Ag	48 Cd 0.56	49 In 3.403	50 Sn 3.722	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La 6	72 Hf 0.12	73 Ta 4.438	74 W 0.012	75 Re 1.4	76 Os 0.655	77 Ir 0.14	78 Pt	79 Au	80 Hg 4.153	81 Tl 2.39	82 Pb 7.193	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt									
		58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu 0.1		
		90 Th 1.368	91 Pa 1.4	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr		

Meissner effect



Superconductors are perfect diamagnets at low fields.
$B = 0$ inside a bulk superconductor.

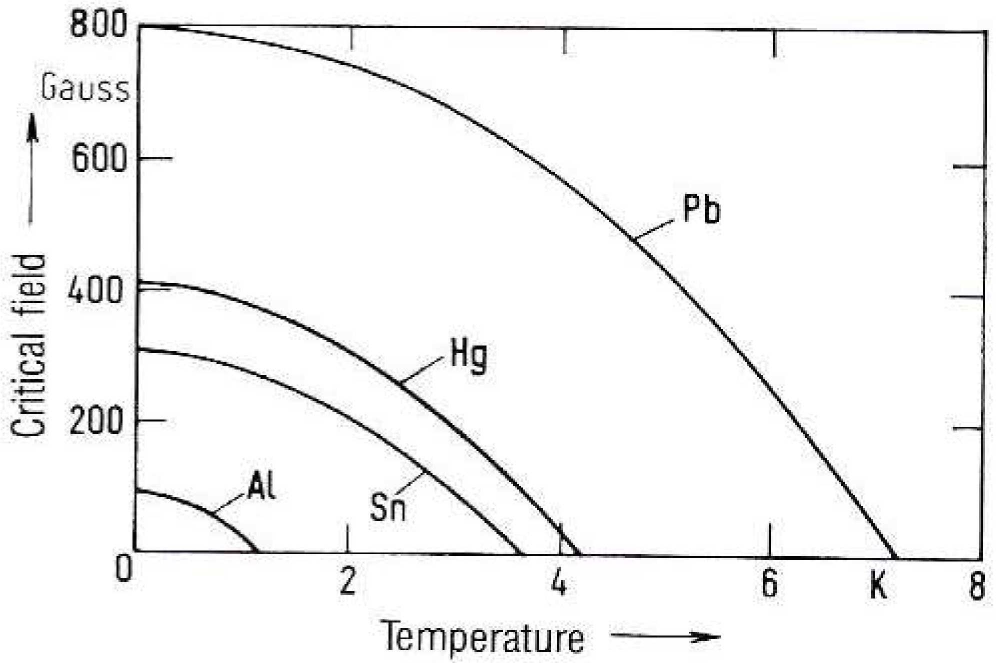
Superconductors are used for magnetic shielding.

Superconductivity

Critical temperature T_c

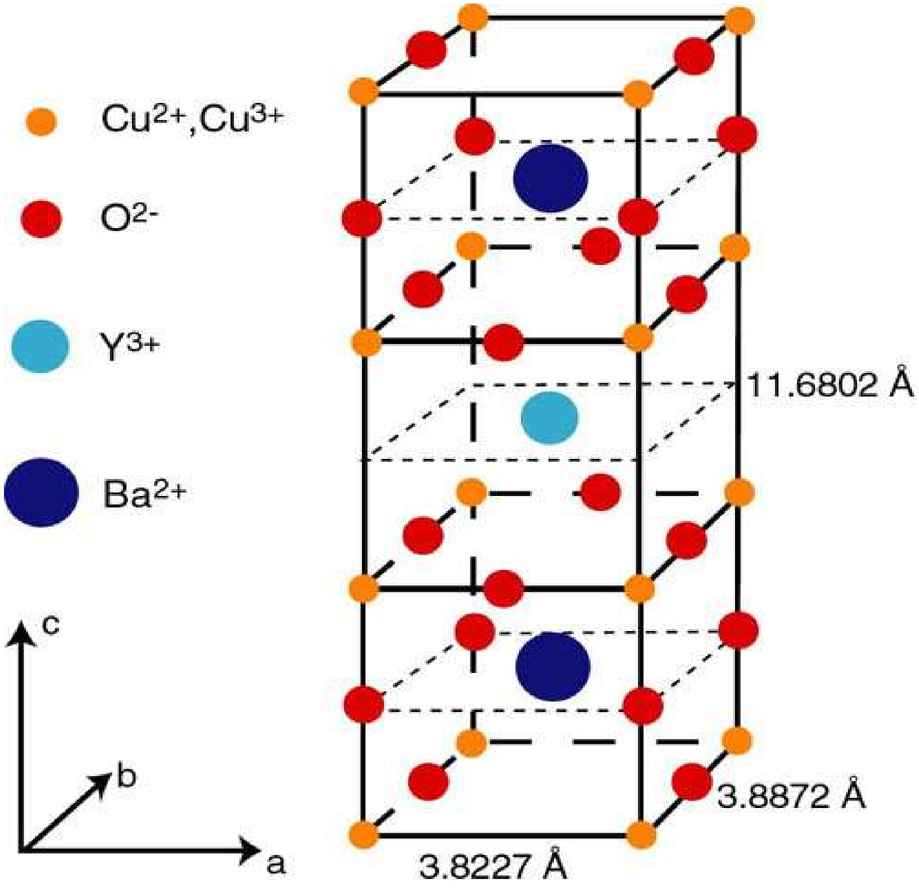
Critical current density J_c

Critical field H_c

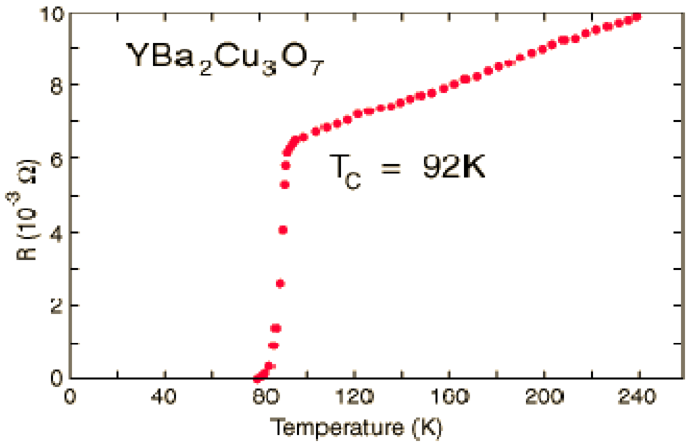


$$n\Delta \approx nk_BT_c \approx \mu_0 H_c^2 \approx \tfrac{1}{2} nmv^2 = \frac{m}{2ne^2} J_c^2$$

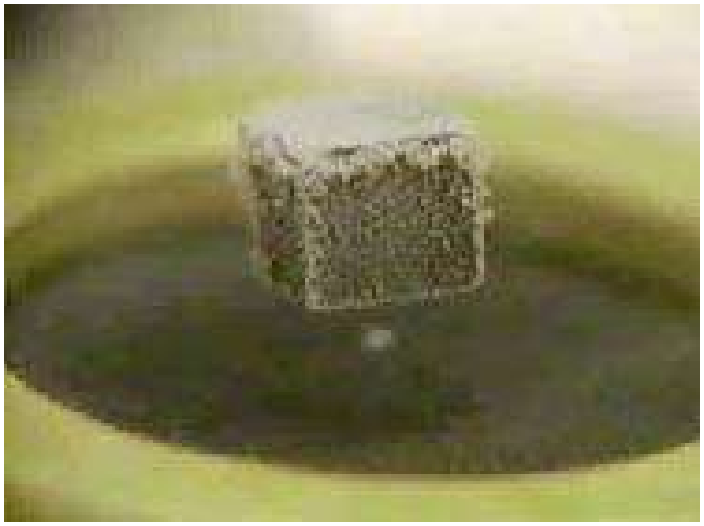
$YBa_2Cu_3O_x$



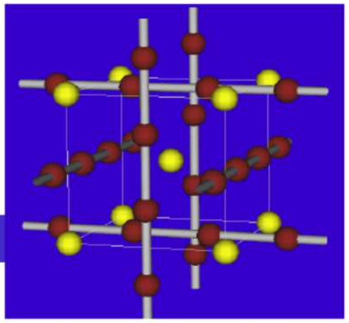
Cu^{2+}, Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

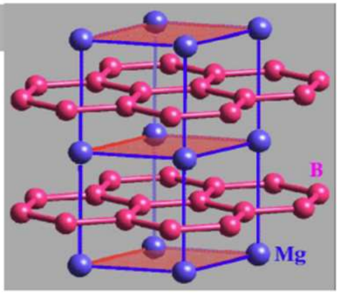


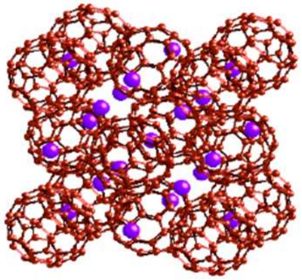
$YBa_2Cu_3O_7$
T_C = 92K
R (10^{-3} Ω)
0
2
4
6
8
10
0
40
80
120
160
200
240
Temperature (K)



	Material	T_c
• Legierung:	NbTi	9,6 K
• Verbindungen:	NbN	16,0 K
Borocarbide:	(Lu/Y)Ni$_2$B$_2$C	16,0 K
"A15"-Strukturen:	Nb$_3$Sn	18,0 K
(= β-Wolfram-Struktur)	Nb$_3$Al	18,7 K
	Nb$_3$Ge	22,5 K
neu:	MgB$_2$	39 K
Fullerene:	Cs$_2$RbC$_{60}$	33 K
+ Druck 15 kbar:	Cs$_3$C$_{60}$	40 K

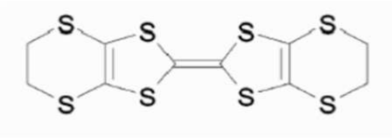


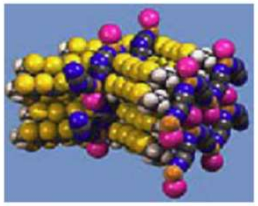




Organische Supraleiter:

(BEDT-TTF)$_2$Cu[N(CN)$_2$]Br 11,2 K

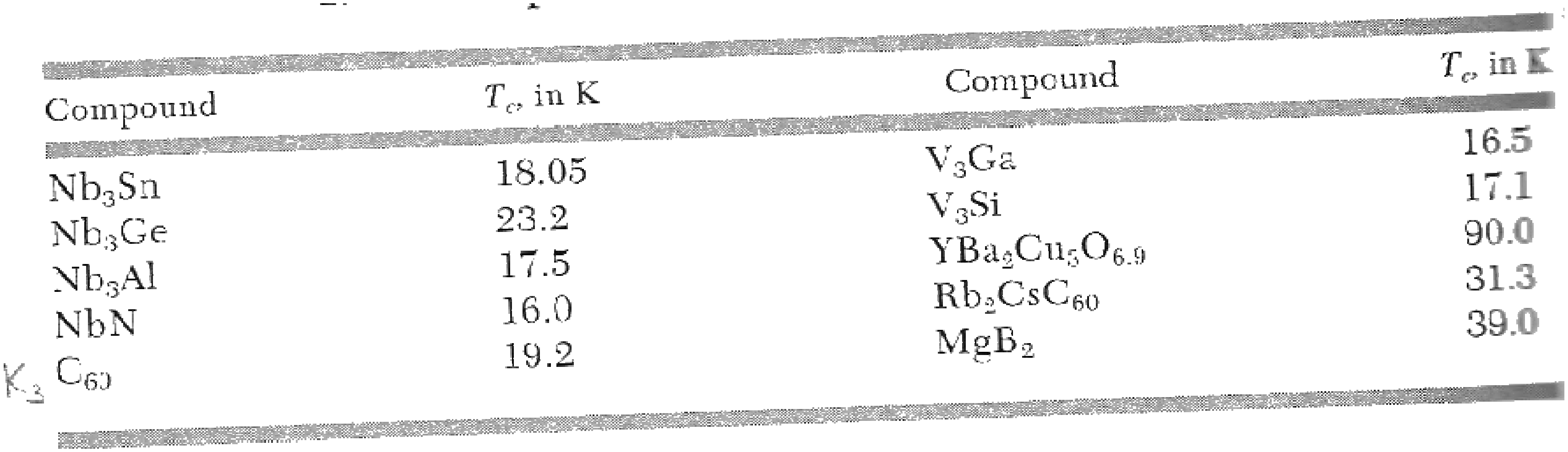




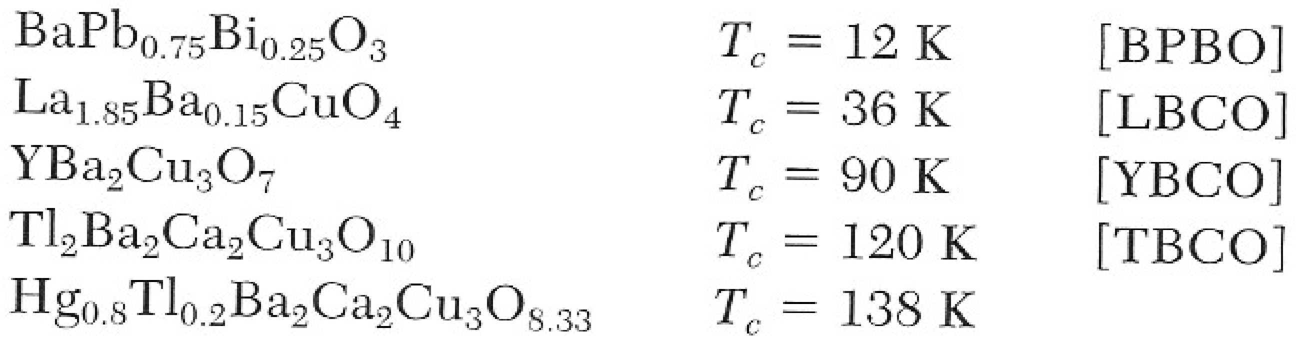
Polymere

hochdotierte Halbleiter

http://www.wmi.badw.de/teaching/Lecturenotes/index.html



Compound	T_c, in K	Compound	T_c, in K
Nb_3Sn	18.05	V_3Ga	16.5
Nb_3Ge	23.2	V_3Si	17.1
Nb_3Al	17.5	$YBa_2Cu_3O_{6.9}$	90.0
NbN	16.0	Rb_2CsC_{60}	31.3
K_3 C_{60}	19.2	MgB_2	39.0



$BaPb_{0.75}Bi_{0.25}O_3$	$T_c = 12$ K	[BPBO]
$La_{1.85}Ba_{0.15}CuO_4$	$T_c = 36$ K	[LBCO]
$YBa_2Cu_3O_7$	$T_c = 90$ K	[YBCO]
$Tl_2Ba_2Ca_2Cu_3O_{10}$	$T_c = 120$ K	[TBCO]
$Hg_{0.8}Tl_{0.2}Ba_2Ca_2Cu_3O_{8.33}$	$T_c = 138$ K	
$(Sn_5In)Ba_4Ca_2Cu_{10}O_y$	$T_c = 212$ K	

Superconductivity

Perfect diamagnetism

Jump in the specific heat like a 2nd order phase transition, not a structural transition

Superconductors are good electrical conductors but poor thermal conductors, electrons no longer conduct heat

There is a dramatic decrease of acoustic attenuation at the phase transition, no electron-phonon scattering

Dissipationless currents - quantum effect

Electrons condense into a single quantum state - low entropy.

Electron decrease their energy by Δ but loose their entropy.

Probability current

Schrödinger equation for a charged particle in an electric and magnetic field is

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{1}{2m}(-i\hbar\nabla-qA)^2\psi+V\psi$$

write out the $(-i\hbar\nabla-qA)^2\psi$ term

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{1}{2m}\left(-\hbar^2\nabla^2+i\hbar qA\nabla+ihq\nabla A+q^2A^2\right)\psi+V\psi$$

write the wave function in polar form

$$\psi=|\psi|e^{i\theta}$$

$$\nabla\psi=\nabla|\psi|e^{i\theta}+i\nabla\theta|\psi|e^{i\theta}$$

$$\nabla^2\psi=\nabla^2|\psi|e^{i\theta}+2i\nabla\theta\nabla|\psi|e^{i\theta}+i\nabla^2\theta|\psi|e^{i\theta}-\left(\nabla\theta\right)^2|\psi|e^{i\theta}$$

Probability current

Schrödinger equation becomes:

$$i\hbar \frac{\partial |\psi|}{\partial t} - \hbar |\psi| \frac{\partial \theta}{\partial t} = \frac{1}{2m}\Big[\text{-}\hbar^2 \left(\nabla^2 |\psi| + 2i\nabla\theta\nabla|\psi| + i\nabla^2\theta|\psi| - (\nabla\theta)^2 |\psi| \right)$$
$$+i\hbar qA\left(\nabla|\psi| + i\nabla\theta|\psi|\right) + ihq\nabla A|\psi| + q^2A^2|\psi|\Big] + V|\psi|$$

Real part:

$$-\hbar|\psi|\frac{\partial\theta}{\partial t} = \frac{\text{-}\hbar^2}{2m}\left(\nabla^2 - \left(\nabla\theta - \frac{q}{\hbar}\vec{A} \right)^2 \right)|\psi| + V|\psi|$$

Imaginary part:

$$\hbar\frac{\partial|\psi|}{\partial t} = \frac{1}{2m}\left[\text{-}\hbar^2\left(2\nabla\theta\nabla|\psi| + i\nabla^2\theta|\psi| - (\nabla\theta)^2|\psi| \right) + 2\hbar qA\nabla|\psi| + \hbar q|\psi|\nabla A \right]$$

Probability current

Imaginary part:

$$\hbar \frac{\partial |\psi|}{\partial t} = \frac{1}{2m}\left[\text{-}\hbar^2 \left(2\nabla\theta\nabla|\psi| + i\nabla^2\theta|\psi| - (\nabla\theta)^2 |\psi| \right) + 2\hbar q A \nabla |\psi| + \hbar q |\psi| \nabla A \right]$$

Multiply by |ψ| and rearrange

$$\frac{\partial}{\partial t}|\psi|^2 + \nabla\cdot\left[\frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)\right] = 0$$

This is a continuity equation for probability

$$\frac{\partial P}{\partial t} + \nabla\cdot\vec{S} = 0$$

The probability current: $\vec{S} = \frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)$

Probability current / supercurrent

The probability current: $\vec{S} = \frac{\hbar}{m}|\psi|^2\left(\nabla\theta - \frac{q}{\hbar}\vec{A}\right)$

This result holds for all charged particles in a magnetic field.

In superconductivity the particles are Cooper pairs $q = -2e$, $m = 2m_e$, $|\psi|^2 = n_{cp}$.

All superconducting electrons are in the same state so

$$\vec{j} = -2en_{cp}\vec{S}$$

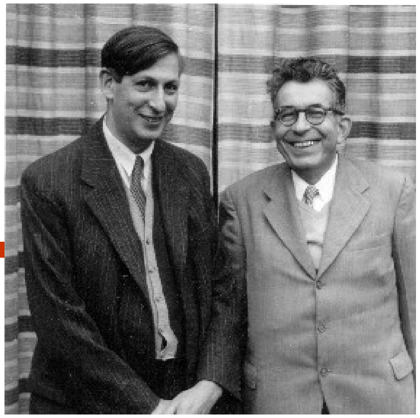
$$\boxed{\vec{j} = \frac{-e\hbar n_{cp}}{m_e}\left(\nabla\theta + \frac{2e}{\hbar}\vec{A}\right)}$$

London gauge $\nabla\theta = 0$

$$\vec{j} = \frac{-2n_{cp}e^2}{m_e}\vec{A} = \frac{-n_s e^2}{m_e}\vec{A}$$

$n_s = 2n_{cp}$

1st London equation


Heinz & Fritz

$$\vec{j} = \frac{-n_s e^2}{m_e} \vec{A}$$

$$\frac{d\vec{j}}{dt} = \frac{-n_s e^2}{m_e} \frac{d\vec{A}}{dt} = \frac{n_s e^2}{m_e} \vec{E}$$

$$\frac{d\vec{A}}{dt} = -\vec{E}$$

First London equation:

$$\boxed{\frac{d\vec{j}}{dt} = \frac{n_s e^2}{m_e} \vec{E}}$$

Classical derivation:

$$-e\vec{E} = m \frac{d\vec{v}}{dt} = -\frac{m}{n_s e} \frac{d\vec{j}}{dt}$$

$$\frac{d\vec{j}}{dt} = \frac{n_s e^2}{m_e} \vec{E}$$

2nd London equation

$$\vec{j} = \frac{-n_s e^2}{m_e} \vec{A}$$

$$\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \nabla \times \vec{A}$$

Second London equation: $$\boxed{\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \vec{B}}$$

Meissner effect

Combine second London equation with Ampere's law

$$\nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \vec{B} \qquad \nabla \times \vec{B} = \mu_0 \vec{j}$$

$$\nabla \times \nabla \times \vec{B} = \frac{-n_s e^2 \mu_0}{m_e} \vec{B}$$

$$\nabla \times \nabla \times \vec{B} = \nabla \left(\nabla \cdot \vec{B} \right) - \nabla^2 \vec{B}$$

Helmholtz equation: $\lambda^2 \nabla^2 \vec{B} = \vec{B}$

London penetration depth: $\lambda = \sqrt{\dfrac{n_s e^2 \mu_0}{m_e}}$

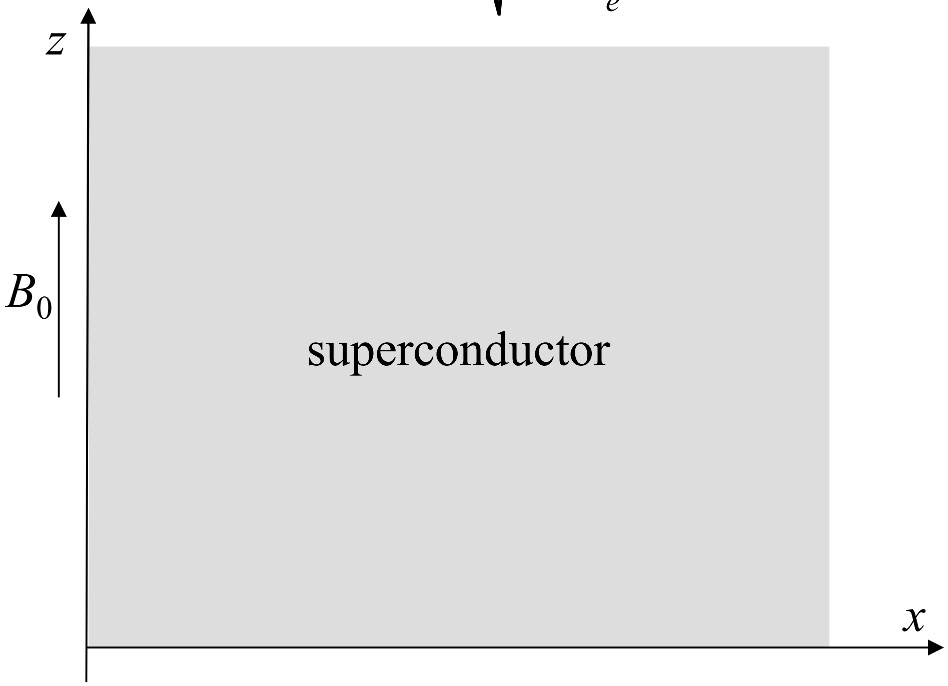
Meissner effect

$$\lambda^2 \nabla^2 \vec{B} = \vec{B} \qquad \lambda = \sqrt{\frac{n_s e^2 \mu_0}{m_e}}$$

solution to Helmholtz equation:

$$\vec{B} = \vec{B}_0 \exp\left(\frac{-x}{\lambda}\right)\hat{z}$$

Al	λ = 50 nm
In	λ = 65 nm
Sn	λ = 50 nm
Pb	λ = 40 nm
Nb	λ = 85 nm



$$\nabla \times \vec{B} = \mu_0 \vec{j} \qquad \vec{j} = \frac{\vec{B}_0}{\mu_0 \lambda}\exp\left(\frac{-x}{\lambda}\right)\hat{y}$$