Quasiparticles

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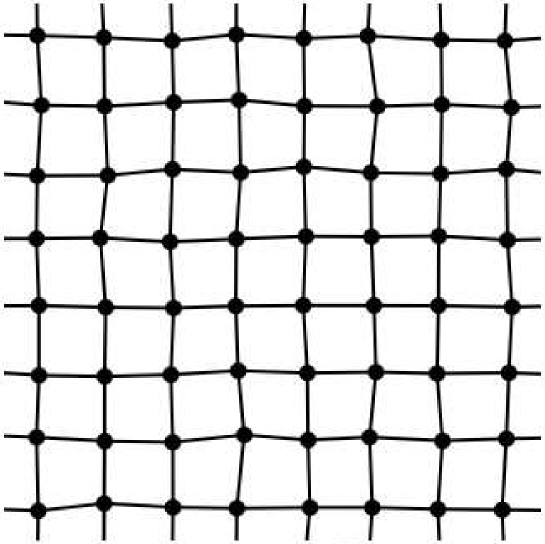
Excitations from equilibrium

phonons
plasmons
magnons
polaritons
(electrons)
Landau's theory of a Fermi liquid

polarons
excitons

Normal Modes and Phonons

At finite temperatures, the atoms in a crystal vibrate. In the simulation below, the atoms move randomly around their equilibrium positions.

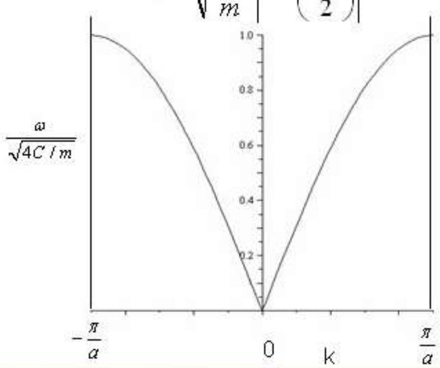
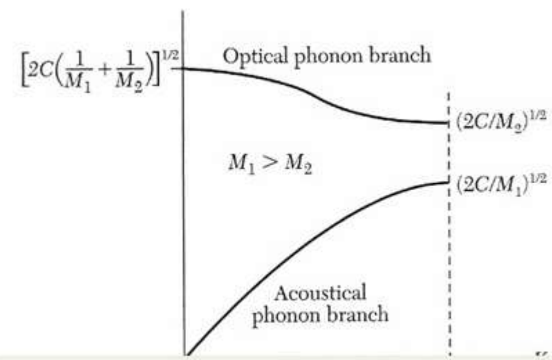


T = 300 K - +

Quasiparticle recipe

1. Write down the classical equations of motion.
2. Linearize the forces.
3. Insert the eigen functions of the translation operator into the linearized equations.
4. The resulting equation is the ω vs. k dispersion relation.

http://lamp.tu-graz.ac.at/~hadley/ss1/phonons/phonontable.html

	Linear Chain $m\frac{d^2u_s}{dt^2} = C(u_{s+1} - 2u_s + u_{s-1})$	**Linear chain 2 masses** $M_1\frac{d^2u_s}{dt^2} = C(v_{s-1} - 2u_s + v_s)$ $M_2\frac{d^2v_s}{dt^2} = C(u_s - 2v_s + u_{s+1})$	**Linear chain 2 spring constants** $M\frac{d^2u_s}{dt^2} = C_1(v_{s-1} - 2u_s + v_s)$ $M\frac{d^2v_s}{dt^2} = C_2(u_s - 2v_s + u_{s+1})$	n…
Eigenfunction solutions	$u_s = A_k e^{i(ksa-\omega t)}$	$u_s = ue^{i(ksa-\omega t)}$ $v_s = ve^{i(ksa-\omega t)}$	$u_s = ue^{i(ksa-\omega t)}$ $v_s = ve^{i(ksa-\omega t)}$	
Dispersion relation	$\omega = \sqrt{\frac{4C}{m}}\left\|\sin\left(\frac{ka}{2}\right)\right\|$ 	$\omega^2 = C\left(\frac{1}{M_1}+\frac{1}{M_2}\right) \pm C\sqrt{\left(\frac{1}{M_1}+\frac{1}{M_2}\right)^2 - \frac{4\sin^2 ka}{M_1M_2}}$ 		

Anharmonic terms

Expand the energy in terms of the normal modes of the linearized problem u_k

$$U = U_0 + \frac{\partial U}{\partial u_k} u_k + \frac{1}{2}\frac{\partial^2 U}{\partial u_j \partial u_k} u_j u_k + \frac{1}{6}\frac{\partial^3 U}{\partial u_i \partial u_j \partial u_k} u_i u_j u_k + \frac{1}{24}\frac{\partial^4 U}{\partial u_h \partial u_i \partial u_j \partial u_k} u_h u_i u_j u_k + \cdots$$

Thermal expansion
Thermal conductivity limited by Umklapp scattering
High temperature limit of specific heat does not approach the Dulong-Petit law

Phonon quasiparticle lifetime

Phonons are the eigenstates of the linearized equations, not the full equations.

Phonons have a finite lifetime that can be calculated by Fermi's golden rule.

$$\Gamma_{i\to f} = \frac{2\pi}{\hbar}\left|\langle f \,|\, H_{ph-ph} \,|\, i\rangle\right|^2 \delta\left(E_f - E_i\right)$$

Occupation is determined by a master equation (not the Bose-Einstein function).

$$\begin{bmatrix} \frac{dP_0}{dt} \\ \frac{dP_1}{dt} \\ \vdots \\ \frac{dP_N}{dt} \end{bmatrix} = \begin{bmatrix} -\sum_{i\neq 0}\Gamma_{0\to i} & \Gamma_{1\to 0} & \cdots & \Gamma_{N\to 0} \\ \Gamma_{0\to 1} & -\sum_{i\neq 1}\Gamma_{1\to i} & \cdots & \Gamma_{N\to 1} \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{0\to N} & \Gamma_{1\to N} & \cdots & -\sum_{i\neq N}\Gamma_{N\to i} \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ \vdots \\ P_N \end{bmatrix}$$

Acoustic attenuation

The amplitude of a monocromatic sound wave decreases as the wave propagates through the crystal as the phonon quasiparticles decay into phonons with other frequencies and directions.