

Optical Properties of Metals

AC Conductivity

For constant voltage, conductors conduct and insulators don't.

For low ac voltages in a conductor, electric field and the electron velocity are in-phase, electric field and electron position are out-of-phase.

For low ac voltages in an insulator, electric field and the electron position are in-phase, electric field and electron velocity are out-of-phase.

At high (optical) frequencies the in-phase and out-of-phase component of the response is described by the dielectric function.

Conductivity / Dielectric function

Harmonic dependence $v = v(\omega)e^{i\omega t}$, $x = x(\omega)e^{i\omega t}$, $E = E(\omega)e^{i\omega t}$

$$v(\omega) = i\omega x(\omega)$$

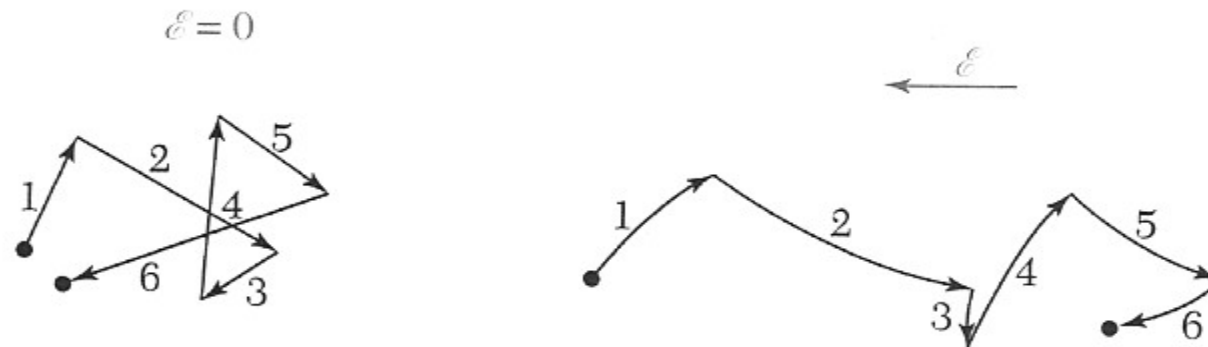
$$\chi(\omega) = \frac{P(\omega)}{\varepsilon_0 E(\omega)} = \frac{-nex(\omega)}{\varepsilon_0 E(\omega)} \quad \sigma(\omega) = \frac{j(\omega)}{E(\omega)} = \frac{-nev(\omega)}{E(\omega)} = \frac{-i\omega nex(\omega)}{E(\omega)}$$

$$\chi(\omega) = \frac{\sigma(\omega)}{i\omega\varepsilon_0}$$

$$\varepsilon(\omega) = 1 + \chi = 1 + \frac{\sigma(\omega)}{i\omega\varepsilon_0}$$

Below about 100 GHz the frequency dependent conductivity is normally used.
Above about 100 GHz the dielectric function is used (optical experiments).

Diffusive transport (low frequencies)



$$\vec{F} = m\vec{a} = -e\vec{E} = m \frac{\vec{v}_d}{\tau_{sc}}$$

$$\tau_{sc} = \frac{\mu m}{e}$$

$$-\frac{e\tau_{sc}}{m} \vec{E} = \vec{v}_d$$

$$-\mu_e \vec{E} = \vec{v}_d$$

$$\sigma = ne\mu = \frac{ne^2\tau}{m}$$

Diffusive metal

The current is related to the electric field

$$j_n = \sigma_{nm} E_m \quad v_n = -\mu_{nm} E_m \leftarrow \text{Steady state solution}$$

The differential equation that describes how the velocity changes in time is:

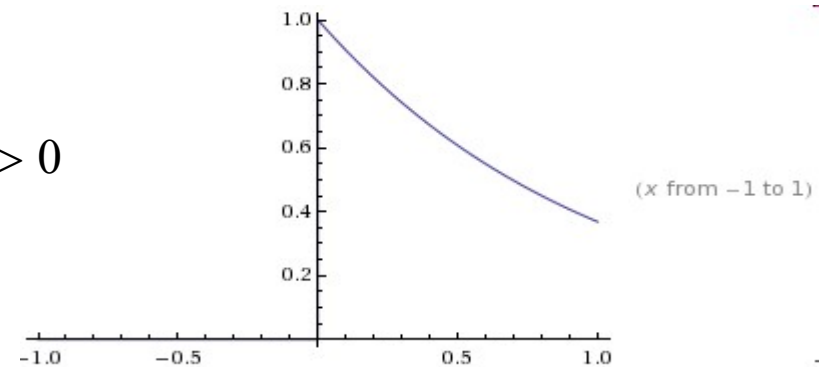
$$m \frac{dv(t)}{dt} + \frac{ev(t)}{\mu} = -eE(t)$$

Inertial term \rightarrow

The impulse response function :

$$g(t) = \frac{1}{m} \exp\left(\frac{-et}{\mu m}\right)$$

$$t > 0$$



Diffusive metal

The differential equation is:

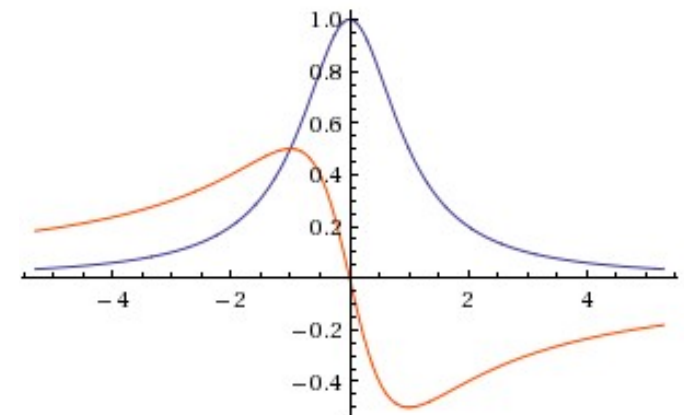
$$m \frac{dv(t)}{dt} + \frac{ev(t)}{\mu} = -eE(t)$$

Assume a harmonic solution $E(\omega)e^{i\omega t}$, $v(\omega)e^{i\omega t}$

$$\left(-\frac{i\omega m}{e} - \frac{1}{\mu} \right) v(\omega) = E(\omega)$$

$$\frac{v(\omega)}{E(\omega)} = \left(-\frac{i\omega m}{e} - \frac{1}{\mu} \right)^{-1} = -\mu(1 + i\omega\tau)^{-1} = \frac{-\mu(1 - i\omega\tau)}{1 + \omega^2\tau^2}$$

$$\sigma(\omega) = \frac{j(\omega)}{E(\omega)} = -ne \frac{v(\omega)}{E(\omega)} = ne\mu \left(\frac{1 - i\omega\tau}{1 + \omega^2\tau^2} \right)$$



$$\tau = \frac{\mu m}{e} \longleftarrow \text{Scattering time}$$

$$\sigma(\text{low } \omega) = ne\mu$$

$$\sigma(\text{high } \omega) = \frac{-ine^2}{\omega m}$$

Diffusive metal

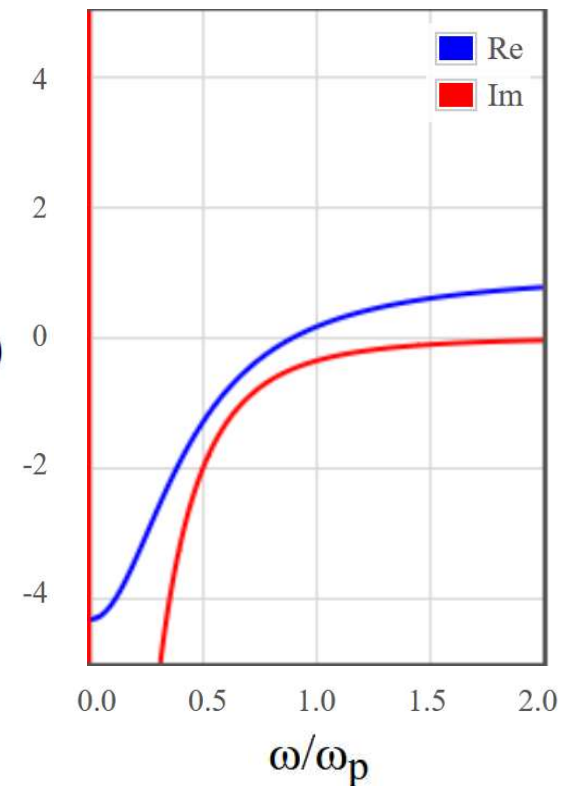
$$\chi(\omega) = \frac{\sigma(\omega)}{i\omega\epsilon_0} = \frac{ne\mu}{i\omega\epsilon_0} \left(\frac{1-i\omega\tau}{1+\omega^2\tau^2} \right)$$

$$\epsilon(\omega) = 1 + \chi = 1 - \frac{ne\mu}{\omega\epsilon_0} \left(\frac{\omega\tau + i}{1 + \omega^2\tau^2} \right)$$

$$\epsilon(\omega) = 1 - \omega_p^2 \left(\frac{\omega\tau^2 + i\tau}{\omega + \omega^3\tau^2} \right) \quad \omega_p^2 = \frac{ne^2}{m\epsilon_0} \quad \epsilon_r(\omega)$$

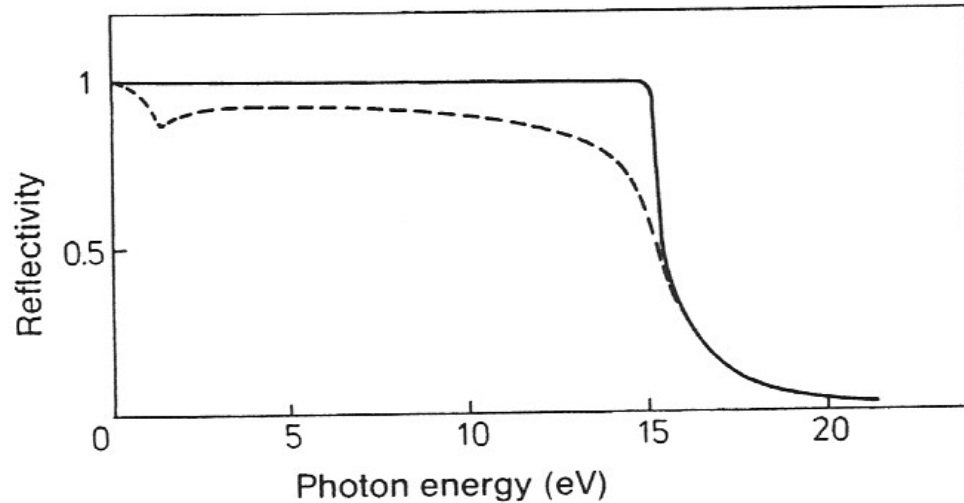
Take the limit as τ goes to infinity

$$\epsilon'(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

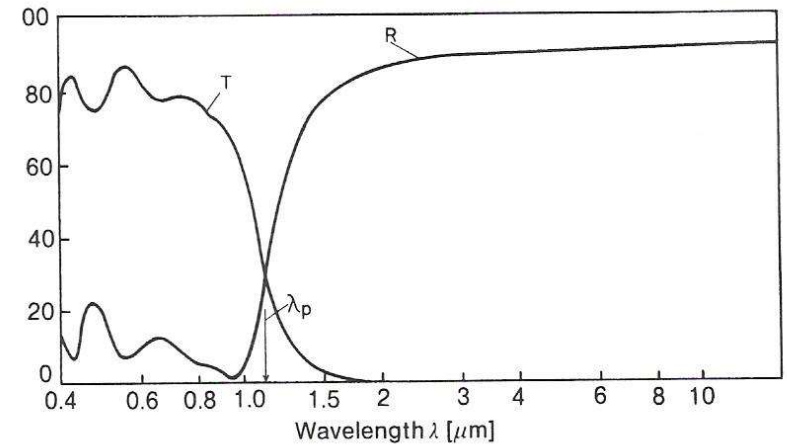


low frequency metal / high frequency insulator

Ibach & Lueth



Aluminum



ITO

Conducting transparent contacts for LEDs and Solar cells

Windows that reflect infrared

Reflection of radio waves from ionosphere

$$\omega_p^2 \approx \frac{ne^2}{\epsilon_0 m}$$



Advanced Solid State Physics

Optical properties of a diffusive metal

- Outline
- Quantization
- Photons
- Electrons
- Magnetic effects and Fermi surfaces
- Linear response
- Transport
- Crystal Physics
- Electron-electron interactions
- Quasiparticles
- Structural phase transitions
- Landau theory of second order phase transitions
- Superconductivity
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It is assumed that electrons in a diffusive metal scatter so often that we can average over the scattering events. The differential equation that describes the motion of the electrons is,

$$m \frac{d\vec{v}}{dt} + \frac{e\vec{v}}{\mu} = -e\vec{E}.$$

Here m is the mass of an electron, \vec{v} is the velocity of the electron, $-e$ is the charge of an electron, and \vec{E} is the electric field. When a constant electric field is applied, the solution is,

$$\vec{v} = -\mu\vec{E}.$$

Thus the (negatively charged) electrons move in the opposite direction as the electric field.

If the electric field is pulsed on, the reponse of the electrons is described by the impulse response function $g(t)$. The impulse response function satisfies the equation,

$$m \frac{dg}{dt} + \frac{eg}{\mu} = -e\delta(t).$$

When the electric field is pulsed on, the electrons suddenly start moving and then their velocity decays exponentially to zero in a time $\tau = m\mu/e$.

$$g(t) = -\frac{e}{m} \exp(-t/\tau).$$

The scattering time τ and the electron density n are the only two parameters that are needed to describe many of the optical properties of a diffusive metal. The form below can be used to input τ and n and then a script calculates and plots the impulse response function, the Fourier transform of the impulse response function, the mobility, the dc conductivity, the frequency dependent complex conductivity, the electric susceptibility, the dielectric function, the plasma frequency, the index of refraction, the extinction coefficient, and the reflectance.

$\tau =$ [s] $n =$ [m^{-3}]

Mobility $\mu = 1.76 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$
 DC conductivity $\sigma_0 = 2.82\text{e}+9 \text{ } \Omega^{-1} \text{ m}^{-1}$
 Plasma frequency $\omega_p = 5.64\text{e}+15 \text{ rad/s}$, $\omega_p\tau = 5.64\text{e}+4$.

Impulse response function



[Click here to begin](#)

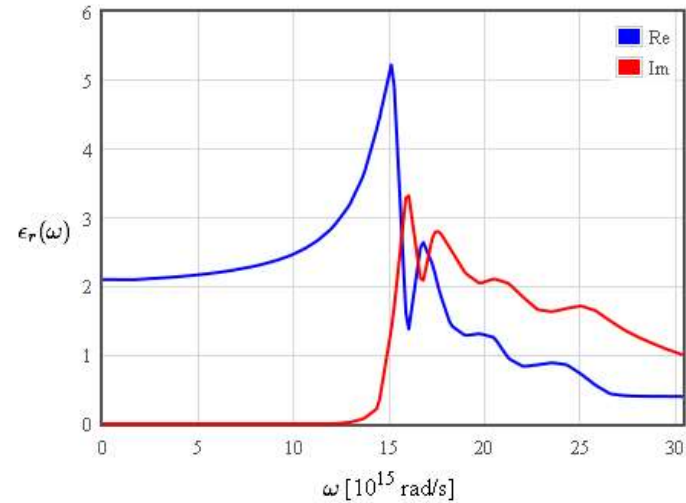
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The optical properties of SiO₂ (glass)

nanophotonics.csic.es

Dielectric function

The relative dielectric constant describes the relationship between the electric displacement \vec{D} and the electric field \vec{E} , $\vec{D} = \epsilon_r \epsilon_0 \vec{E} = \vec{P} + \epsilon_0 \vec{E}$.



There are two conventions for dielectric function. Either it is assumed that the time dependence of \vec{D} , \vec{P} , and \vec{E} is $\exp(-i\omega t)$ and the plot of the dielectric function looks as it is shown above, or it is assumed that the time dependence of \vec{D} , \vec{P} , and \vec{E} is $\exp(i\omega t)$ and the imaginary part of the has the opposite sign as in the plot above. Here we will assume a time dependence of $\exp(-i\omega t)$.

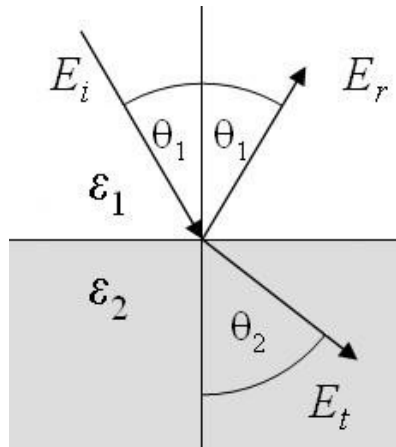
Electric susceptibility

The electric susceptibility χ_E describes the relationship between the polarization \vec{P} and the electric field \vec{E} , $\vec{P} = \epsilon_0 \chi_E \vec{E}$.

$$\chi_E = \epsilon_r - 1$$



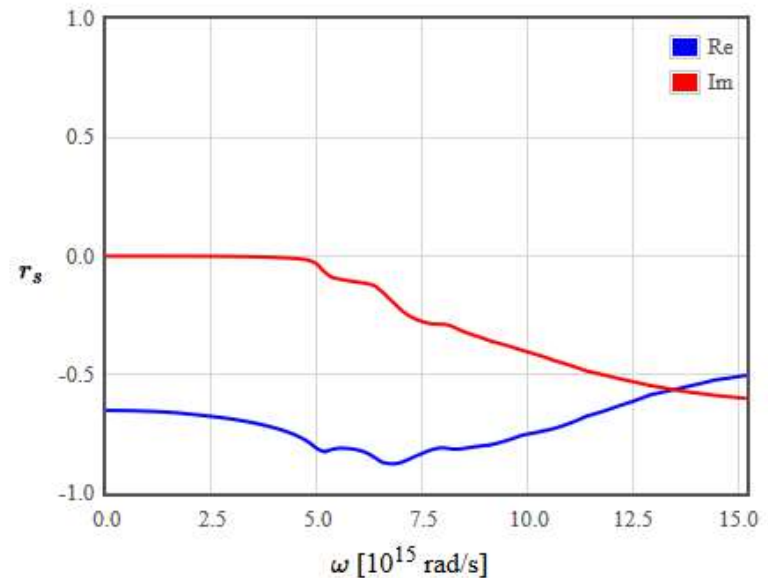
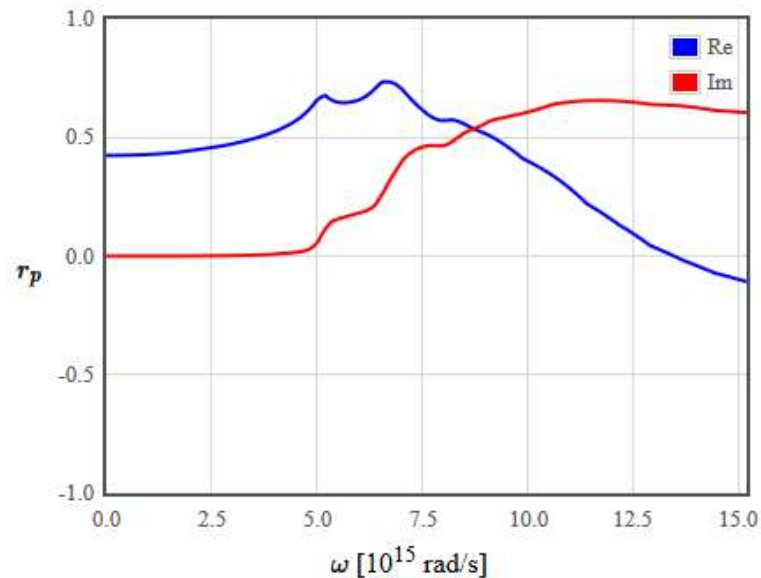
Ellipsometry



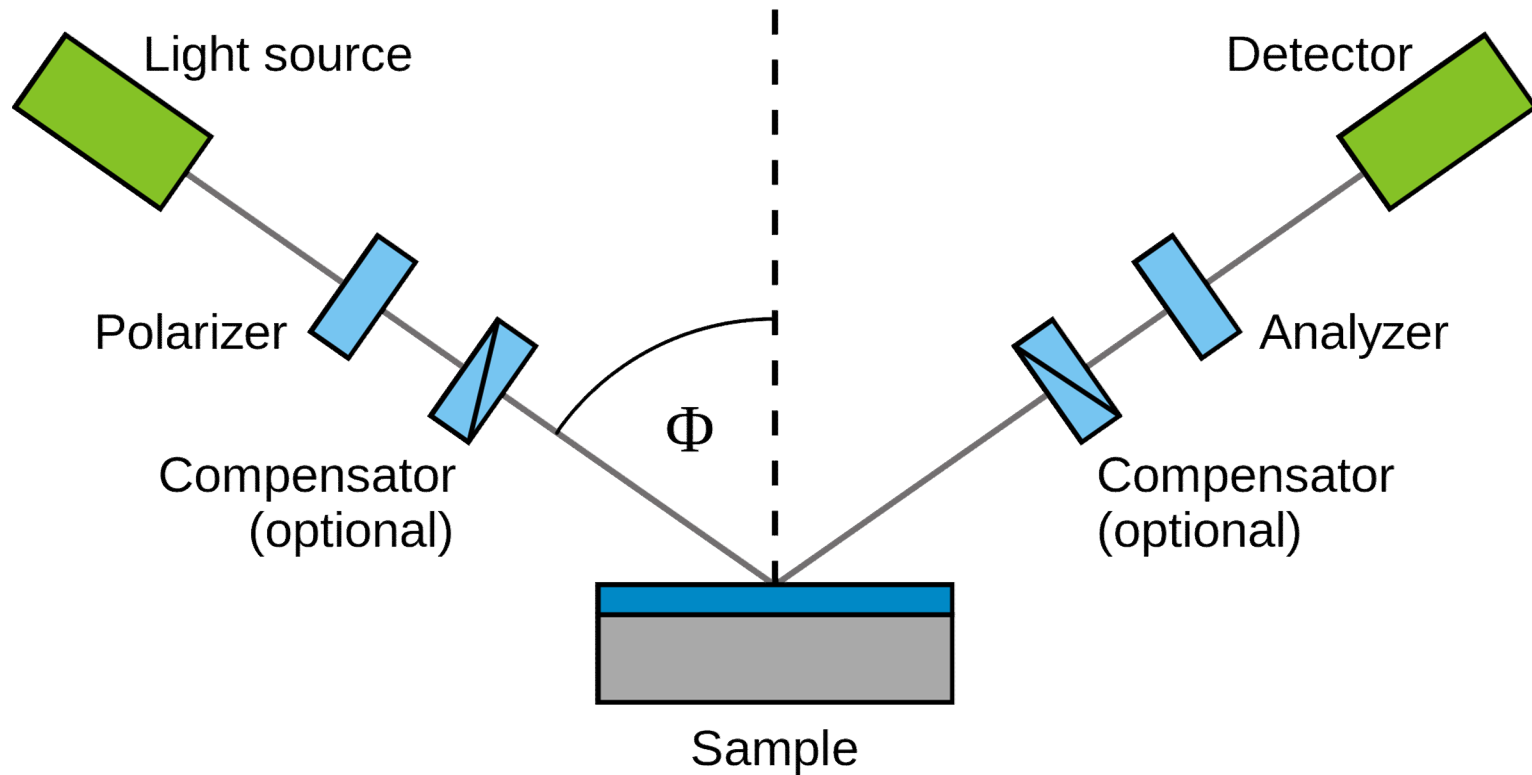
$$r_p = \frac{E_{rp}}{E_{ip}} = \frac{\sqrt{\epsilon_2} \cos \theta_1 - \sqrt{\epsilon_1} \cos \theta_2}{\sqrt{\epsilon_2} \cos \theta_1 + \sqrt{\epsilon_1} \cos \theta_2}$$

$$r_s = \frac{E_{sr}}{E_{si}} = \frac{\sqrt{\epsilon_2} \cos \theta_2 - \sqrt{\epsilon_1} \cos \theta_1}{\sqrt{\epsilon_1} \cos \theta_1 + \sqrt{\epsilon_2} \cos \theta_2}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$



Ellipsometry

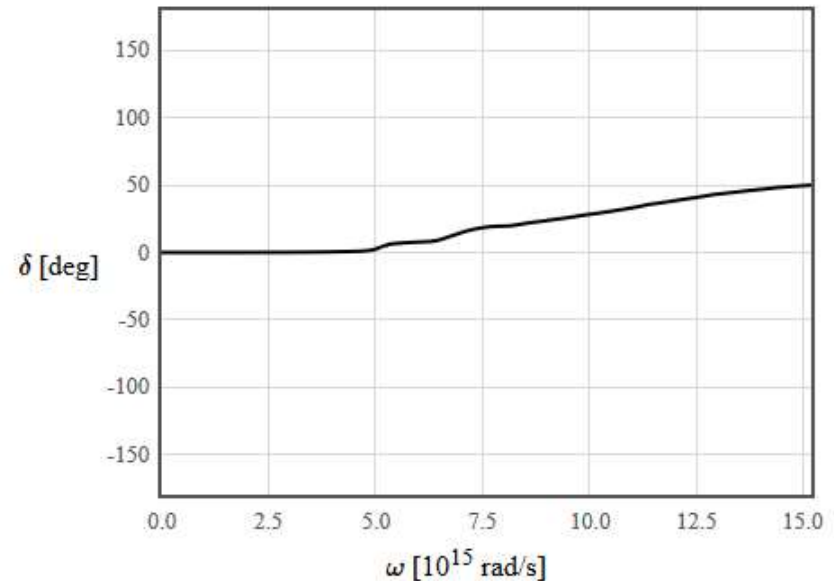
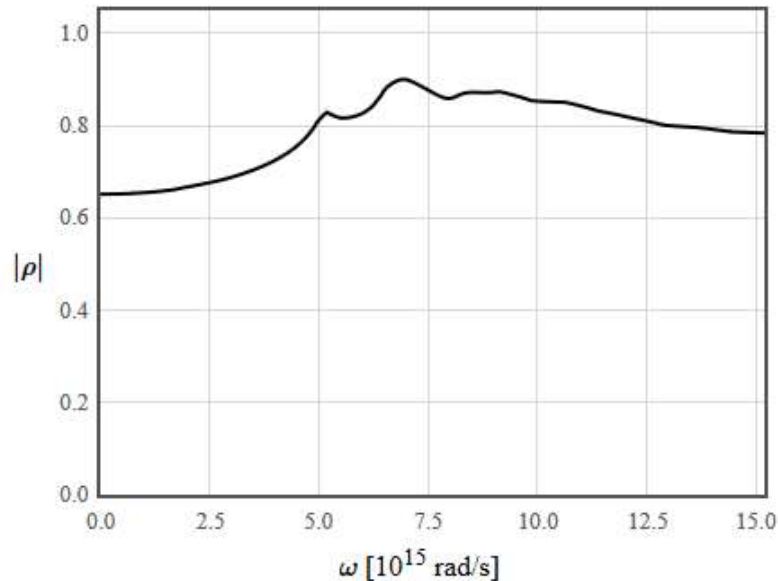


Ellipsometry measures the change of polarization upon reflection. The measured signal depends on the thickness and the dielectric constant.

<http://en.wikipedia.org/wiki/Ellipsometry>

Ellipsometry

$$\rho = \frac{r_p}{r_s} = |\rho|e^{i\delta}$$



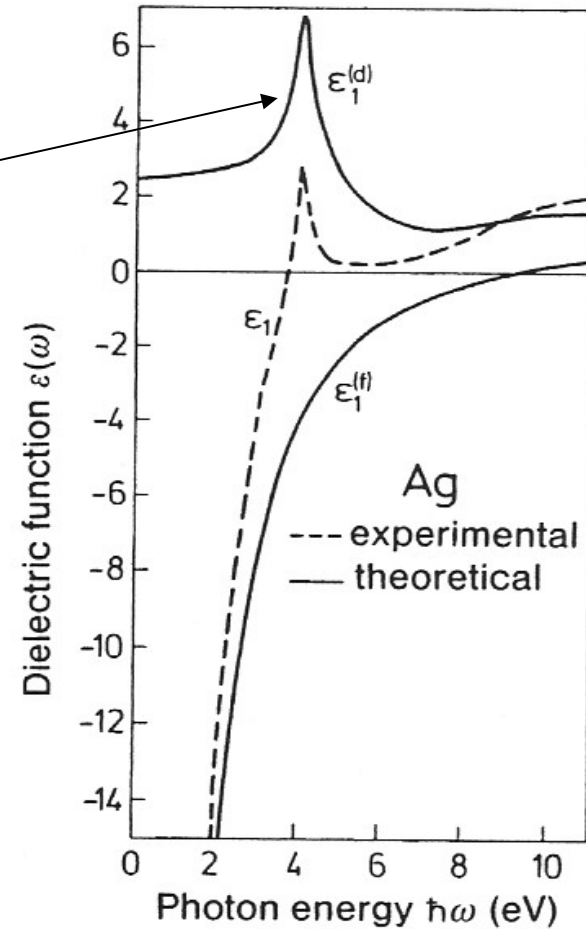
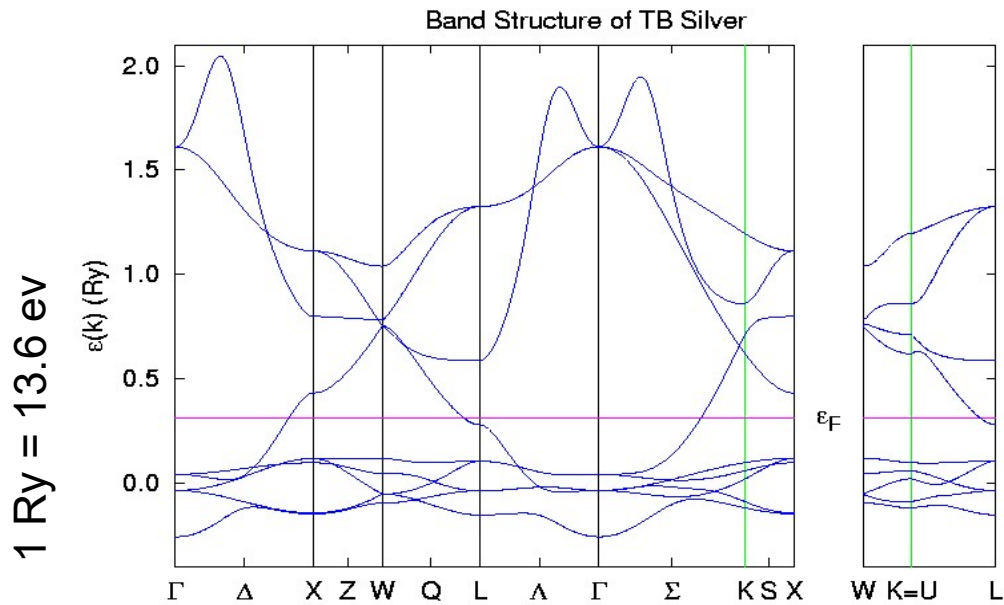
The ratio of the two reflected polarizations is insensitive to instabilities of light source or atmospheric absorption.

Intraband transitions

When the bands are parallel, there is a peak in the absorption (ϵ'')

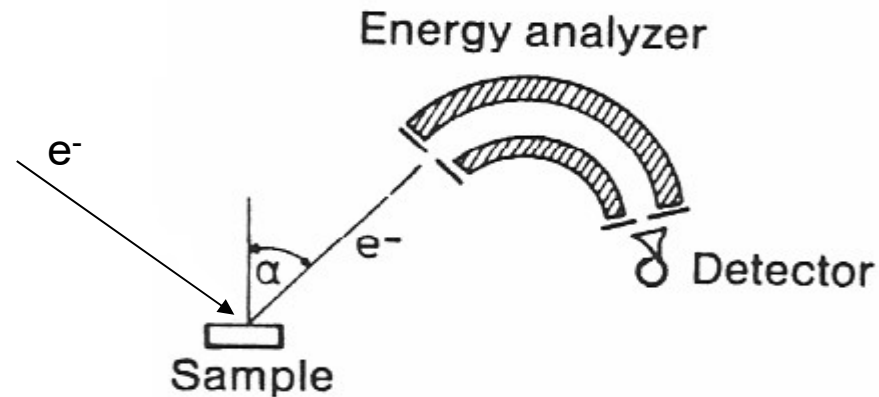
$$\hbar\omega = E_c(\vec{k}) - E_v(\vec{k})$$

Intraband (d-band) absorption



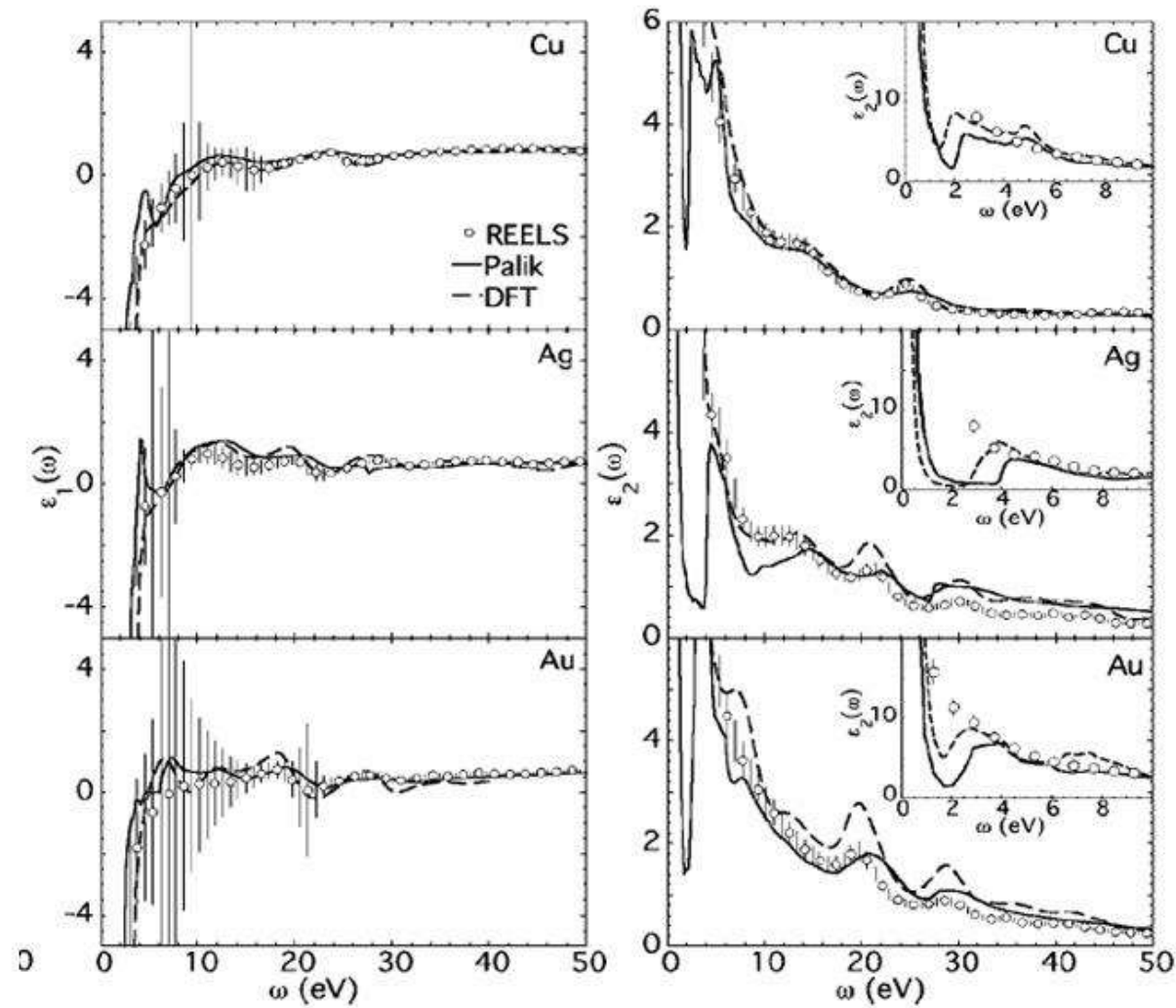
Ibach & Lueeth

Reflection Electron Energy Loss Spectroscopy



Fast electrons moving through the solid generate a time dependent electric field. If the polarization moves out of phase with this field, energy will be lost. This is detected in the reflected electrons.

Dielectric function of Cu, Ag, and Au obtained from reflection electron energy loss spectra, optical measurements, and density functional theory



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