15. Magnetism
/ Superconductivity

Nov 25, 2019
Bloch wall

Anisotropy energy depends on the number of spins pointing in the hard direction

\[ KNa \]

\( Na = \text{thickness of wall} \)

anisotropy constant J/m\(^3\)

Total energy per unit area:

\[ E = \frac{JS^2 \pi^2}{2Na^2} + KNa \] [J/m\(^2\)]

smaller for large \( N \)  smaller for small \( N \)

\[ \frac{dE}{dN} = 0 \Rightarrow - \frac{JS^2 \pi^2}{2N^2 a^2} + Ka = 0 \]

\[ N = \sqrt{\frac{JS^2 \pi^2}{2Ka^3}} \]

\( N \approx 300 \) for iron
**Soft magnetic materials**

- **Soft magnets**
- **Transformers**
- **Magnetic shielding**
- **Ferrites have low eddy current losses**

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>INITIAL RELATIVE PERMEABILITY ($\mu_r$ AT $B \sim 0$)</th>
<th>HYSTERESIS LOSS JOULE/m$^3$ PER CYCLE</th>
<th>B [T] SATURATION INDUCTION, WEBER/m$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commercial iron ingot</td>
<td>250</td>
<td>500</td>
<td>2.16</td>
</tr>
<tr>
<td>Fe-4% Si, random</td>
<td>500</td>
<td>50–150</td>
<td>1.95</td>
</tr>
<tr>
<td>Fe-3% Si, oriented</td>
<td>15,000</td>
<td>35–140</td>
<td>2.0</td>
</tr>
<tr>
<td>45 Permalloy</td>
<td>2,700</td>
<td>120</td>
<td>1.6</td>
</tr>
<tr>
<td>(45% Ni-55% Fe)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mumetal (75% Ni-5% Cu-2% Cr-18% Fe)</td>
<td>30,000</td>
<td>20</td>
<td>0.8</td>
</tr>
<tr>
<td>Supermalloy (79% Ni-15% Fe-5% Mo-0.5% Ma)</td>
<td>100,000</td>
<td>2</td>
<td>0.79</td>
</tr>
</tbody>
</table>

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**Equations**

\[
B = \mu_0 \left( H + M \right)
\]

\[
B = \mu_r \mu_0 H \quad \quad M = \chi H
\]

\[
\mu_r = 1 + \chi
\]
Hard magnetic materials

**Coercive field**

**Remnant Field**

<table>
<thead>
<tr>
<th>Magnet</th>
<th>$B_s$ (T)</th>
<th>$H_{ci}$ (kA/m)</th>
<th>$(BH)_{max}$ (kJ/m³)</th>
<th>$T_c$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nd$<em>2$Fe$</em>{14}$B (sintered)</td>
<td>1.0–1.4</td>
<td>750–2000</td>
<td>200–440</td>
<td>310–400</td>
</tr>
<tr>
<td>Nd$<em>2$Fe$</em>{14}$B (bonded)</td>
<td>0.6–0.7</td>
<td>600–1200</td>
<td>60–100</td>
<td>310–400</td>
</tr>
<tr>
<td>SmCo$_5$ (sintered)</td>
<td>0.8–1.1</td>
<td>600–2000</td>
<td>120–200</td>
<td>720</td>
</tr>
<tr>
<td>Sm(Co,Fe,Cu,Zr)$_7$ (sintered)</td>
<td>0.9–1.15</td>
<td>450–1300</td>
<td>150–240</td>
<td>800</td>
</tr>
<tr>
<td>Alnico (sintered)</td>
<td>0.6–1.4</td>
<td>275</td>
<td>10–88</td>
<td>700–860</td>
</tr>
</tbody>
</table>

Permanent magnets, magnetron, motors, generators
ferrites can also be hard magnets

Defects are introduced to pin the Bloch walls in a hard magnet.
Single domain particles

Small 10 - 100 nm particles have single domains.

Elongated particles have the magnetization along the long axis.

Single domains are used for magnetic recording. Long crystals can be magnetized in either of the two directions along the long axis.

Shape anisotropy.
Hard magnets

Grains too small to contain Bloch walls must be flipped entirely by the field.

Alnico: 8-12% Al, 15-26% Ni, 5-24% Co, up to 6% Cu, up to 1% Ti, rest is Fe
Applications of hard magnets

Motors, generators, speakers, microphone
Giant magnetoresistance

$E$

$D(E)$

spin up

spin down

spin value
GMR sensors in read-heads for hard-disk drives

Shipment of GMR-read-heads (1997-2007): 5 billion ($10^9$)

Peter Gruenberg Nobel Lecture 2007:
From Spinwaves to Giant Magnetoresistance (GMR) and Beyond
Superconductivity

Primary characteristic: zero resistance at dc

There is a critical temperature $T_c$ above which superconductivity disappears

About 1/3 of all metals are superconductors

Metals are usually superconductors OR magnetic, not both

Good conductors are bad superconductors

Kittel chapter 10
Superconductivity was discovered in 1911.

Heike Kammerling-Onnes

Superconductivity was discovered in 1911

Nobel Lecture 1913
Superconductivity

Heike Kammerling-Onnes

Superconductivity was discovered in 1911

Nobel Lecture 1913
### Critical temperature

#### Superconducting $T_c$

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| 1 | H |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 3 | Li | 4 | Be | 0.026 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 11 | Na | 12 | Mg |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 19 | K | 20 | Ca | 21 | Sc | 22 | Ti | 0.39 | 23 | V | 5.38 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 37 | Rb | 38 | Sr | 39 | Y | 40 | Zr | 0.546 | 41 | Nb | 9.5 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 55 | Ce | 56 | Ba | 57 | La | 6 |   |   | 72 | Hf | 0.12 | 73 | Ta | 4.438 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 87 | Fr | 88 | Ra | 89 | Ac | 104 | Pd |   | 105 | Db |   | 106 | Sg |   | 107 | Bh |   | 108 | Hs |   | 109 | Mt |   |   |   |   |   |   |   |   |

### Notes
- The critical temperature $T_c$ is the temperature at which a material transitions into a superconducting state.
- The table lists elements and their critical temperatures, with some elements highlighted in blue indicating superconductivity at room temperature or higher.
- Elements such as Hg and Au are listed, with critical temperatures at 4.15 K and 2.39 K, respectively.
- The use of color coding helps to visually identify elements with high critical temperatures.
Superconductivity

Antiaromatic molecules are unstable and highly reactive

No measurable decay in current after 2.5 years. $\rho < 10^{-25} \ \Omega m.$
Meissner effect

Superconductors are perfect diamagnets at low fields. $B = 0$ inside a bulk superconductor.

Superconductors are used for magnetic shielding.
Superconductivity

Critical temperature $T_c$

Critical current density $J_c$

Critical field $H_c$

\[ n\Delta \approx nk_B T_c \approx \mu_0 H_c^2 \approx \frac{1}{2} nm v^2 = \frac{m}{2ne^2} J_c^2 \]
$\text{YBa}_2\text{Cu}_3\text{O}_x$

- $\text{Cu}^{2+},\text{Cu}^{3+}$
- $\text{O}^{2-}$
- $\text{Y}^{3+}$
- $\text{Ba}^{2+}$

$T_C = 92K$

$R(10^3 \Omega)$ vs Temperature (K)
Material \( T_c \)

- **Legierung:**
  - NbTi \( 9.6 \) K
- **Verbindungen:**
  - NbN \( 16.0 \) K
  - (Lu/Y)\( \text{Ni}_2\text{B}_2\text{C} \) \( 16.0 \) K
  - \( \text{Nb}_3\text{Sn} \) \( 18.0 \) K
  - \( \text{Nb}_3\text{Al} \) \( 18.7 \) K
  - \( \text{Nb}_3\text{Ge} \) \( 22.5 \) K
- **neu:**
  - MgB\(_2\) \( 39 \) K
- **Fullerene:**
  - \( \text{Cs}_2\text{RbC}_{60} \) \( 33 \) K
  - \( \text{Cs}_3\text{C}_{60} \) \( 40 \) K

**Organische Supraleiter:**

\( (\text{BEDT-TTF})_2\text{Cu[N(CN)\(_2\)]Br} \) \( 11.2 \) K

**Polymere hochdotierte Halbleiter**

[http://www.wmi.badw.de/teaching/Lecturenotes/index.html](http://www.wmi.badw.de/teaching/Lecturenotes/index.html)
<table>
<thead>
<tr>
<th>Compound</th>
<th>$T_c$ in K</th>
<th>Compound</th>
<th>$T_c$ in K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nb$_3$Sn</td>
<td>18.05</td>
<td>V$_3$Ga</td>
<td>16.5</td>
</tr>
<tr>
<td>Nb$_3$Ge</td>
<td>23.2</td>
<td>V$_3$Si</td>
<td>17.1</td>
</tr>
<tr>
<td>Nb$_3$Al</td>
<td>17.5</td>
<td>YBa$_2$Cu$_3$O$_6.9$</td>
<td>90.0</td>
</tr>
<tr>
<td>NbN</td>
<td>16.0</td>
<td>Rb$<em>2$CsC$</em>{60}$</td>
<td>31.3</td>
</tr>
<tr>
<td>K$<em>3$C$</em>{60}$</td>
<td>19.2</td>
<td>MgB$_2$</td>
<td>39.0</td>
</tr>
</tbody>
</table>

BaPb$_{0.75}$Bi$_{0.25}$O$_3$ \hspace{1cm} $T_c = 12$ K \hspace{1cm} [BPBO]
La$_{1.85}$Ba$_{0.15}$CuO$_4$ \hspace{1cm} $T_c = 36$ K \hspace{1cm} [LBCO]
YBa$_2$Cu$_3$O$_7$ \hspace{1cm} $T_c = 90$ K \hspace{1cm} [YBCO]
Tl$_2$Ba$_2$Ca$_2$Cu$_3$O$_{10}$ \hspace{1cm} $T_c = 120$ K \hspace{1cm} [TBCO]
Hg$_{0.8}$Tl$_{0.2}$Ba$_2$Ca$_2$Cu$_3$O$_{8.33}$ \hspace{1cm} $T_c = 138$ K
(Sn$_5$In)Ba$_4$Ca$_2$Cu$_{10}$O$_y$ \hspace{1cm} $T_c = 212$ K
Superconductivity

Critical temperature $T_c$

Critical current density $J_c$

Critical field $H_c$

$$n\Delta \approx nk_B T_c \approx \mu_0 H_c^2 \approx \frac{1}{2} nm v^2 = \frac{m}{2ne^2} J_c^2$$
Superconductivity

Perfect diamagnetism

Jump in the specific heat like a 2nd order phase transition, not a structural transition

Superconductors are good electrical conductors but poor thermal conductors, electrons no longer conduct heat

There is a dramatic decrease of acoustic attenuation at the phase transition, no electron-phonon scattering

Dissipationless currents - quantum effect

Electrons condense into a single quantum state - low entropy.
Electron decrease their energy by $\Delta$ but loose their entropy.
Schrödinger equation for a charged particle in an electric and magnetic field is

\[ i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m} (-i\hbar \nabla - qA)^2 \psi + V\psi \]

write out the \((-i\hbar \nabla - qA)^2 \psi\) term

\[ i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m} \left( -\hbar^2 \nabla^2 + i\hbar qA \nabla + i\hbar q \nabla A + q^2 A^2 \right) \psi + V\psi \]

write the wave function in polar form

\[ \psi = |\psi| e^{i\theta} \]

\[ \nabla \psi = \nabla |\psi| e^{i\theta} + i\nabla \theta |\psi| e^{i\theta} \]

\[ \nabla^2 \psi = \nabla^2 |\psi| e^{i\theta} + 2i\nabla \theta \nabla |\psi| e^{i\theta} + i\nabla^2 \theta |\psi| e^{i\theta} - (\nabla \theta)^2 |\psi| e^{i\theta} \]
Schrödinger equation becomes:

\[ i\hbar \frac{\partial |\psi|}{\partial t} - \hbar |\psi| \frac{\partial \theta}{\partial t} = \frac{1}{2m} \left[ -\hbar^2 \left( \nabla^2 |\psi| + 2i\nabla \theta \nabla |\psi| + i\nabla^2 \theta |\psi| - (\nabla \theta)^2 |\psi| \right) \right. \]

\[ + i\hbar q A \left( \nabla |\psi| + i\nabla \theta |\psi| \right) + \hbar q \nabla A |\psi| + q^2 A^2 |\psi| \] + V |\psi|

Real part:

\[ -\hbar |\psi| \frac{\partial \theta}{\partial t} = -\frac{\hbar^2}{2m} \left( \nabla^2 \left( \nabla \theta - \frac{q}{\hbar} A \right)^2 \right) |\psi| + V |\psi| \]

Imaginary part:

\[ \hbar \frac{\partial |\psi|}{\partial t} = \frac{1}{2m} \left[ -\hbar^2 \left( 2\nabla \theta \nabla |\psi| + i\nabla^2 \theta |\psi| - (\nabla \theta)^2 |\psi| \right) \right. \]

\[ + 2\hbar q A \nabla |\psi| + \hbar q |\psi| \nabla A \]
Probability current

Imaginary part:

\[ \frac{\hbar}{\partial t} \frac{\partial |\psi|}{\partial t} = \frac{1}{2m} \left[ -\hbar^2 \left( 2\nabla \theta \nabla |\psi| + i \nabla^2 \theta |\psi| - (\nabla \theta)^2 |\psi| \right) + 2\hbar q A \nabla |\psi| + \hbar q |\psi| \nabla A \right] \]

Multiply by \(|\psi|\) and rearrange

\[ \frac{\partial}{\partial t} |\psi|^2 + \nabla \cdot \left[ \frac{\hbar}{m} |\psi|^2 \left( \nabla \theta - \frac{q}{\hbar} \vec{A} \right) \right] = 0 \]

This is a continuity equation for probability

\[ \frac{\partial P}{\partial t} + \nabla \cdot \vec{S} = 0 \]

The probability current:

\[ \vec{S} = \frac{\hbar}{m} |\psi|^2 \left( \nabla \theta - \frac{q}{\hbar} \vec{A} \right) \]
The probability current:  \[ \vec{S} = \frac{\hbar}{m} |\psi|^2 \left( \nabla \theta - \frac{q}{\hbar} \vec{A} \right) \]

This result holds for all charged particles in a magnetic field.

In superconductivity the particles are Cooper pairs \( q = -2e, m = 2m_e, |\psi|^2 = n_{cp} \).

All superconducting electrons are in the same state so

\[ \vec{j} = -2en_{cp} \vec{S} \]

London gauge \( \nabla \theta = 0 \)

\[ \vec{j} = \frac{-e\hbar n_{cp}}{m_e} \left( \nabla \theta + \frac{2e}{\hbar} \vec{A} \right) \]

\[ \vec{j} = \frac{-2n_{cp} e^2}{m_e} \vec{A} = \frac{-n_s e^2}{m_e} \vec{A} \quad n_s = 2n_{cp} \]
1st London equation

\[ \vec{j} = \frac{-n_se^2}{m_e} \vec{A} \]

\[ \frac{\ddot{j}}{dt} = \frac{-n_se^2}{m_e} \frac{\ddot{A}}{dt} = \frac{n_se^2}{m_e} \vec{E} \quad \frac{d\vec{A}}{dt} = -\vec{E} \]

First London equation:

\[ \frac{\ddot{j}}{dt} = \frac{n_se^2}{m_e} \vec{E} \]

Classical derivation:

\[ -e\vec{E} = m \frac{d\vec{v}}{dt} = - \frac{m}{n_se} \frac{d\dot{j}}{dt} \]

\[ \frac{\ddot{j}}{dt} = \frac{n_se^2}{m_e} \vec{E} \]
2nd London equation

\[ \vec{j} = \frac{-n_s e^2}{m_e} \vec{A} \]

\[ \nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \nabla \times \vec{A} \]

Second London equation:

\[ \nabla \times \vec{j} = \frac{-n_s e^2}{m_e} \vec{B} \]