

# 14. Magnetism

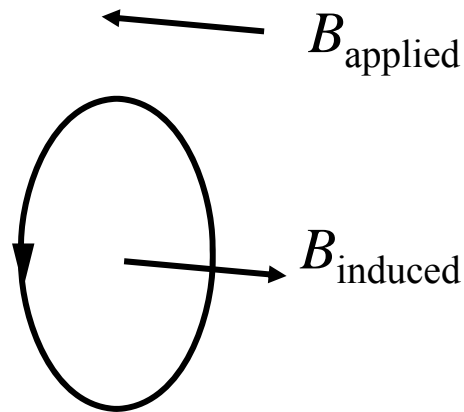
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Nov 18, 2019

# Diamagnetism

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A free electron in a magnetic field will travel in a circle



The magnetic created by the current loop is opposite the applied field.

# Diamagnetism

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Dissipationless currents are induced in a diamagnet that generate a field that opposes an applied magnetic field.

Current flow without dissipation is a quantum effect. There are no lower lying states to scatter into. This creates a current that generates a field that opposes the applied field.

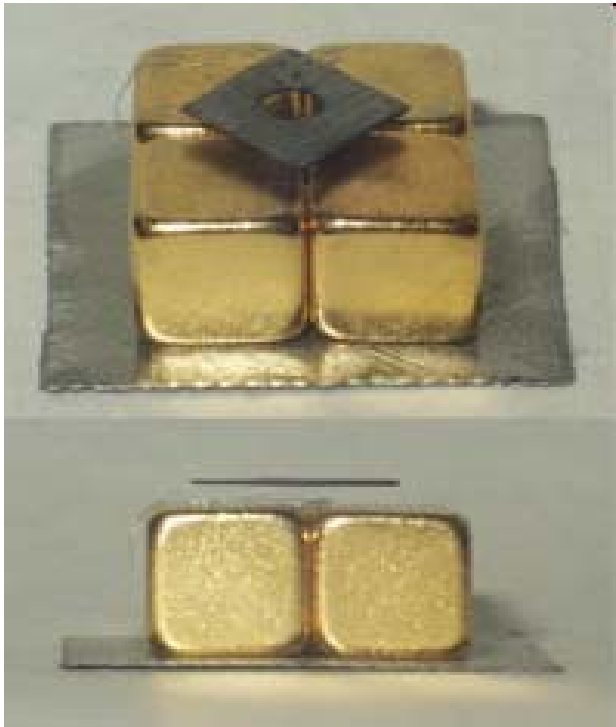
$\chi = -1$  superconductor (perfect diamagnet)

$\chi \sim -10^{-6} - 10^{-5}$  normal materials

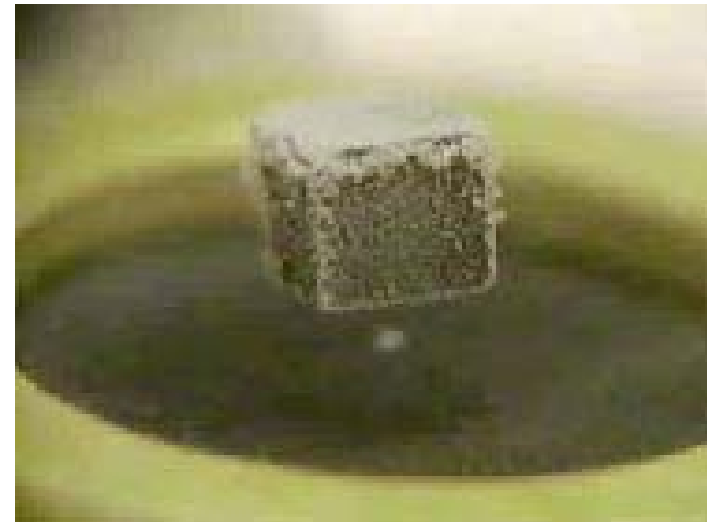
Diamagnetism is always present but is often overshadowed by some other magnetic effect.

# Levitating diamagnets

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Levitating pyrolytic carbon



NOT: Lenz's law

$$V = -\frac{d\Phi}{dt}$$

# Levitating frogs

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$\chi$  for water is  $-9.05 \times 10^{-6}$



16 Tesla magnet at the Nijmegen High Field Magnet Laboratory

<http://www.hfml.ru.nl/froglev.html>

# Andre Geim



2000 Ig Nobel Prize for  
levitating a frog with a  
magnet



The Nobel Prize in Physics 2010  
Andre Geim, Konstantin Novoselov

The Nobel Prize in Physics 2010

Nobel Prize Award Ceremony

Andre Geim



Biographical

Nobel Lecture

Banquet Speech

Interview

Nobel Diploma

Photo Gallery

Other Resources

Konstantin Novoselov

**Andre Geim**

**Born:** 1958, Sochi, Russia

**Affiliation at the time of the award:**  
University of Manchester,  
Manchester, United Kingdom

**Prize motivation:** "for  
groundbreaking experiments  
regarding the two-dimensional  
material graphene"



# Diamagnetism

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A dissipationless current is induced by a magnetic field that opposes the applied field.

$$\vec{M} = \chi\vec{H}$$

## **Diamagnetic susceptibility**

Copper	$-9.8 \times 10^{-6}$
Diamond	$-2.2 \times 10^{-5}$
Gold	$-3.6 \times 10^{-5}$
Lead	$-1.7 \times 10^{-5}$
Nitrogen	$-5.0 \times 10^{-9}$
Silicon	$-4.2 \times 10^{-6}$
water	$-9.0 \times 10^{-6}$
bismuth	$-1.6 \times 10^{-4}$

Most stable molecules have a closed shell configuration and are diamagnetic.

# Paramagnetism

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Materials that have a magnetic moment are paramagnetic.

An applied field aligns the magnetic moments in the material making the field in the material larger than the applied field.

The internal field is zero at zero applied field (random magnetic moments).

$$\vec{M} = \chi \vec{H}$$

## **Paramagnetic susceptibility**

Aluminum	$2.3 \times 10^{-5}$
Calcium	$1.9 \times 10^{-5}$
Magnesium	$1.2 \times 10^{-5}$
Oxygen	$2.1 \times 10^{-6}$
Platinum	$2.9 \times 10^{-4}$
Tungsten	$6.8 \times 10^{-5}$



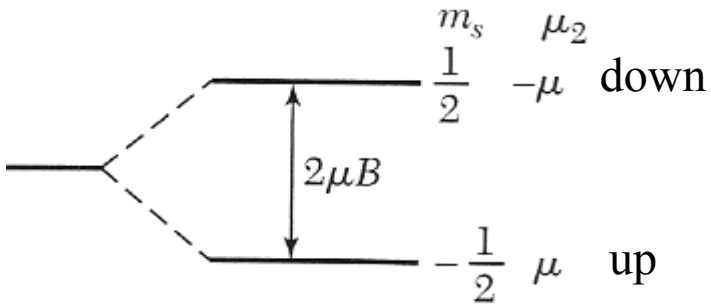
# Boltzmann factors

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To take the average value of quantity  $A$

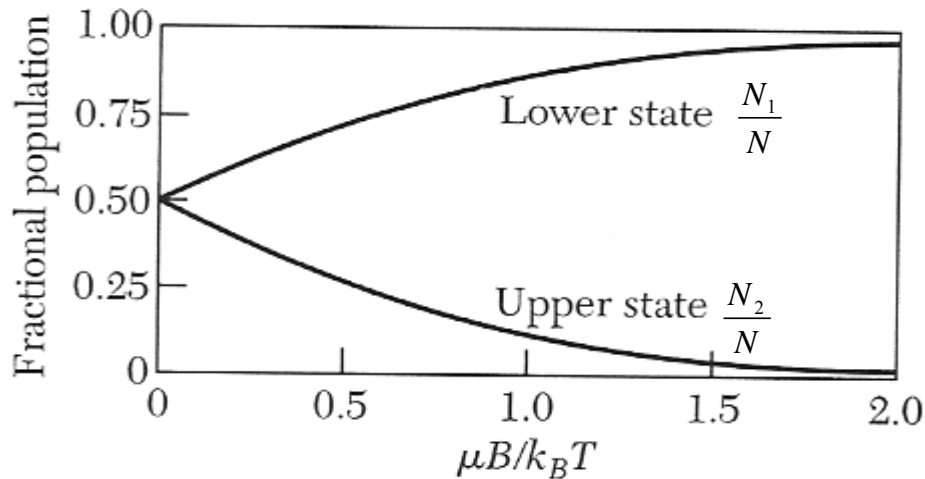
$$\langle A \rangle = \frac{\sum_i A_i e^{-E_i/k_B T}}{\sum_i e^{-E_i/k_B T}}$$

# Spin populations



$$\frac{N_1}{N} = \frac{\exp(\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$

$$\frac{N_2}{N} = \frac{\exp(-\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$

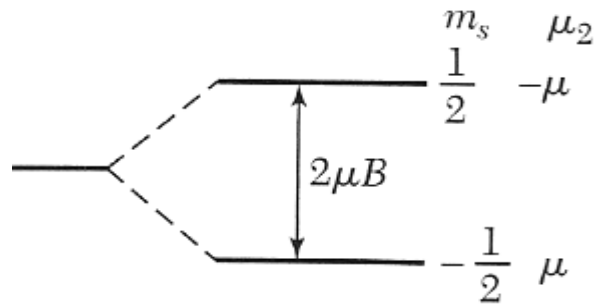


$$M = (N_1 - N_2)\mu / V$$

$$= n\mu \frac{\exp(\mu B / k_B T) - \exp(-\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$

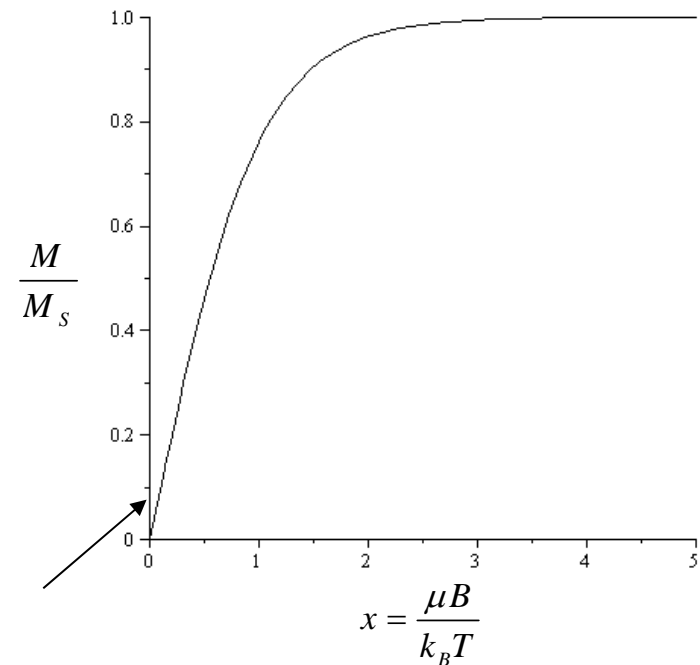
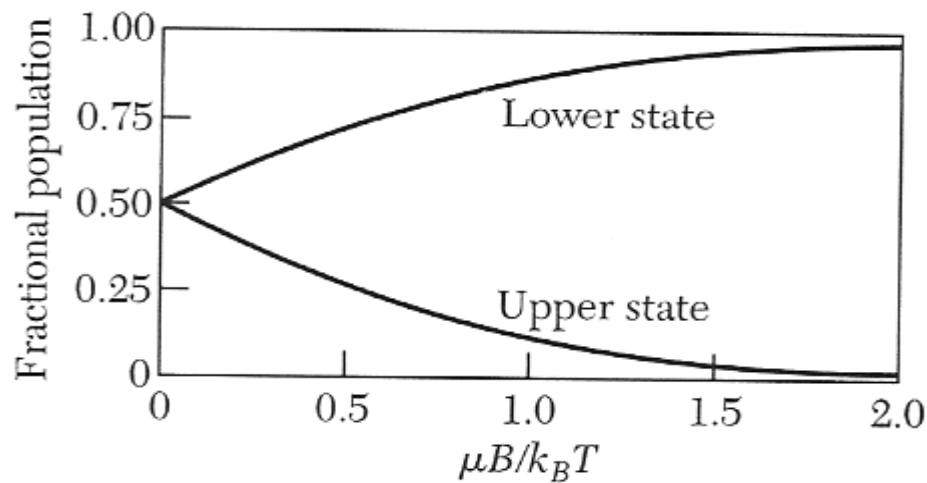
$$= n\mu \tanh\left(\frac{\mu B}{k_B T}\right)$$

# Paramagnetism, spin 1/2

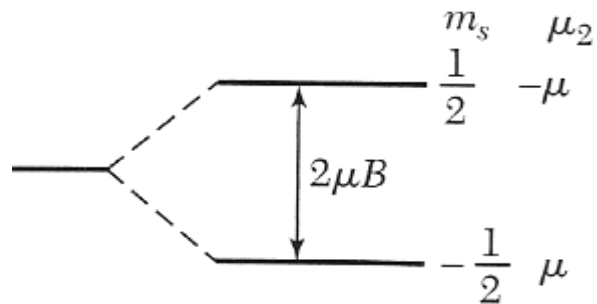


$$M = n\mu \tanh\left(\frac{\mu B}{k_B T}\right) \approx \frac{n\mu^2 B}{k_B T} = \frac{CB}{T} \quad \text{Curie law}$$

for  $\mu B \ll k_B T$

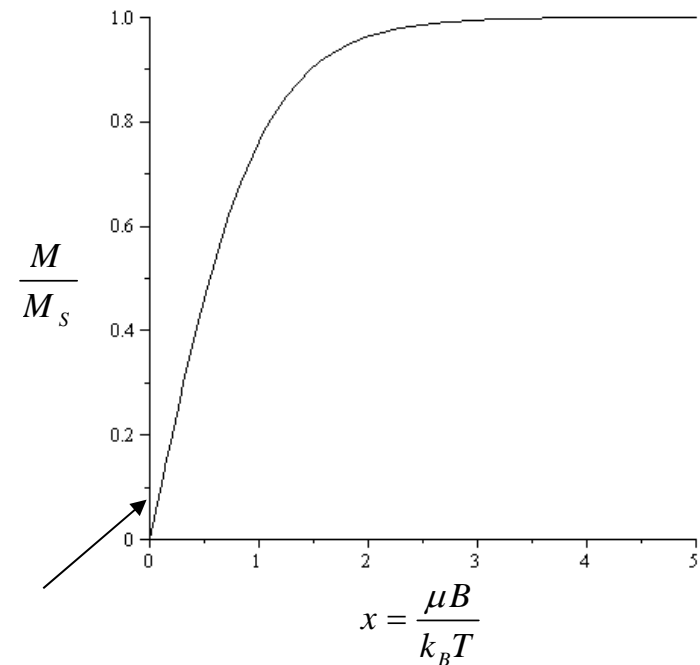
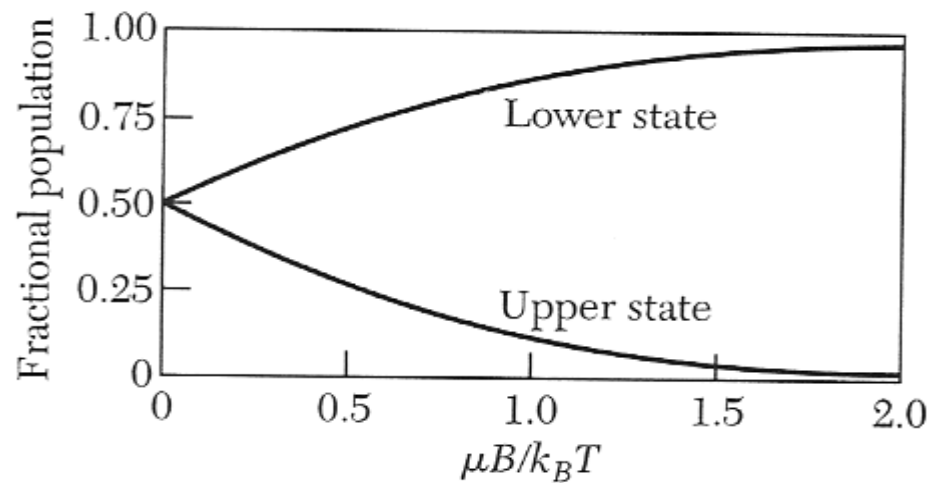


# Paramagnetism, spin 1/2



$$M = n\mu \tanh\left(\frac{\mu B}{k_B T}\right) \approx \frac{n\mu^2 B}{k_B T} = \frac{CB}{T}$$

for  $\mu B \ll k_B T$  Curie law

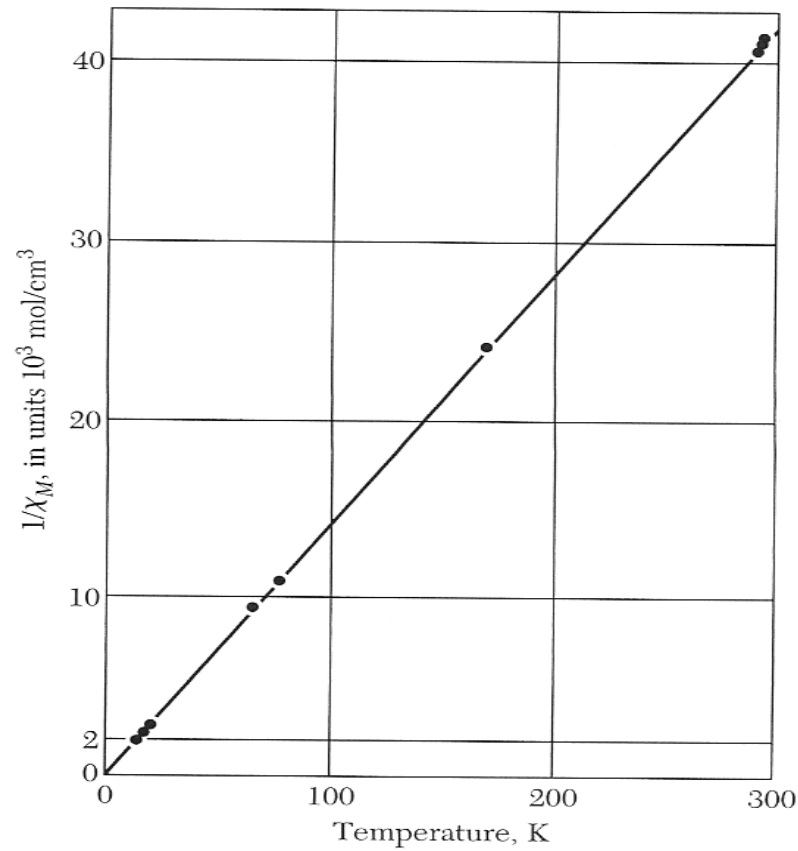


# Curie law

for  $\mu B \ll k_B T$   $M = CB / T$

$$\chi \propto \left. \frac{dM}{dB} \right|_{B=0} = \frac{C}{T}$$

$C$  is the Curie constant



# Atomic physics

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In atomic physics, the possible values of the magnetic moment of an atom in the direction of the applied field can only take on certain values.

Total angular momentum

$$J = L + S \quad \text{Orbital } L + \text{ spin } S \text{ angular momentum}$$

Magnetic quantum number

$$m_J = -J, -J + 1, \dots, J - 1, J$$

Allowed values of the magnetic moment in the z direction

$$\mu_z = m_j g_J \mu_B$$

Lande g factor  $\swarrow$   $\nwarrow$  Bohr magneton

$$g_J \approx \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

Period	Hydrogen																Helium				
1	$\left\langle \psi_{Cu3d^{10}4s^1} \middle  H \middle  \psi_{Cu3d^{10}4s^1} \right\rangle < \left\langle \psi_{Cu3d^94s^2} \middle  H \middle  \psi_{Cu3d^94s^2} \right\rangle$ $\left\langle \psi_{Cu3d^{10}4s^1} \middle  \psi_{Cu3d^{10}4s^1} \right\rangle < \left\langle \psi_{Cu3d^94s^2} \middle  \psi_{Cu3d^94s^2} \right\rangle$																2				
2	Lithium 3 <b>Li</b> 6.94	Beryllium 4 <b>Be</b> 9.0122														Boron 5 <b>B</b> 10.81	Carbon 6 <b>C</b> 12.011	Nitrogen 7 <b>N</b> 14.007	Oxygen 8 <b>O</b> 15.999	Fluorine 9 <b>F</b> 18.998	Neon 10 <b>Ne</b> 20.180
3	Sodium 11 <b>Na</b> 22.990	Magnesium 12 <b>Mg</b> 24.305														Aluminum 13 <b>Al</b> 26.982	Silicon 14 <b>Si</b> 28.085	Phosphorus 15 <b>P</b> 30.974	Sulfur 16 <b>S</b> 32.06	Chlorine 17 <b>Cl</b> 35.45	Argon 18 <b>Ar</b> 39.95
4	Potassium 19 <b>K</b> 39.098	Calcium 20 <b>Ca</b> 40.078	Scandium 21 <b>Sc</b> 44.956	Titanium 22 <b>Ti</b> 47.867	Vanadium 23 <b>V</b> 50.942	Chromium 24 <b>Cr</b> 51.996	Manganese 25 <b>Mn</b> 54.938	Iron 26 <b>Fe</b> 55.845	Cobalt 27 <b>Co</b> 58.933	Nickel 28 <b>Ni</b> 58.693	Copper 29 <b>Cu</b> 63.546	Zinc 30 <b>Zn</b> 65.38	Gallium 31 <b>Ga</b> 69.723	Germanium 32 <b>Ge</b> 72.630	Arsenic 33 <b>As</b> 74.922	Selenium 34 <b>Se</b> 78.971	Bromine 35 <b>Br</b> 79.904	Krypton 36 <b>Kr</b> 83.798			
5	Rubidium 37 <b>Rb</b> 85.468	Strontium 38 <b>Sr</b> 87.62	Yttrium 39 <b>Y</b> 88.906	Zirconium 40 <b>Zr</b> 91.224	Niobium 41 <b>Nb</b> 92.906	Molybdenum 42 <b>Mo</b> 95.95	Technetium 43 <b>Tc</b> [97]	Ruthenium 44 <b>Ru</b> 101.07	Rhodium 45 <b>Rh</b> 102.91	Palladium 46 <b>Pd</b> 106.42	Silver 47 <b>Ag</b> 107.87	Cadmium 48 <b>Cd</b> 112.41	Indium 49 <b>In</b> 114.82	Tin 50 <b>Sn</b> 118.71	Antimony 51 <b>Sb</b> 121.76	Tellurium 52 <b>Te</b> 127.60	Iodine 53 <b>I</b> 126.90	Xenon 54 <b>Xe</b> 131.29			
6	Caesium 55 <b>Cs</b> 132.91	Barium 56 <b>Ba</b> 137.33	Lanthanum 57 <b>La</b> 138.91	Hafnium 72 <b>Hf</b> 178.49	Tantalum 73 <b>Ta</b> 180.95	Tungsten 74 <b>W</b> 183.84	Rhenium 75 <b>Re</b> 186.21	Osmium 76 <b>Os</b> 190.23	Iridium 77 <b>Ir</b> 192.22	Platinum 78 <b>Pt</b> 195.08	Gold 79 <b>Au</b> 196.97	Mercury 80 <b>Hg</b> 200.59	Thallium 81 <b>Tl</b> 204.38	Lead 82 <b>Pb</b> 207.2	Bismuth 83 <b>Bi</b> 208.98	Polonium 84 <b>Po</b> [209]	Astatine 85 <b>At</b> [210]	Radon 86 <b>Rn</b> [222]			
7	Francium 87 <b>Fr</b> [223]	Radium 88 <b>Ra</b> [226]	Actinium 89 <b>Ac</b> [227]	Rutherfordium 104 <b>Rf</b> [267]	Dubnium 105 <b>Db</b> [268]	Seaborgium 106 <b>Sg</b> [269]	Bohrium 107 <b>Bh</b> [270]	Hassium 108 <b>Hs</b> [269]	Meitnerium 109 <b>Mt</b> [278]	Darmstadtium 110 <b>Ds</b> [281]	Roentgenium 111 <b>Rg</b> [282]	Copernicium 112 <b>Cn</b> [285]	Nihonium 113 <b>Nh</b> [286]	Flerovium 114 <b>Fl</b> [289]	Moscovium 115 <b>Mc</b> [290]	Livermorium 116 <b>Lv</b> [293]	Tennessee 117 <b>Ts</b> [294]	Oganesson 118 <b>Og</b> [294]			
				* Cerium 58 <b>Ce</b> 140.12																	
				* Praseodymium 59 <b>Pr</b> 140.91																	
				* Neodymium 60 <b>Nd</b> 144.24																	
				* Promethium 61 <b>Pm</b> [145]																	
				* Samarium 62 <b>Sm</b> 150.36																	
				* Europium 63 <b>Eu</b> 151.96																	
				* Gadolinium 64 <b>Gd</b> 157.25																	
				* Terbium 65 <b>Tb</b> 158.93																	
				* Dysprosium 66 <b>Dy</b> 162.50																	
				* Holmium 67 <b>Ho</b> 164.93																	
				* Erbium 68 <b>Er</b> 167.26																	
				* Thulium 69 <b>Tm</b> 168.93																	
				* Ytterbium 70 <b>Yb</b> 173.05																	
				* Lutetium 71 <b>Lu</b> 174.97																	
				** Thorium 90 <b>Th</b> 232.04																	
				** Protactinium 91 <b>Pa</b> 231.04																	
				** Uranium 92 <b>U</b> 238.03																	
				** Neptunium 93 <b>Np</b> [237]																	
				** Plutonium 94 <b>Pu</b> [244]																	
				** Americium 95 <b>Am</b> [243]																	
				** Curium 96 <b>Cm</b> [247]																	
				** Berkelium 97 <b>Bk</b> [247]																	
				** Californium 98 <b>Cf</b> [251]																	
				** Einsteinium 99 <b>Es</b> [252]																	
				** Fermium 100 <b>Fm</b> [257]																	
				** Mendelevium 101 <b>Md</b> [258]																	
				** Nobelium 102 <b>No</b> [259]																	
				** Lawrencium 103 <b>Lr</b> [266]																	

# Brillouin functions

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Average value of the magnetic quantum number

$$\langle m_J \rangle = \frac{\sum_{-J}^J m_J e^{-E(m_J)/k_B T}}{\sum_{-J}^J e^{-E(m_J)/k_B T}} = \frac{\sum_{-J}^J m_J e^{m_J g_J \mu_B B / k_B T}}{\sum_{-J}^J e^{m_J g_J \mu_B B / k_B T}} = \frac{1}{Z} \frac{dZ}{dx}$$

Lande g factor

$$x = g_J \mu_B B / k_B T$$

Bohr magneton

$$Z = \sum_{-J}^J e^{m_J x} = \frac{\sinh\left(\left(2J + 1\right) \frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right)}$$



# Brillouin functions

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Average value of the magnetic quantum number

$$\langle m_J \rangle = \frac{\sum_{-J}^J m_J e^{-E(m_J)/k_B T}}{\sum_{-J}^J e^{-E(m_J)/k_B T}} = \frac{\sum_{-J}^J m_J e^{m_J g_J \mu_B B / k_B T}}{\sum_{-J}^J e^{m_J g_J \mu_B B / k_B T}} = \frac{1}{Z} \frac{dZ}{dx}$$

Lande g factor

$$x = g_J \mu_B B / k_B T$$

Bohr magneton

$$Z = \sum_{-J}^J e^{m_J x} = e^{Jx} (1 + e^{-x} + e^{-2x} + \dots) - e^{-(J+1)x} (1 + e^{-x} + e^{-2x} + \dots)$$

$$= \frac{e^{Jx} - e^{-(J+1)x}}{1 - e^{-x}} = \frac{e^{-\frac{x}{2}} e^{(J+\frac{1}{2})x} - e^{-(J+\frac{1}{2})x}}{e^{-\frac{x}{2}} (e^{\frac{x}{2}} - e^{-\frac{x}{2}})} = \frac{\sinh\left((2J+1)\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right)}$$

# Brillouin functions

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$$Z = \sum_{-J}^J e^{-m_J x} = \frac{\sinh\left(\left(2J+1\right)\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right)}$$

$$M = n g_J \mu_B \langle m_J \rangle = n g_J \mu_B \frac{1}{Z} \frac{dZ}{dx}$$

Brillouin function

$$M = n g \mu_B J \left( \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} \frac{g \mu_B J B}{k_B T}\right) - \frac{1}{2J} \coth\left(\frac{1}{2J} \frac{g \mu_B J B}{k_B T}\right) \right)$$

# Pauli paramagnetism

Paramagnetic contribution due to free electrons.

Electrons have an intrinsic magnetic moment  $\mu_B$ .

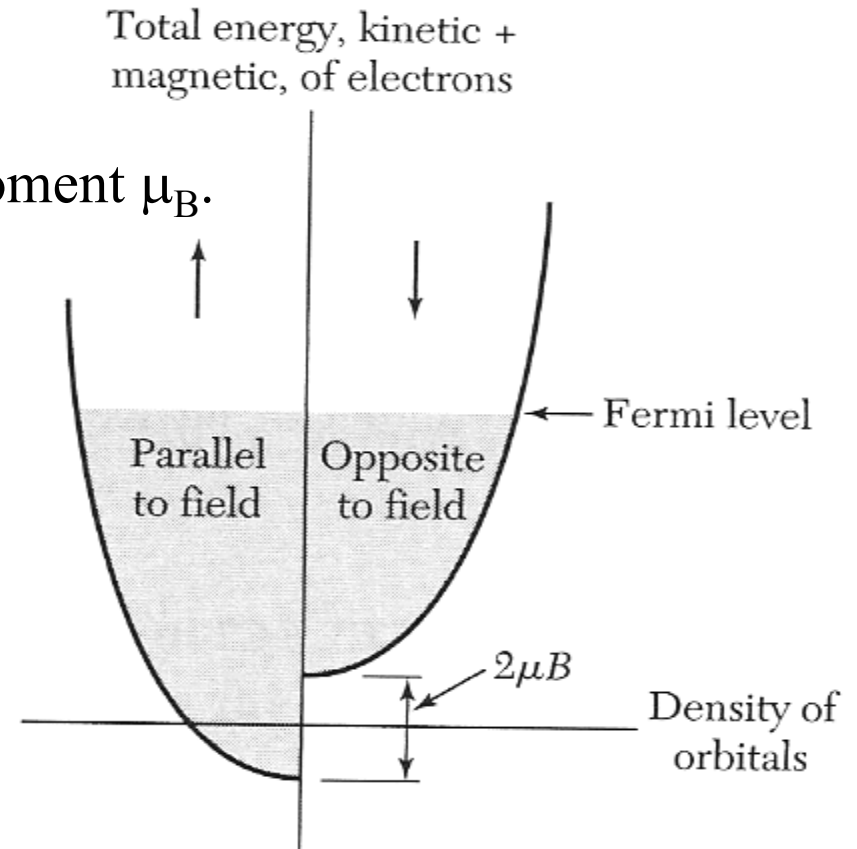
$$n_+ \approx \frac{1}{2}n + \frac{1}{2}\mu_B BD(E_F)$$

$$n_- \approx \frac{1}{2}n - \frac{1}{2}\mu_B BD(E_F)$$

$$M = \mu_B(n_+ - n_-)$$

$$M = \mu_B^2 D(E_F) B = \mu_0 \mu_B^2 D(E_F) H$$

$$\chi = \frac{dM}{dH} = \mu_0 \mu_B^2 D(E_F)$$



If  $E_F$  is 1 eV, a field of  $B = 17000$  T is needed to align all of the spins.

Pauli paramagnetism is much smaller than the paramagnetism due to atomic moments and almost temperature independent because  $D(E_F)$  doesn't change very much with temperature.

# Hund's rules (f - shell)

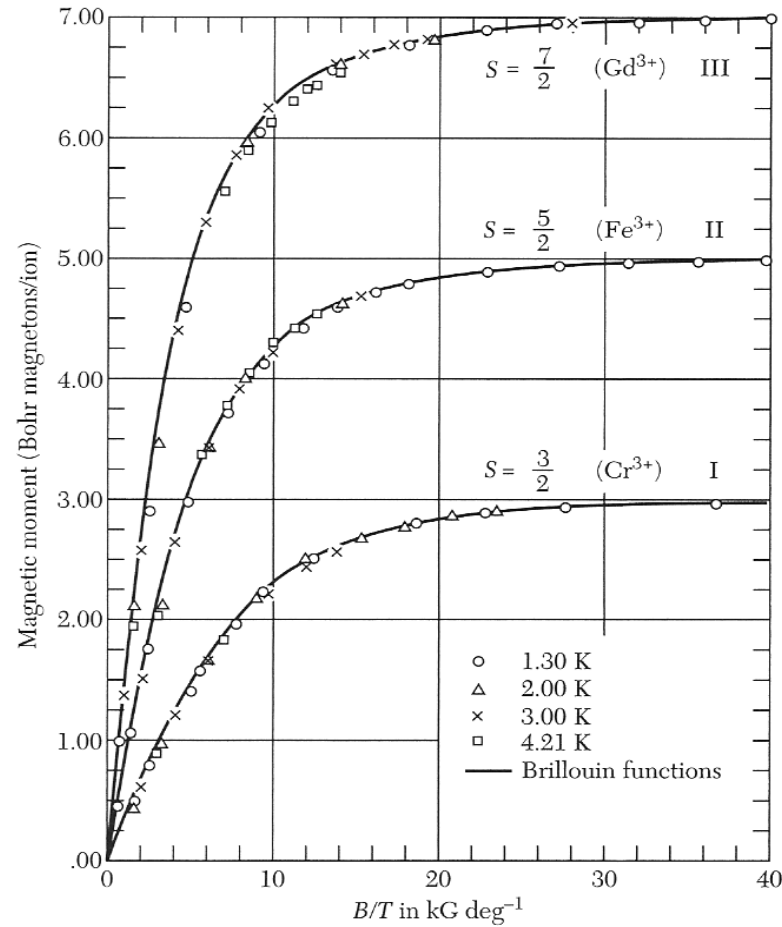
$n$	$l_z = 3, 2, 1, 0, -1, -2, -3$	$S$	$L =  \sum l_z $	$J$
1	↓	1/2	3	5/2
2	↓ ↓	1	5	4
3	↓ ↓ ↓	3/2	6	9/2
4	↓ ↓ ↓ ↓	2	6	4
5	↓ ↓ ↓ ↓ ↓	5/2	5	5/2
6	↓ ↓ ↓ ↓ ↓ ↓	3	3	0
7	↓ ↓ ↓ ↓ ↓ ↓ ↓	7/2	0	7/2
8	↑↑ ↑ ↑ ↑ ↑ ↑	3	3	6
9	↑↑ ↑↑ ↑ ↑ ↑ ↑ ↑	5/2	5	15/2
10	↑↑ ↑↑ ↑↑ ↑ ↑ ↑ ↑ ↑	2	6	8
11	↑↑ ↑↑ ↑↑ ↓↓ ↑ ↑ ↑	3/2	6	15/2
12	↑↑ ↑↑ ↓↓ ↓↓ ↓↓ ↑ ↑	1	5	6
13	↑↑ ↓↓ ↓↓ ↓↓ ↓↓ ↓↓ ↑	1/2	3	7/2
14	↑↑ ↓↓ ↓↓ ↓↓ ↓↓ ↓↓ ↓↓ ↓	0	0	0

$J = |L - S|$

$J = L + S$

The half filled shell and completely filled shell have zero total angular mo.

# Paramagnetism

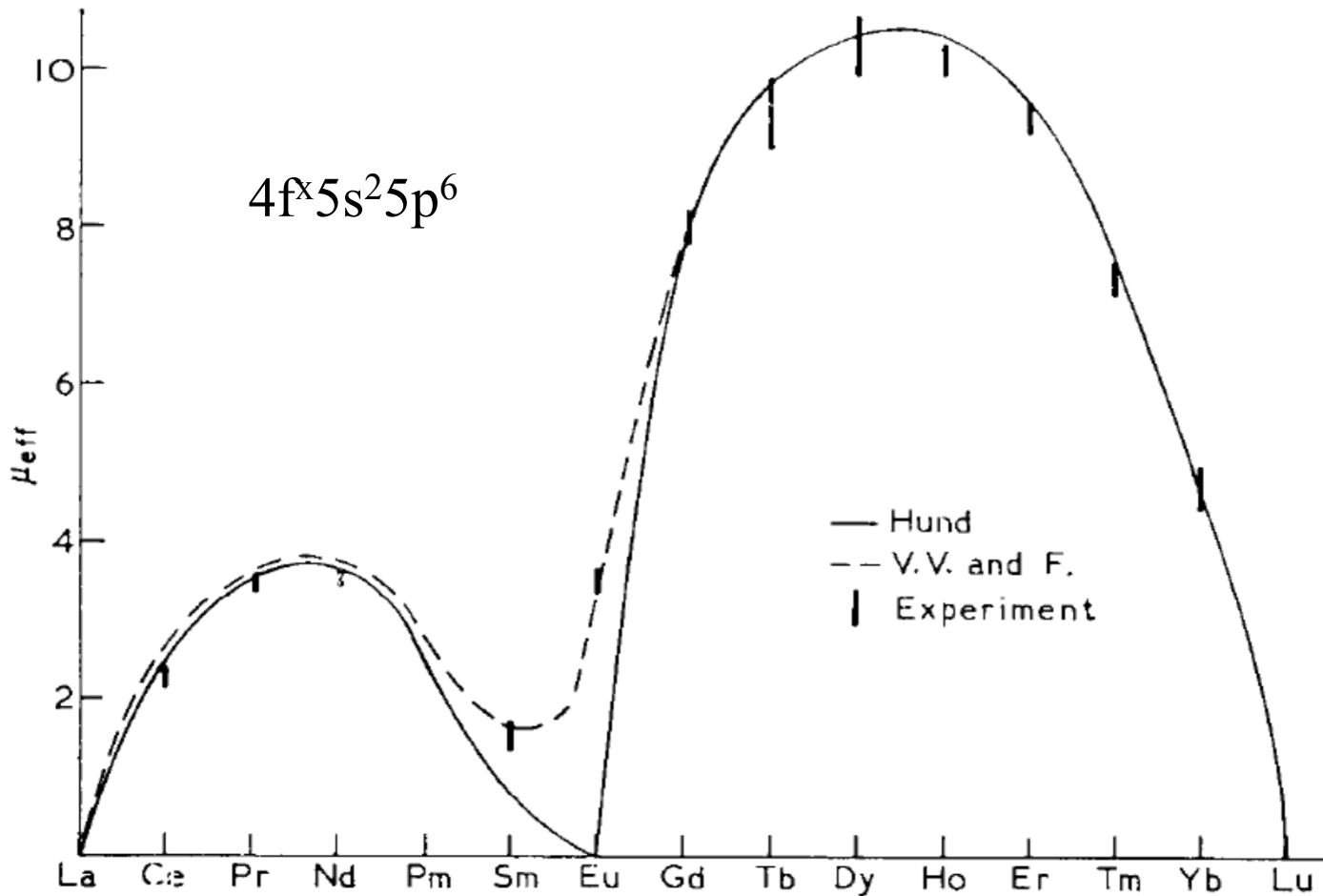


$$M = Ng\mu_B J \left( \frac{2J+1}{2J} \coth \left( \frac{2J+1}{2J} \frac{g\mu_B JB}{k_B T} \right) - \frac{1}{2J} \coth \left( \frac{1}{2J} \frac{g\mu_B JB}{k_B T} \right) \right)$$

# Quantum Mechanics: The Key to Understanding Magnetism

## John H. van Vleck

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# Ferromagnetism

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Below a critical temperature (called the Curie temperature) a magnetization spontaneously appears in a ferromagnet even in the absence of a magnetic field.

Iron, nickel, and cobalt are ferromagnetic.

Ferromagnetism overcomes the magnetic dipole-dipole interactions. It arises from the Coulomb interactions of the electrons. The energy that is gained when the spins align is called the exchange energy.

# Schrödinger equation for two particles

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$$-\frac{\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2)\psi + V_1(\vec{r}_1)\psi + V_2(\vec{r}_2)\psi + V_{1,2}(\vec{r}_1, \vec{r}_2)\psi = E\psi$$

$\psi(\vec{r}_1, \vec{r}_2) = \psi_1(\vec{r}_1)\psi_2(\vec{r}_2)$  is a solution to the noninteracting Hamiltonian,  $V_{1,2} = 0$

$$\psi_A(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2) - \psi_1(\vec{r}_2)\psi_2(\vec{r}_1)) \begin{pmatrix} \uparrow\uparrow \\ \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ \downarrow\downarrow \end{pmatrix}$$

$$\psi_S(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2) + \psi_1(\vec{r}_2)\psi_2(\vec{r}_1)) \frac{1}{\sqrt{2}}(\uparrow(\vec{r}_1)\downarrow(\vec{r}_2) - \downarrow(\vec{r}_1)\uparrow(\vec{r}_2))$$



# Exchange (Austauschwechselwirkung)

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$$\psi_A(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2) - \psi_1(\vec{r}_2)\psi_2(\vec{r}_1))$$

$$\begin{aligned} \langle \psi_A | H | \psi_A \rangle &= \frac{1}{2} [\langle \psi_1(\vec{r}_1)\psi_2(\vec{r}_2) | H | \psi_1(\vec{r}_1)\psi_2(\vec{r}_2) \rangle - \langle \psi_1(\vec{r}_1)\psi_2(\vec{r}_2) | H | \psi_1(\vec{r}_2)\psi_2(\vec{r}_1) \rangle \\ &\quad - \langle \psi_1(\vec{r}_2)\psi_2(\vec{r}_1) | H | \psi_1(\vec{r}_1)\psi_2(\vec{r}_2) \rangle + \langle \psi_1(\vec{r}_2)\psi_2(\vec{r}_1) | H | \psi_1(\vec{r}_2)\psi_2(\vec{r}_1) \rangle] \end{aligned}$$

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$$\psi_S(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}(\psi_1(\vec{r}_1)\psi_2(\vec{r}_2) + \psi_1(\vec{r}_2)\psi_2(\vec{r}_1))$$

$$\begin{aligned} \langle \psi_S | H | \psi_S \rangle &= \frac{1}{2} [\langle \psi_1(\vec{r}_1)\psi_2(\vec{r}_2) | H | \psi_1(\vec{r}_1)\psi_2(\vec{r}_2) \rangle + \langle \psi_1(\vec{r}_1)\psi_2(\vec{r}_2) | H | \psi_1(\vec{r}_2)\psi_2(\vec{r}_1) \rangle \\ &\quad + \langle \psi_1(\vec{r}_2)\psi_2(\vec{r}_1) | H | \psi_1(\vec{r}_1)\psi_2(\vec{r}_2) \rangle + \langle \psi_1(\vec{r}_2)\psi_2(\vec{r}_1) | H | \psi_1(\vec{r}_2)\psi_2(\vec{r}_1) \rangle] \end{aligned}$$

The difference in energy between the  $\psi_A$  and  $\psi_S$  is twice the **exchange energy**.

# Exchange

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The exchange energy can only be defined when you speak of multi-electron wavefunctions. It is the difference in energy between the symmetric solution and the antisymmetric solution. There is only a difference when the electron-electron term is included. Coulomb repulsion determines the exchange energy.

In ferromagnets, the antisymmetric state has a lower energy. Thus the state with parallel spins has lower energy.

In antiferromagnets, the symmetric state has a lower energy. Neighboring spins are antiparallel.

Ordered states have a lower entropy than free electrons.

# Mean field theory (Molekularfeldtheorie)

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Heisenberg Hamiltonian  $H = -\sum_{i,j} J_{i,j} \vec{S}_i \cdot \vec{S}_j - g \mu_B \vec{B} \cdot \sum_i \vec{S}_i$

Exchange energy

Mean field approximation

$$H_{MF} = \sum_i \vec{S}_i \cdot \left( \sum_{\delta} J_{i,\delta} \langle \vec{S} \rangle + g \mu_B \vec{B} \right)$$

$\delta$  sums over the neighbors of spin  $i$

Looks like a magnetic field  $B_{MF}$

$$\vec{B}_{MF} = \frac{1}{g \mu_B} \sum_{\delta} J_{i,\delta} \langle \vec{S} \rangle$$

magnetization  $\longrightarrow \vec{M} = g \mu_B \frac{N}{V} \langle \vec{S} \rangle$

eliminate  $\langle S \rangle$

# Mean field theory

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$$\vec{B}_{MF} = \frac{V}{Ng^2\mu_B^2} zJ\vec{M}$$

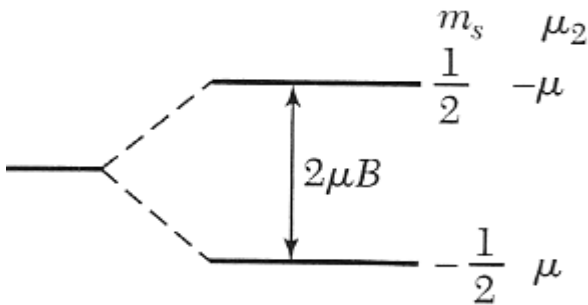
$z$  is the number of nearest neighbors

In mean field, the energy of the spins is

$$E = \pm \frac{1}{2} g \mu_B (B_{MF} + B_a)$$

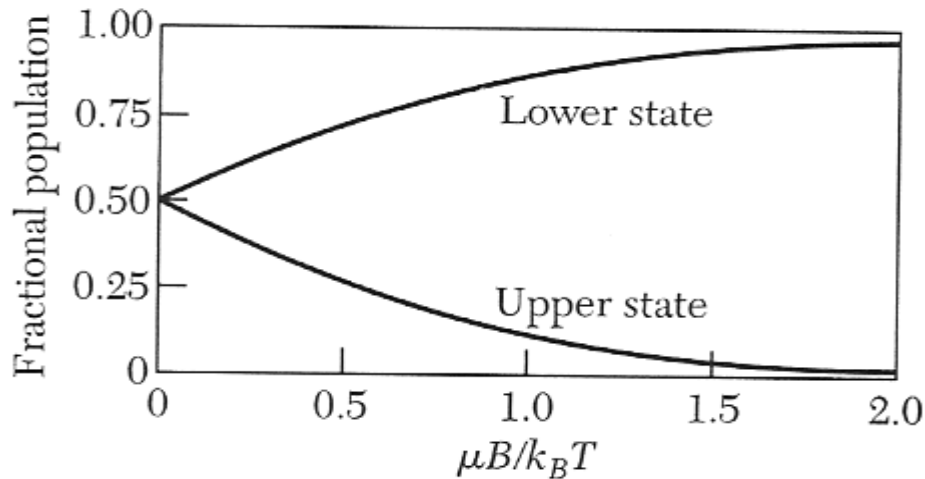
We calculated the populations of the spins in the paramagnetism section

# Spin populations



$$\frac{N_1}{N} = \frac{\exp(\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$

$$\frac{N_2}{N} = \frac{\exp(-\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$



$$M = (N_1 - N_2)\mu$$

$$= N \mu \frac{\exp(\mu B / k_B T) - \exp(-\mu B / k_B T)}{\exp(\mu B / k_B T) + \exp(-\mu B / k_B T)}$$

$$= N \mu \tanh\left(\frac{\mu B}{k_B T}\right)$$

# Mean field theory

---

$$M = \frac{1}{2} g \mu_B \frac{N}{V} \tanh \left( \frac{g \mu_B (B_{MF} + B_a)}{2k_B T} \right)$$

For zero applied field

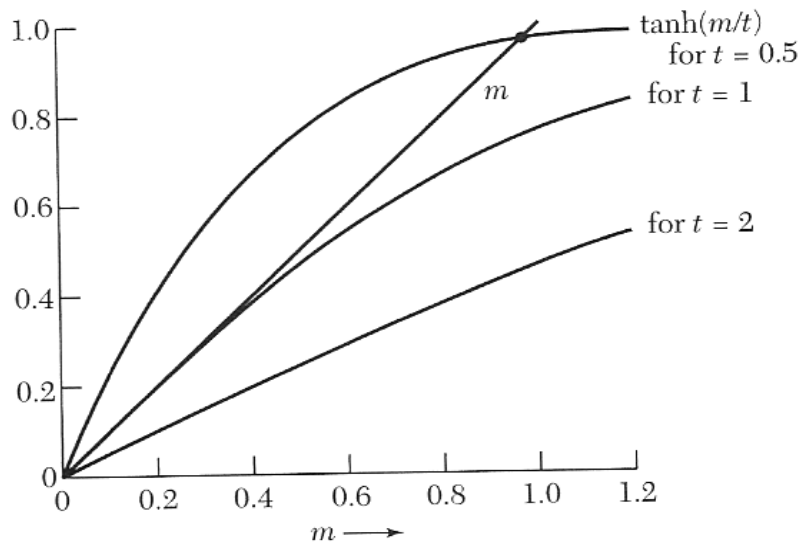
$$M = M_s \tanh \left( \frac{T_c}{T} \frac{M}{M_s} \right)$$

$$M_s = \frac{N}{2V} g \mu_B \quad \text{and} \quad T_c = \frac{z}{4k_B} J$$

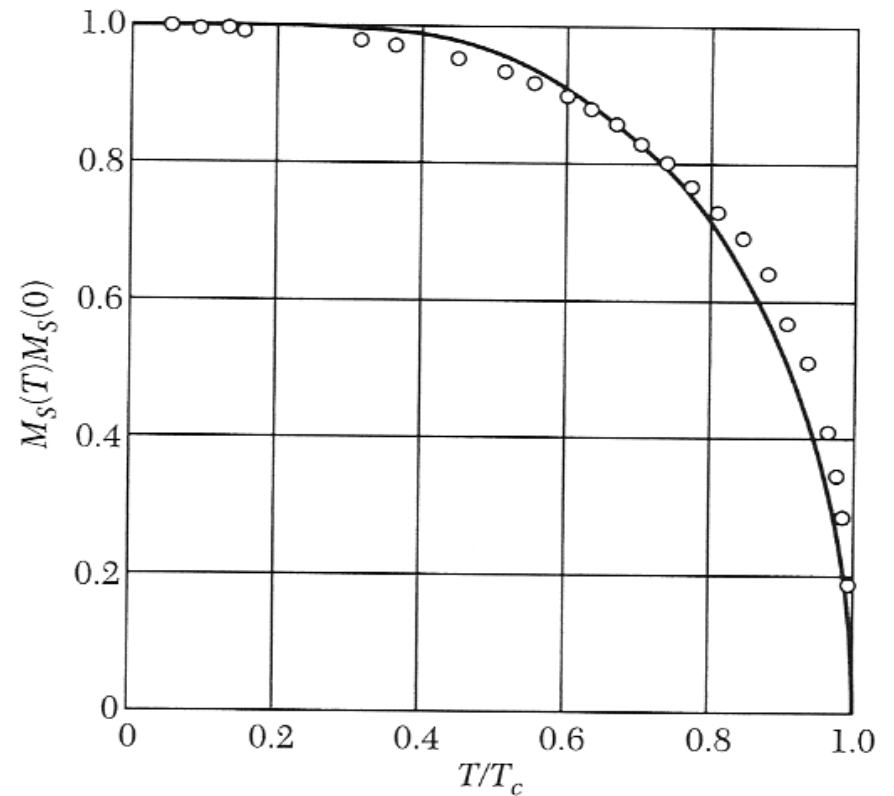
$M_s$  = saturation magnetization       $T_c$  = Curie temperature

# Mean field theory

$$M = M_S \tanh\left(\frac{T_c}{T} \frac{M}{M_S}\right)$$



$$m = \tanh\left(\frac{m}{t}\right)$$



Experimental points for Ni.

$$M_S = \frac{N}{2V} g \mu_B \quad \text{and} \quad T_c = \frac{z}{4k_B} J$$

Source: Kittel

# Ferromagnetism

---

## Material Curie temp. (K)

Co	1388	
Fe	1043	
FeOFe <sub>2</sub> O <sub>3</sub>	858	
NiOFe <sub>2</sub> O <sub>3</sub>	858	
CuOFe <sub>2</sub> O <sub>3</sub>	728	
MgOFe <sub>2</sub> O <sub>3</sub>	713	
MnBi	630	
Ni	627	
MnSb	587	
MnOFe <sub>2</sub> O <sub>3</sub>	573	
Y <sub>3</sub> Fe <sub>5</sub> O <sub>12</sub>	560	
CrO <sub>2</sub>	386	
MnAs	318	
Gd	292	
Dy	88	
EuO	69	Electrical insulator
Nd <sub>2</sub> Fe <sub>14</sub> B	353	$M_s = 10 M_s(\text{Fe})$
Sm <sub>2</sub> Co <sub>17</sub>	700	rare earth magnets

$$M_s = \frac{N}{2V} g \mu_B$$

$$T_c = \frac{z}{4k_B} J$$



# Curie - Weiss law

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$$M = \frac{1}{2} g \mu_B \frac{N}{V} \tanh \left( \frac{g \mu_B (B_{MF} + B_a)}{2k_B T} \right) \quad \vec{B}_{MF} = \frac{V}{Ng^2 \mu_B^2} zJ\vec{M}$$

Above  $T_c$  we can expand the hyperbolic tangent  $\tanh(x) \approx x$  for  $x \ll 1$

$$M \approx \frac{1}{4} g^2 \mu_B^2 \frac{N}{Vk_B T} \left( \frac{V}{Ng^2 \mu_B^2} zJM + B_a \right)$$

Solve for  $M$

$$M \approx \frac{g^2 \mu_B^2 N}{4Vk_B} \frac{B_a}{T - T_c} \quad T_c = \frac{z}{4k_B} J$$

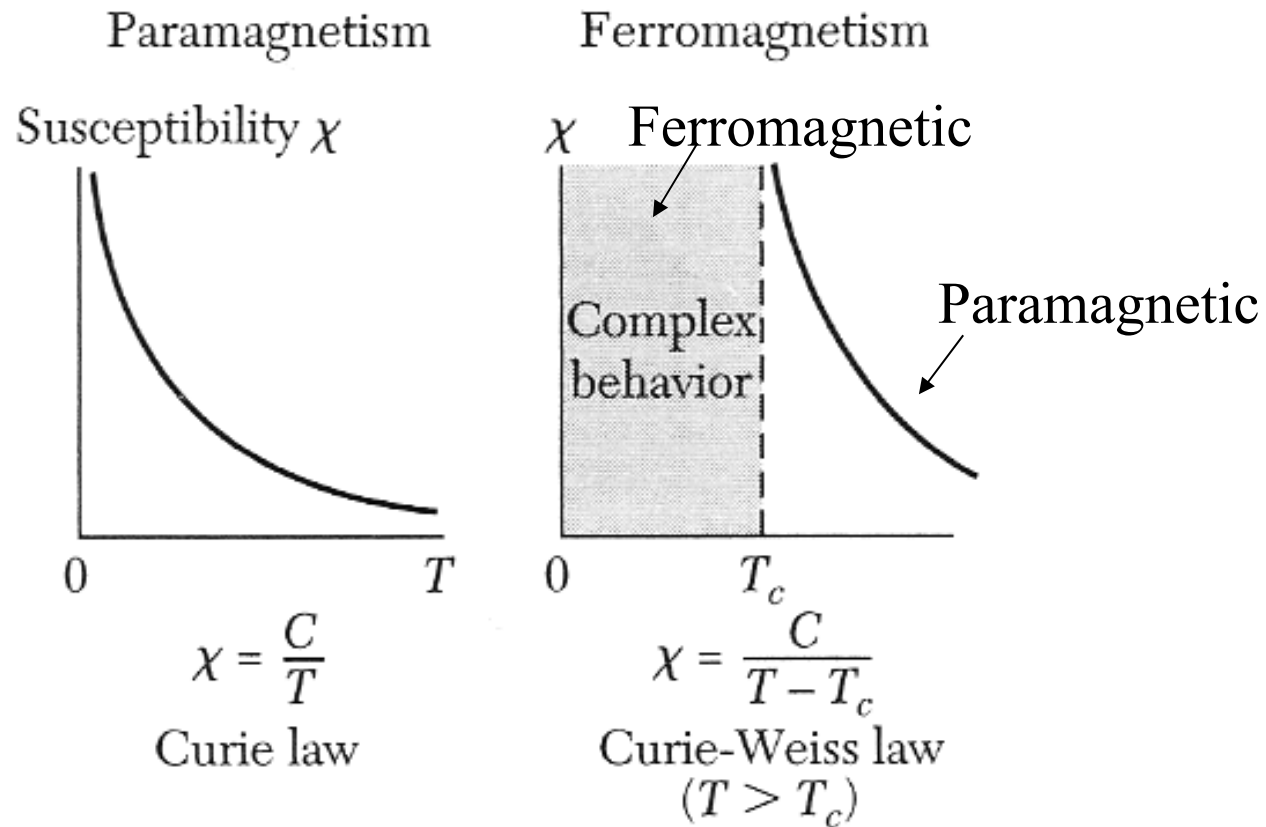
Curie Weiss Law  $\chi = \frac{dM}{dH} \approx \frac{C}{T - T_c}$

Critical fluctuations near  $T_c$

# Ferromagnets are paramagnetic above $T_c$

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Source: Kittel



Critical fluctuations near  $T_c$ .

# Magnetization of a Magnetite Single Crystal Near the Curie Point\*

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(Received January 20, 1956)

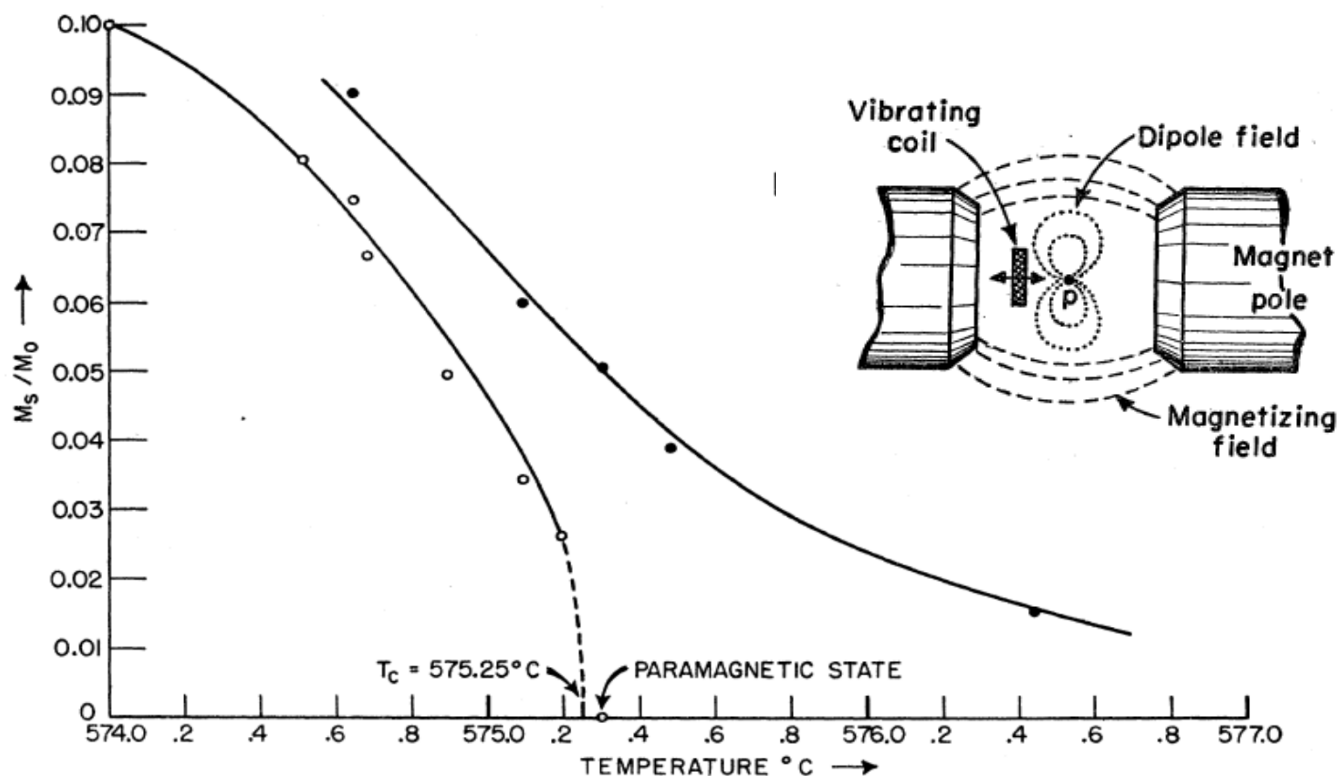


FIG. 9.  $M_s/M_0$  vs  $T$  in the [111] direction near the Curie point for single-crystal magnetite.

FIG. 2. Principle of the vibrating-coil magnetometer.

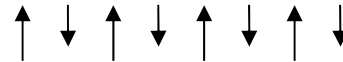
# Magnetic ordering

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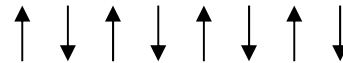
Ferromagnetism



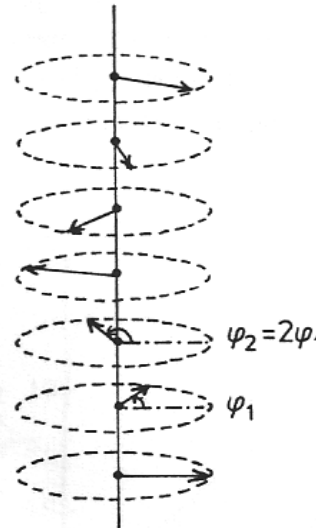
Ferrimagnetism



Antiferromagnetism



Helimagnetism



All ordered magnetic states  
have excitations called  
magnons