16. Dielectrics / Metals

Nov. 26, 2018

Dielectrics

Dielectrics used as electrical insulators should not conduct.

Large breakdown field.

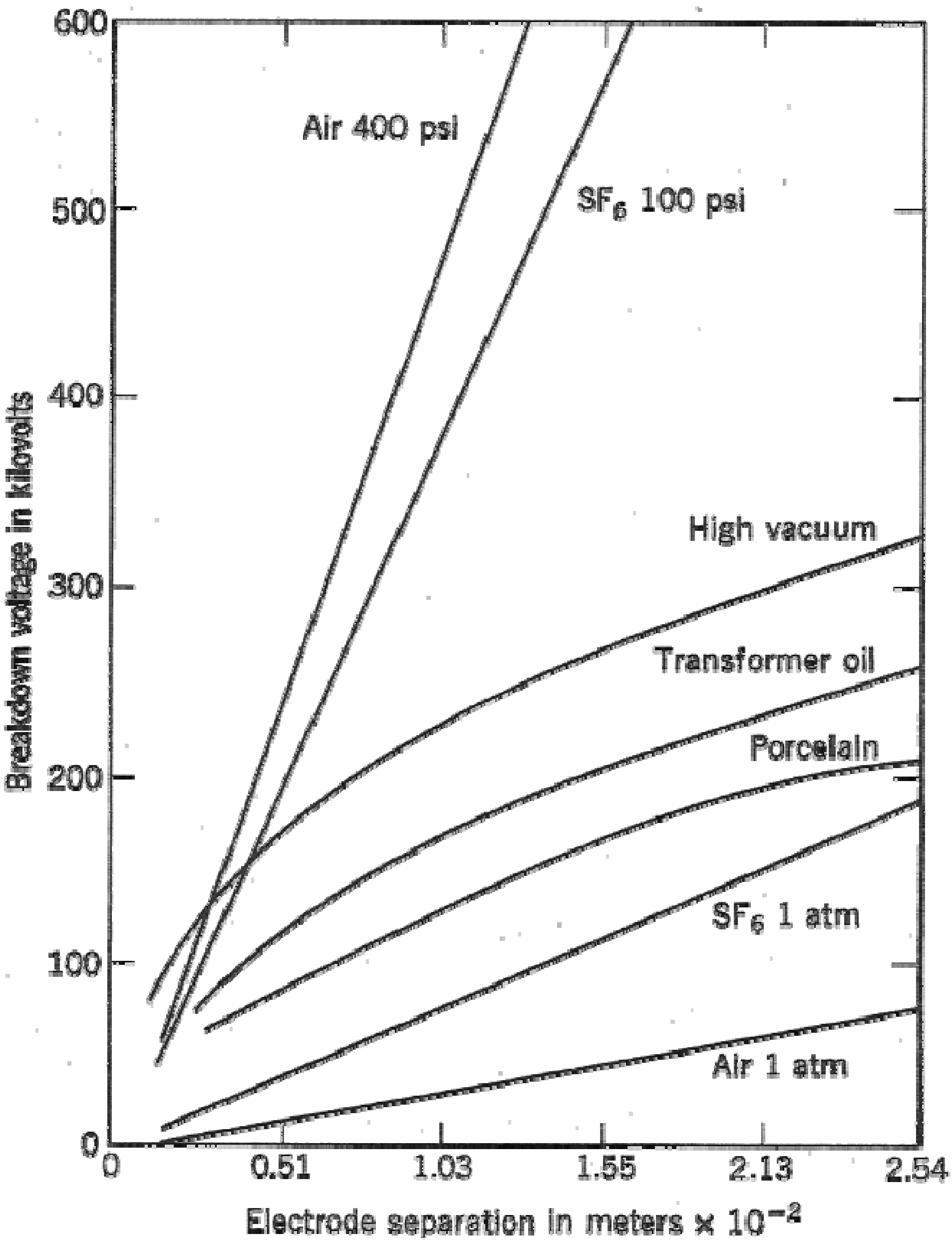
Low AC losses.

Sometimes a low dielectric constant is desired (CMOS interconnects)

Sometimes a high dielectric constant is desired (supercapacitors).

Breakdown field

Typically 10^5-10^6 V/cm



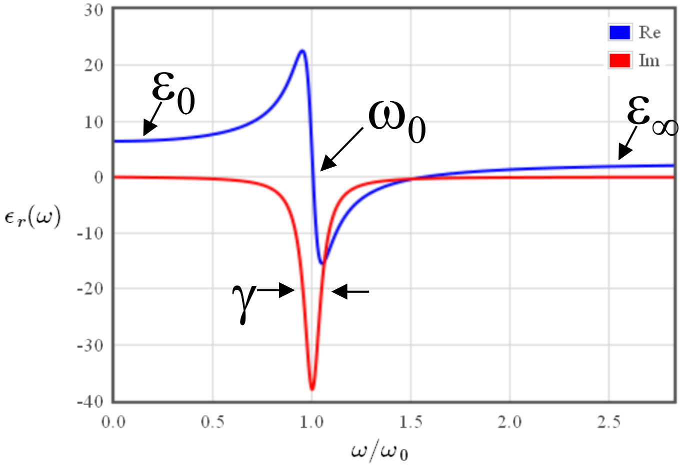
AC losses - loss tangent

In an ideal capacitor, current leads voltage by 90°.

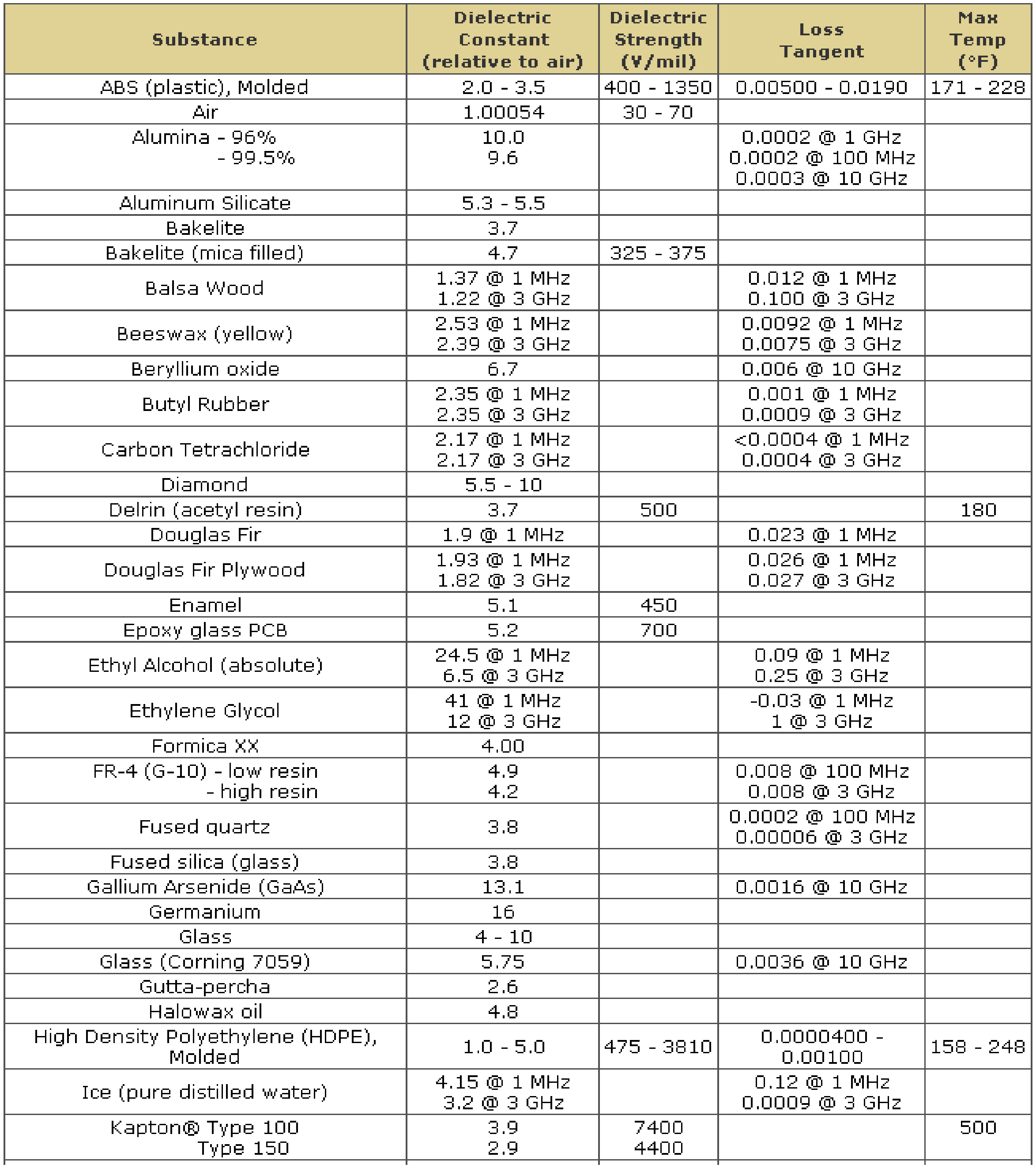
Because the dielectric constant is complex, in real materials current leads voltage by 90° - δ.

$$\text{Power loss} = \frac{\omega \varepsilon_1 V_0^2}{2} \tan \delta$$

Becomes more of an issue at high frequencies (microwaves)



Loss tangent



Substance	Dielectric Constant (relative to air)	Dielectric Strength (V/mil)	Loss Tangent	Max Temp (°F)
ABS (plastic), Molded	2.0 - 3.5	400 - 1350	0.00500 - 0.0190	171 - 228
Air	1.00054	30 - 70		
Alumina - 96% - 99.5%	10.0 9.6		0.0002 @ 1 GHz 0.0002 @ 100 MHz 0.0003 @ 10 GHz	
Aluminum Silicate	5.3 - 5.5			
Bakelite	3.7			
Bakelite (mica filled)	4.7	325 - 375		
Balsa Wood	1.37 @ 1 MHz 1.22 @ 3 GHz		0.012 @ 1 MHz 0.100 @ 3 GHz	
Beeswax (yellow)	2.53 @ 1 MHz 2.39 @ 3 GHz		0.0092 @ 1 MHz 0.0075 @ 3 GHz	
Beryllium oxide	6.7		0.006 @ 10 GHz	
Butyl Rubber	2.35 @ 1 MHz 2.35 @ 3 GHz		0.001 @ 1 MHz 0.0009 @ 3 GHz	
Carbon Tetrachloride	2.17 @ 1 MHz 2.17 @ 3 GHz		<0.0004 @ 1 MHz 0.0004 @ 3 GHz	
Diamond	5.5 - 10			
Delrin (acetyl resin)	3.7	500		180
Douglas Fir	1.9 @ 1 MHz		0.023 @ 1 MHz	
Douglas Fir Plywood	1.93 @ 1 MHz 1.82 @ 3 GHz		0.026 @ 1 MHz 0.027 @ 3 GHz	
Enamel	5.1	450		
Epoxy glass PCB	5.2	700		
Ethyl Alcohol (absolute)	24.5 @ 1 MHz 6.5 @ 3 GHz		0.09 @ 1 MHz 0.25 @ 3 GHz	
Ethylene Glycol	41 @ 1 MHz 12 @ 3 GHz		-0.03 @ 1 MHz 1 @ 3 GHz	
Formica XX	4.00			
FR-4 (G-10) - low resin - high resin	4.9 4.2		0.008 @ 100 MHz 0.008 @ 3 GHz	
Fused quartz	3.8		0.0002 @ 100 MHz 0.00006 @ 3 GHz	
Fused silica (glass)	3.8			
Gallium Arsenide (GaAs)	13.1		0.0016 @ 10 GHz	
Germanium	16			
Glass	4 - 10			
Glass (Corning 7059)	5.75		0.0036 @ 10 GHz	
Gutta-percha	2.6			
Halowax oil	4.8			
High Density Polyethylene (HDPE), Molded	1.0 - 5.0	475 - 3810	0.0000400 - 0.00100	158 - 248
Ice (pure distilled water)	4.15 @ 1 MHz 3.2 @ 3 GHz		0.12 @ 1 MHz 0.0009 @ 3 GHz	
Kapton® Type 100 Type 150	3.9 2.9	7400 4400		500

http://www.rfcafe.com/references/electrical/dielectric-constants-strengths.htm

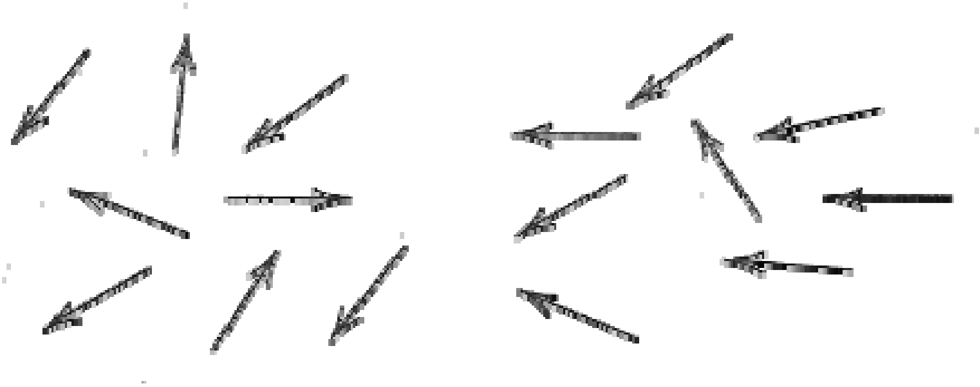
Polarizability

Overdamped modes	• Orientation polarizability • Space charge polarizability
Underdamped modes	• Ionic polarizability • Electronic polarizability

Orientation (dipolar) Polarizability

For materials (gases, liquids, solids) with a permanent dipole moment.

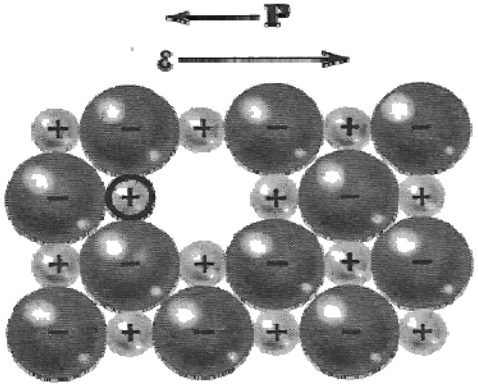
The theory is very similar to paramagnetism.

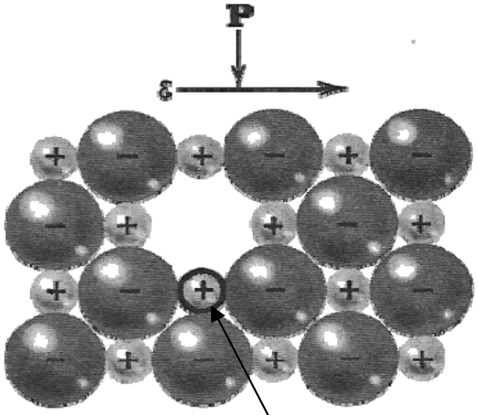


$$\chi \propto \frac{1}{T}$$ Curie law

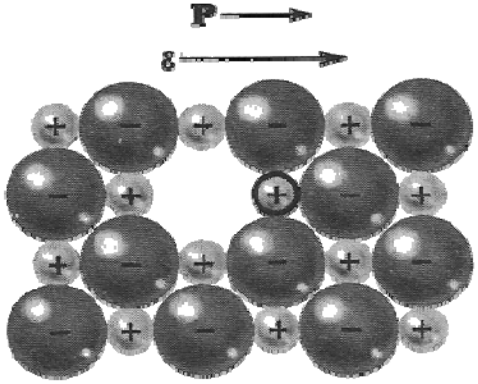
Orientation Polarizability

Ion jumps.

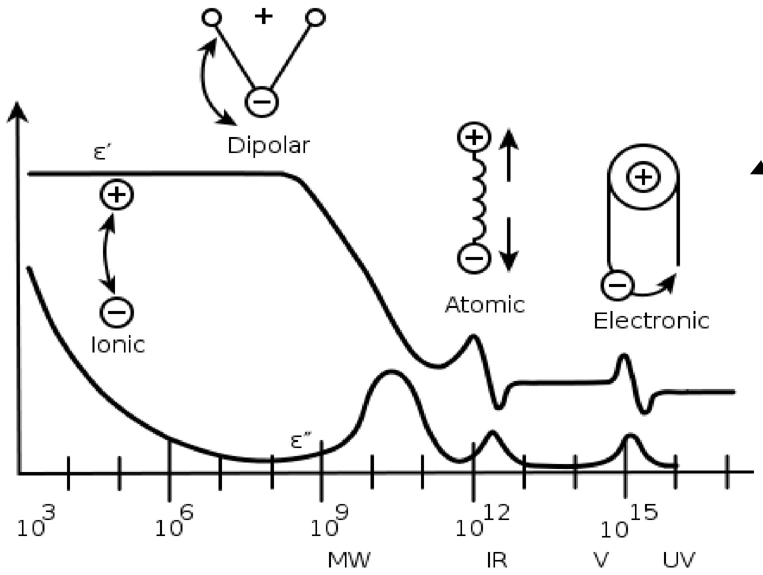




doubly ionized

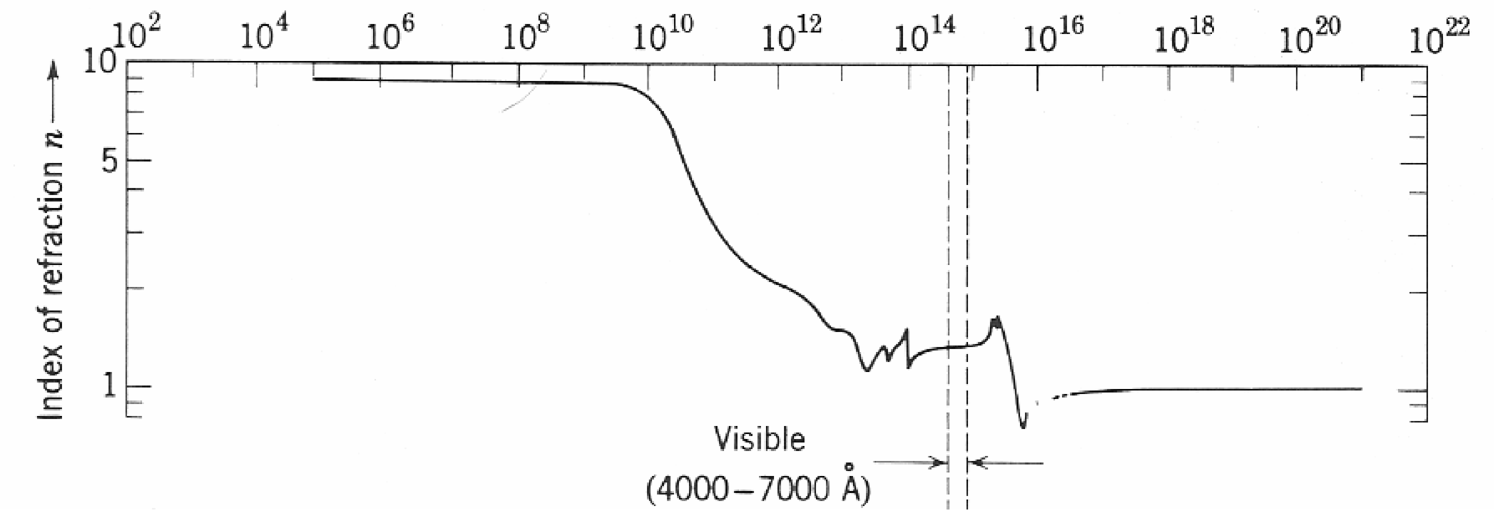


Water



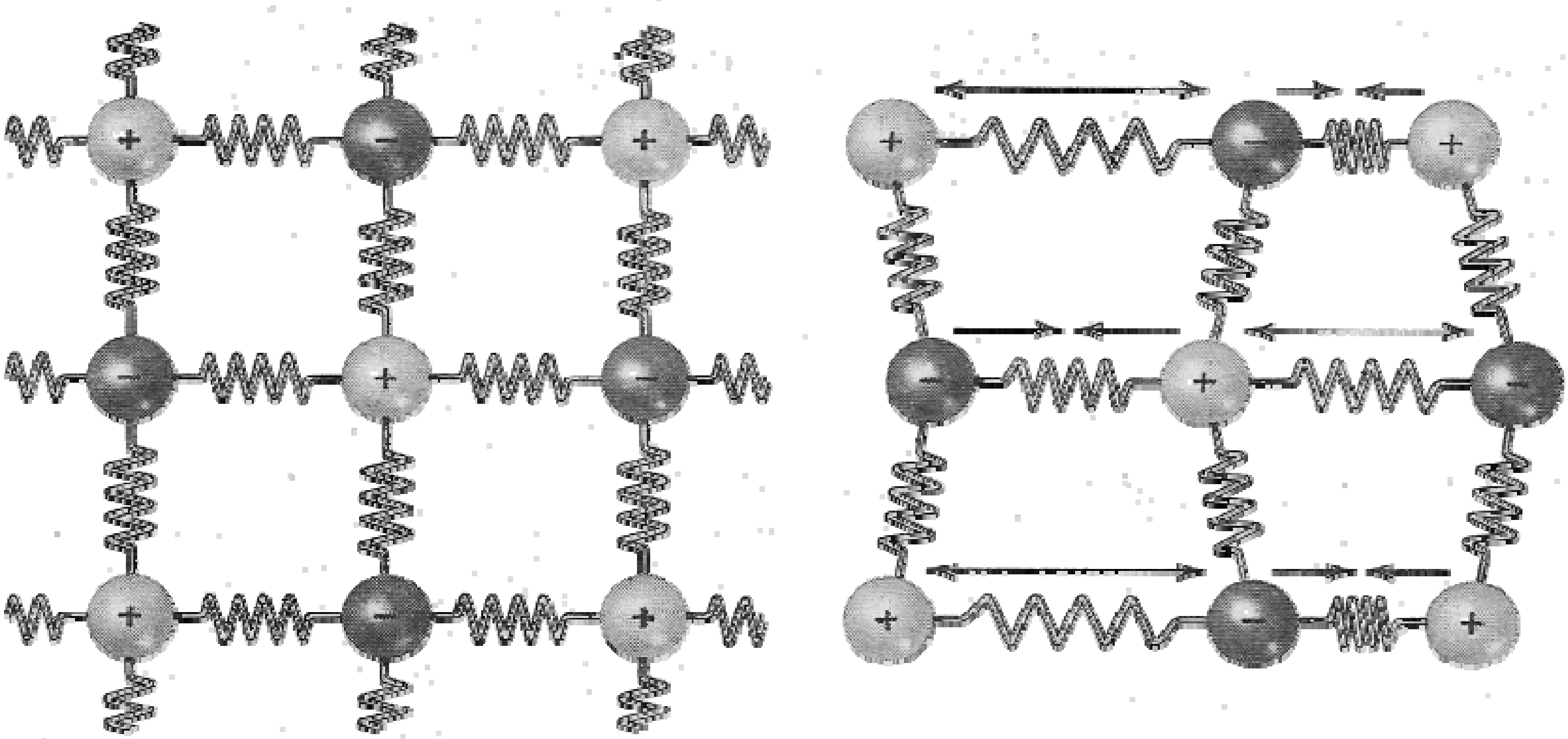
Schematic dielectric function of water from Wikipedia

Source: Classical Electrodynamics, J.D. Jackson



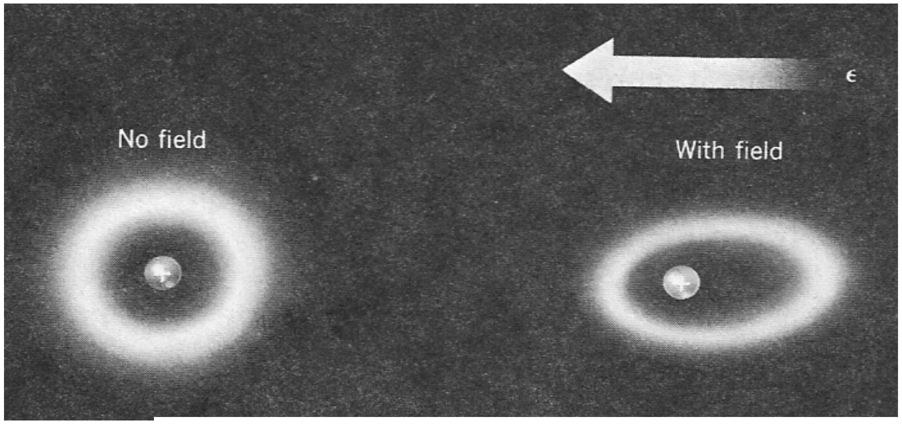
Ionic Polarizability

Displacement of ions of opposite sign. Only in ionic substances.



This is an underdamped mode in the infrared.

Electronic polarizability (all materials)



dipole moments

$$\vec{P} = N\vec{p} = N\alpha\vec{E}$$

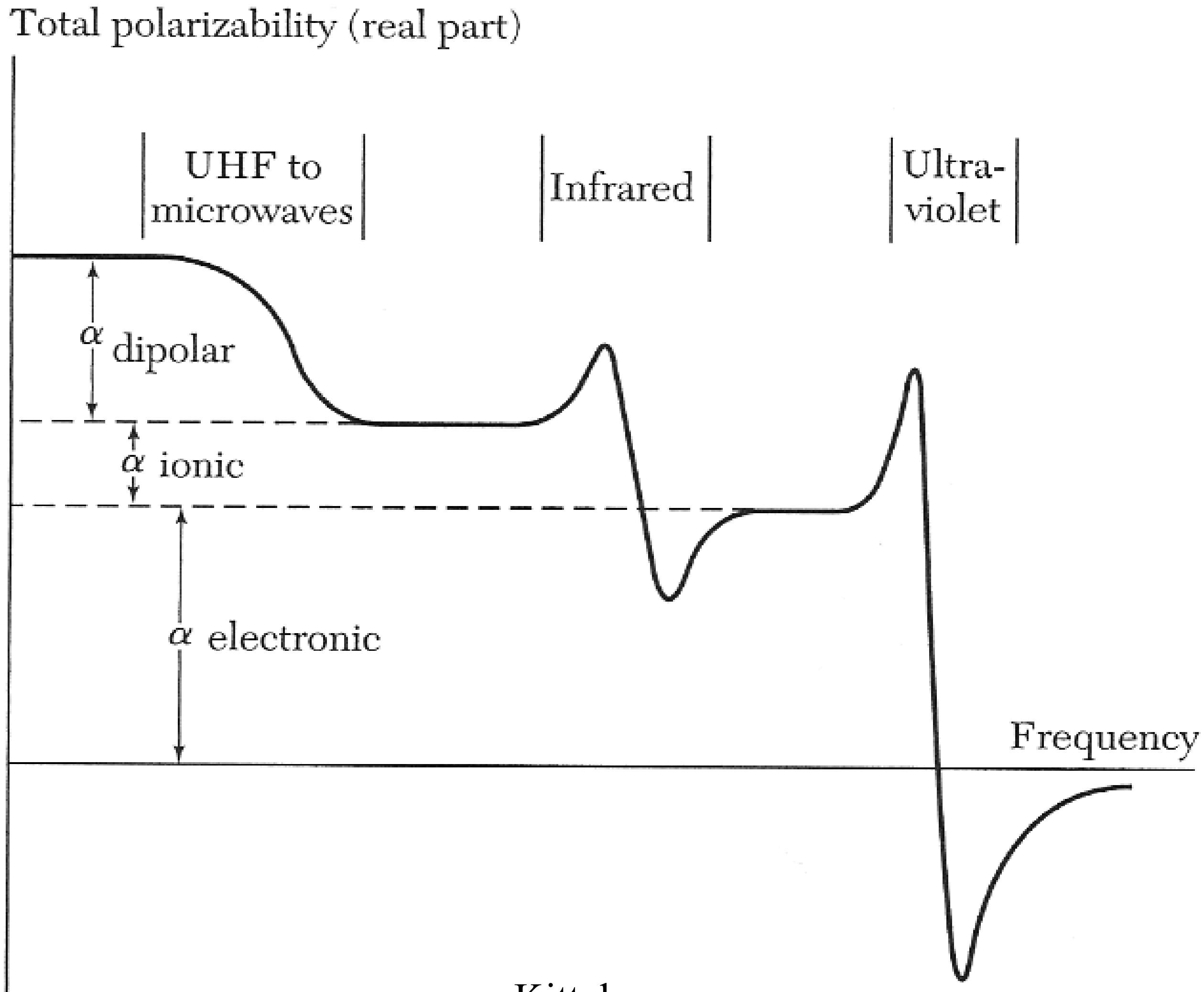
density

polarizability

Table 1 Electronic polarizabilities of atoms and ions, in 10^{-24} cm^3

			He	Li^{+}	Be^{2+}	B^{3+}	C^{4+}
Pauling			0.201	0.029	0.008	0.003	0.0013
JS				0.029			
	O^{2-}	F^{-}	Ne	Na^{+}	Mg^{2+}	Al^{3+}	Si^{4+}
Pauling	3.88	1.04	0.390	0.179	0.094	0.052	0.0165
JS-(TKS)	(2.4)	0.858		0.290			
	S^{2-}	Cl^{-}	Ar	K^{+}	Ca^{2+}	Se^{3+}	Ti^{4+}
Pauling	10.2	3.66	1.62	0.83	0.47	0.286	0.185
JS-(TKS)	(5.5)	2.947		1.133	(1.1)		(0.19)
	Se^{2-}	Br^{-}	Kr	Rb^{+}	Sr^{2+}	Y^{3+}	Zr^{4+}
Pauling	10.5	4.77	2.46	1.40	0.86	0.55	0.37
JS-(TKS)	(7.)	4.091		1.679	(1.6)		
	Te^{2-}	I^{-}	Xe	Cs^{+}	Ba^{2+}	La^{3+}	Ce^{4+}
Pauling	14.0	7.10	3.99	2.42	1.55	1.04	0.73
JS-(TKS)	(9.)	6.116		2.743	(2.5)		

Polarizability

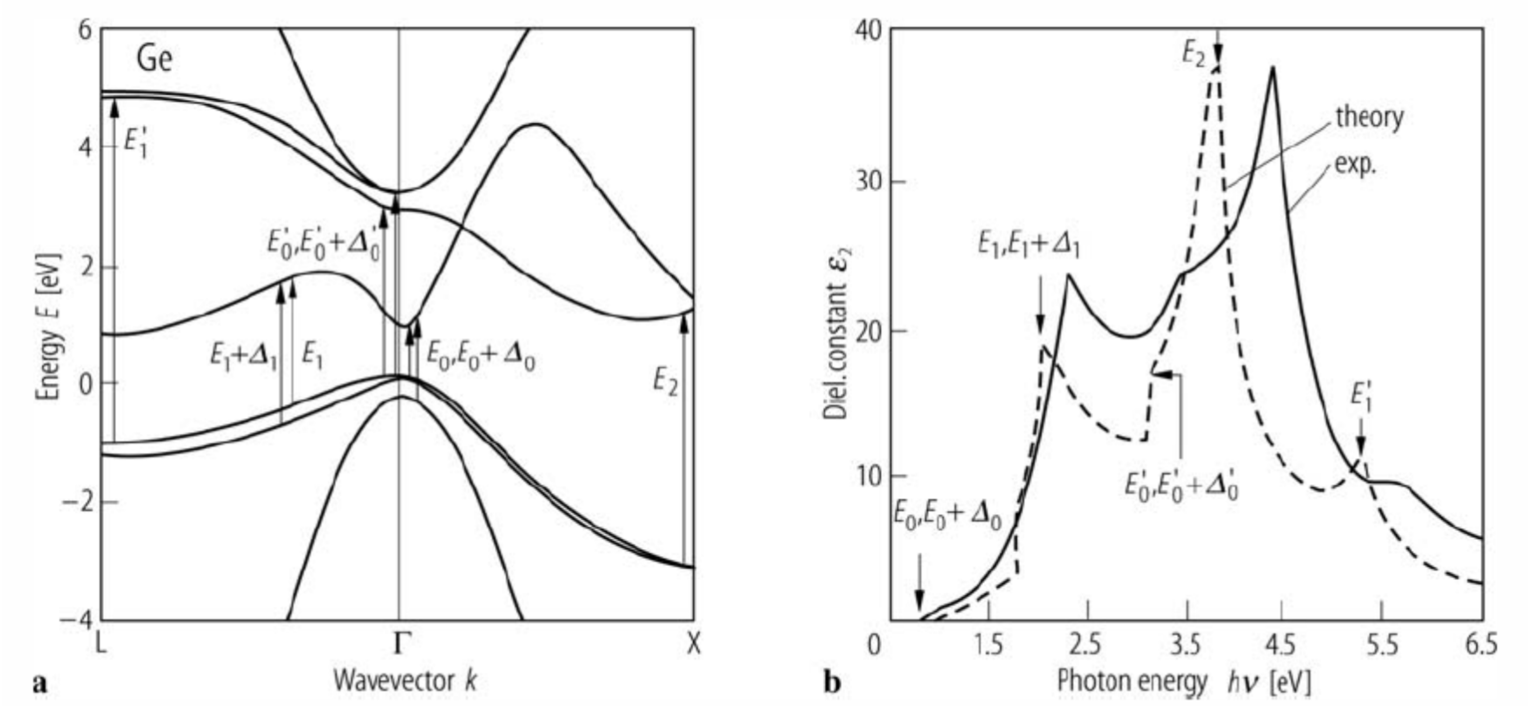


Kittel

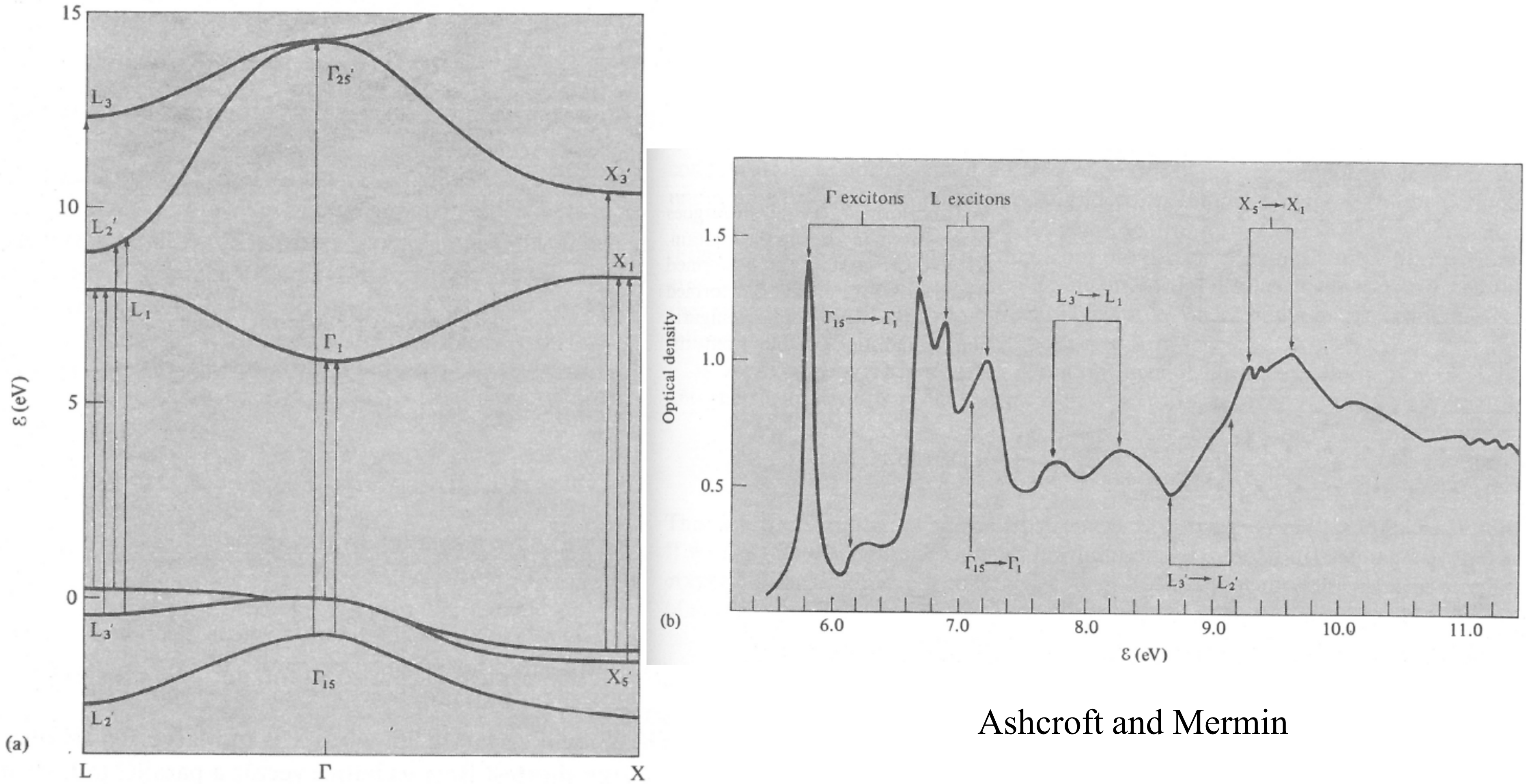
Inter- and intraband transitions

When the bands are parallel, there is a peak in the absorption (ε")

$$\hbar\omega = E_c(\vec{k}) - E_v(\vec{k})$$



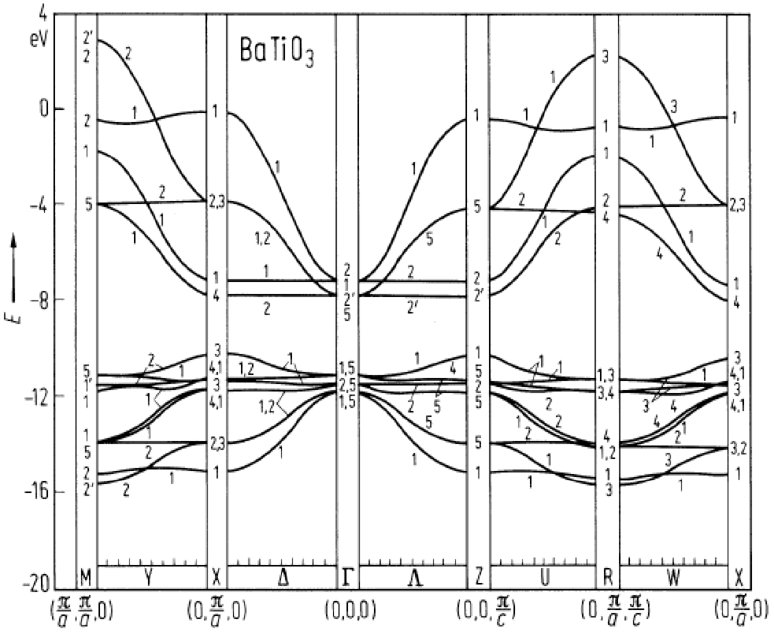
Optical spectroscopy has developed into the most important experimental tool for band structure determination. - Kittel

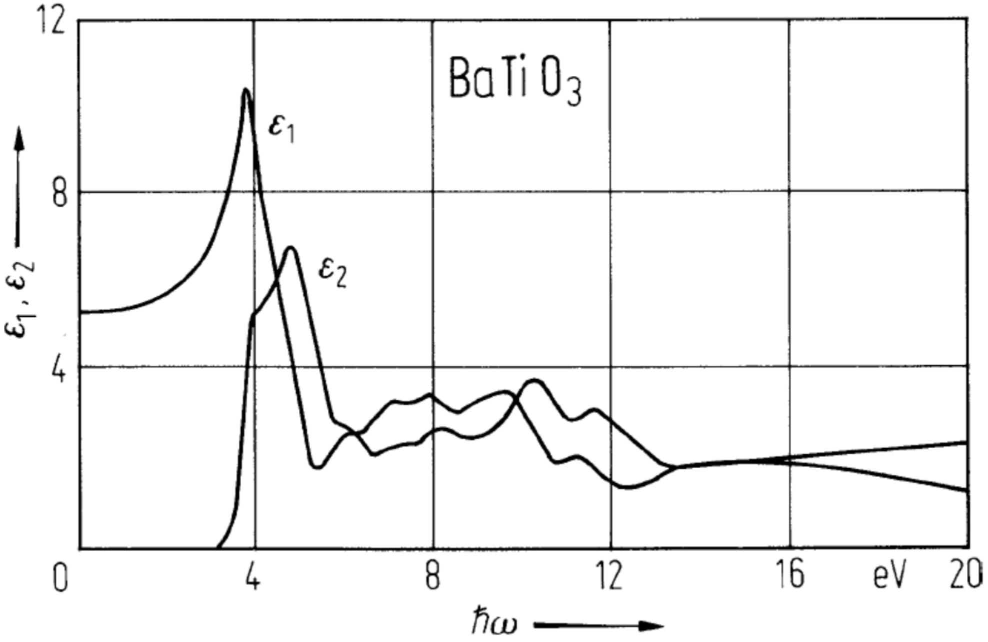


Ashcroft and Mermin

Figure 30.11
(a) The band structure of KI as inferred by J. C. Phillips (*Phys. Rev.* **136**, A1705 (1964) from its optical absorption spectrum. (b) The exciton spectrum associated with the various valence and conduction band maxima and minima. (After J. E. Eby, K. J. Teegarden, and D. B. Dutton, *Phys. Rev.* **116**, 1099 (1959), as summarized by J. C. Phillips, "Fundamental Optical Spectra of Solids," in *Solid State Physics*, vol. 18, Academic Press, New York, 1966.)

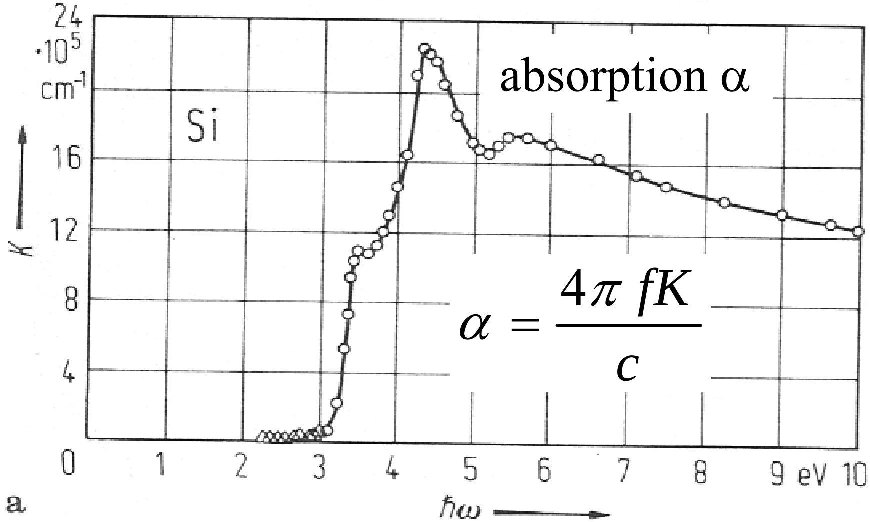
Dielectric function of $BaTiO_3$





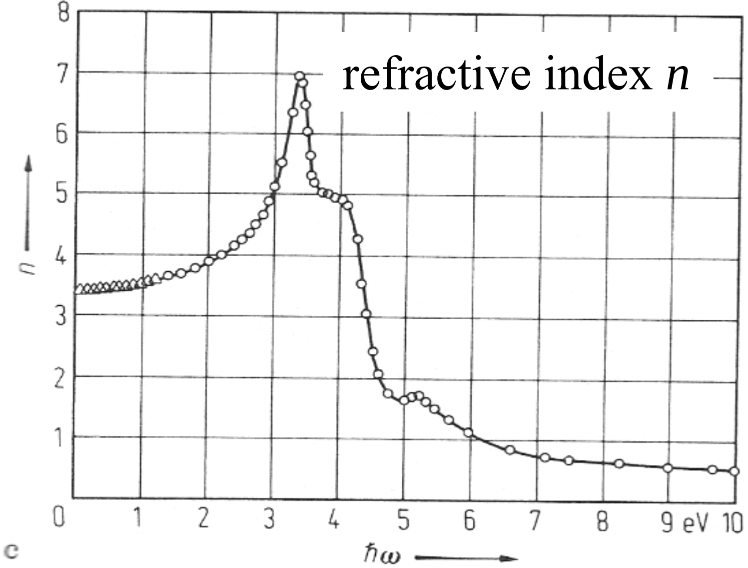
Dielectric function of silicon

$$\sqrt{\varepsilon(\omega)} = n(\omega) + iK(\omega)$$

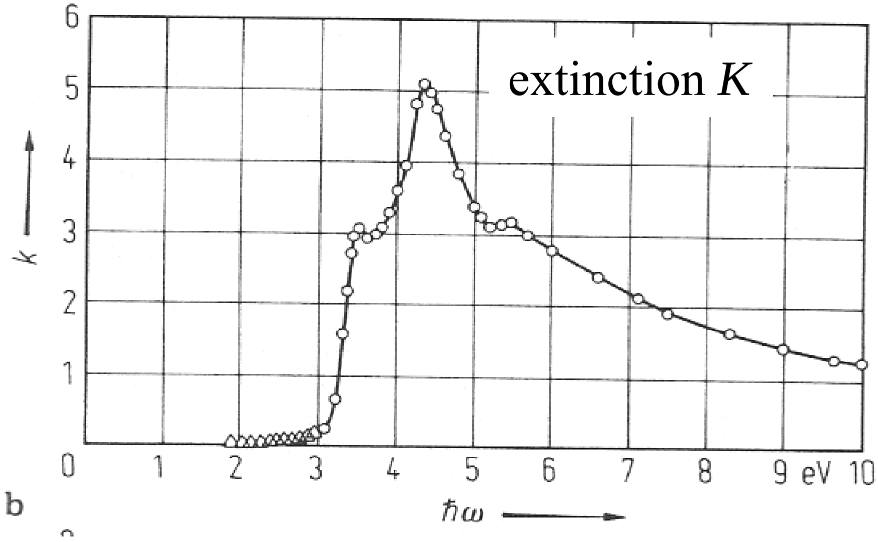


absorption α

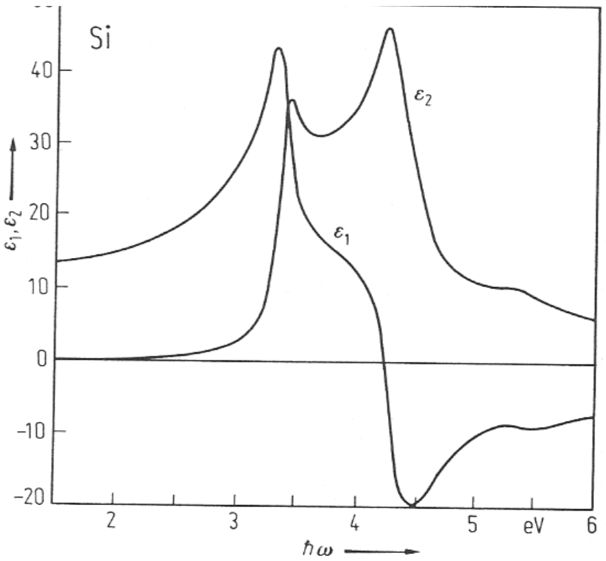
$$\alpha = \frac{4\pi fK}{c}$$



refractive index n



extinction K



AC Conductivity

For constant voltage, conductors conduct and insulators don't.

For low ac voltages in a conductor, electric field and the electron velocity are in-phase, electric field and electron position are out-of-phase.

For low ac voltages in an insulator, electric field and the electron position are in-phase, electric field and electron velocity are out-of-phase.

At high (optical) frequencies the in-phase and out-of-phase component of the response is described by the dielectric function.

Conductivity / Dielectric function

Harmonic dependence $v = v(\omega)e^{i\omega t}, \quad x = x(\omega)e^{i\omega t}, \quad E = E(\omega)e^{i\omega t}$

$$v(\omega) = i\omega x(\omega)$$

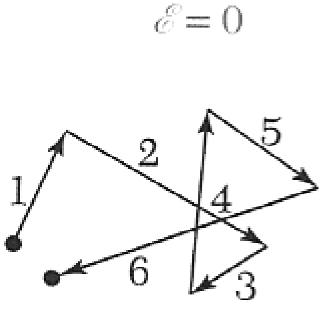
$$\chi(\omega) = \frac{P(\omega)}{\varepsilon_0 E(\omega)} = \frac{-nex(\omega)}{\varepsilon_0 E(\omega)} \qquad \sigma(\omega) = \frac{j(\omega)}{E(\omega)} = \frac{-nev(\omega)}{E(\omega)} = \frac{-i\omega nex(\omega)}{E(\omega)}$$

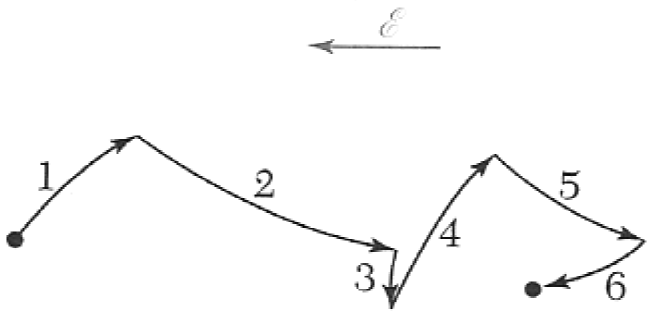
$$\chi(\omega) = \frac{\sigma(\omega)}{i\omega\varepsilon_0}$$

$$\boxed{\varepsilon(\omega) = 1 + \chi = 1 + \frac{\sigma(\omega)}{i\omega\varepsilon_0}}$$

Below about 100 GHz the frequency dependent conductivity is normally used.
Above about 100 GHz the dielectric function is used (optical experiments).

Diffusive transport (low frequencies)





$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{\vec{v}_d}{\tau_{\text{sc}}}$$

$$\tau_{\text{sc}} = \frac{\mu m}{e}$$

$$-\frac{e\tau_{\text{sc}}}{m}\vec{E} = \vec{v}_d$$

$$\sigma = ne\mu = \frac{ne^2\tau}{m}$$

$$-\mu_e\vec{E} = \vec{v}_d$$

Diffusive metal

The current is related to the electric field

$$j_n = \sigma_{nm} E_m \qquad v_n = -\mu_{nm} E_m$$

Steady state solution

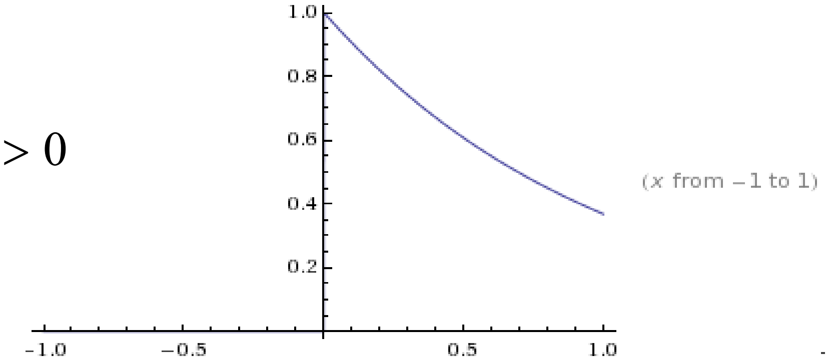
The differential equation that describes how the velocity changes in time is:

$$m\frac{dv(t)}{dt} + \frac{ev(t)}{\mu} = -eE(t)$$

Inertial term

The impulse response function :

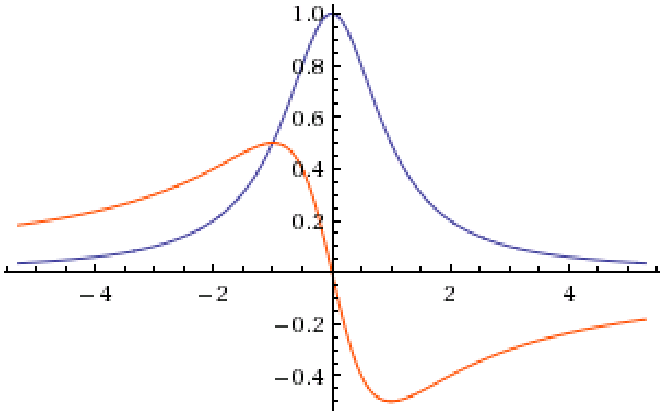
$$g(t) = \frac{1}{m}\exp\left(\frac{-et}{\mu m}\right) \qquad t > 0$$



Diffusive metal

The differential equation is:

$$m\frac{dv(t)}{dt}+\frac{ev(t)}{\mu}=-eE(t)$$



Assume a harmonic solution $E(\omega)e^{i\omega t}$, $v(\omega)e^{i\omega t}$

$$\left(-\frac{i\omega m}{e}-\frac{1}{\mu}\right)v(\omega)=E(\omega)$$

$$\frac{v(\omega)}{E(\omega)}=\left(-\frac{i\omega m}{e}-\frac{1}{\mu}\right)^{-1}=-\mu\left(1+i\omega\tau\right)^{-1}=\frac{-\mu\left(1-i\omega\tau\right)}{1+\omega^2\tau^2}$$

$$\sigma(\omega)=\frac{j(\omega)}{E(\omega)}=-ne\frac{v(\omega)}{E(\omega)}=ne\mu\left(\frac{1-i\omega\tau}{1+\omega^2\tau^2}\right)$$

$$\tau=\frac{\mu m}{e}\longleftarrow\text{Scattering time}$$

$$\sigma(\text{low }\omega)=ne\mu$$

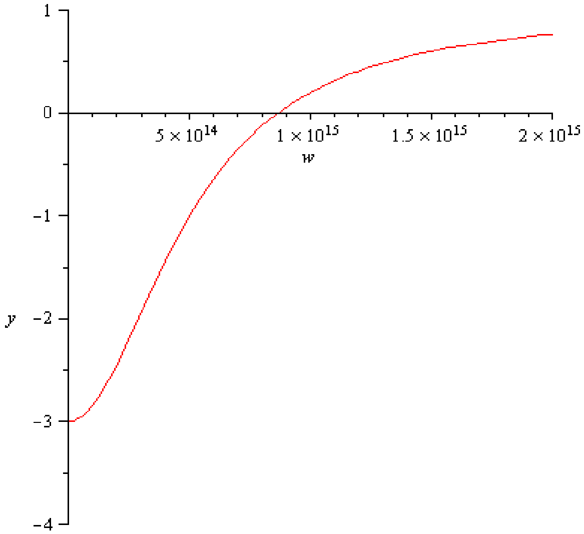
$$\sigma(\text{high }\omega)=\frac{-ine^2}{\omega m}$$

Diffusive metal

$$\chi(\omega)=\frac{\sigma(\omega)}{i\omega\varepsilon_0}=\frac{ne\mu}{i\omega\varepsilon_0}\left(\frac{1-i\omega\tau}{1+\omega^2\tau^2}\right)$$

$$\varepsilon(\omega)=1+\chi=1-\frac{ne\mu}{\omega\varepsilon_0}\left(\frac{\omega\tau+i}{1+\omega^2\tau^2}\right)$$

$$\varepsilon(\omega)=1-\omega_p^2\left(\frac{\omega\tau^2+i\tau}{\omega+\omega^3\tau^2}\right)\qquad \omega_p^2=\frac{ne^2}{m\varepsilon_0}$$

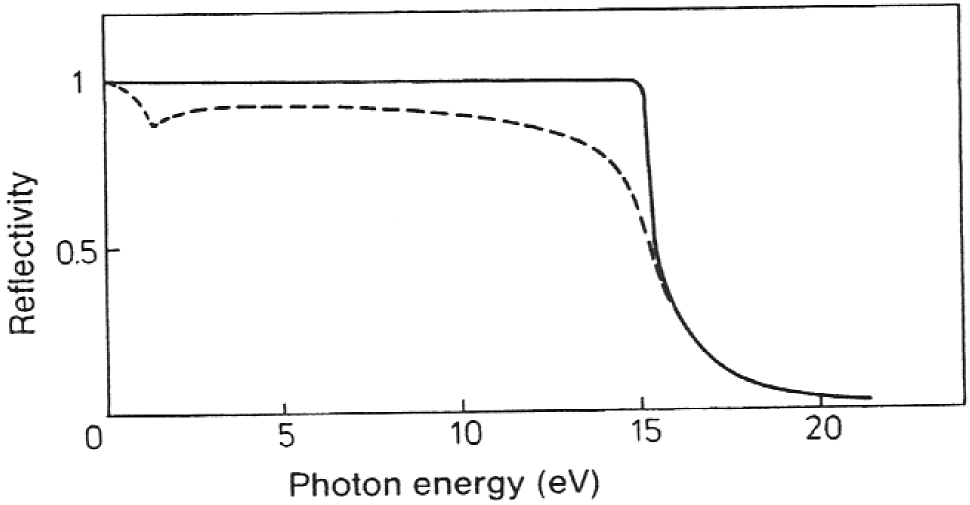


Take the limit as τ goes to infinity

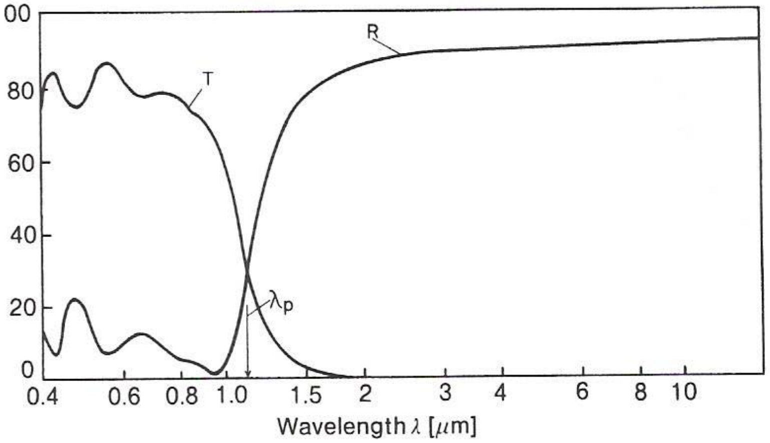
$$\varepsilon'(\omega)=1-\frac{\omega_p^2}{\omega^2}\qquad \varepsilon''(\omega)=\begin{matrix}0 & \text{for} & \omega>0\\ \infty & \text{for} & \omega=0\end{matrix}$$

low frequency metal / high frequency insulator

Ibach & Lueth



Aluminum



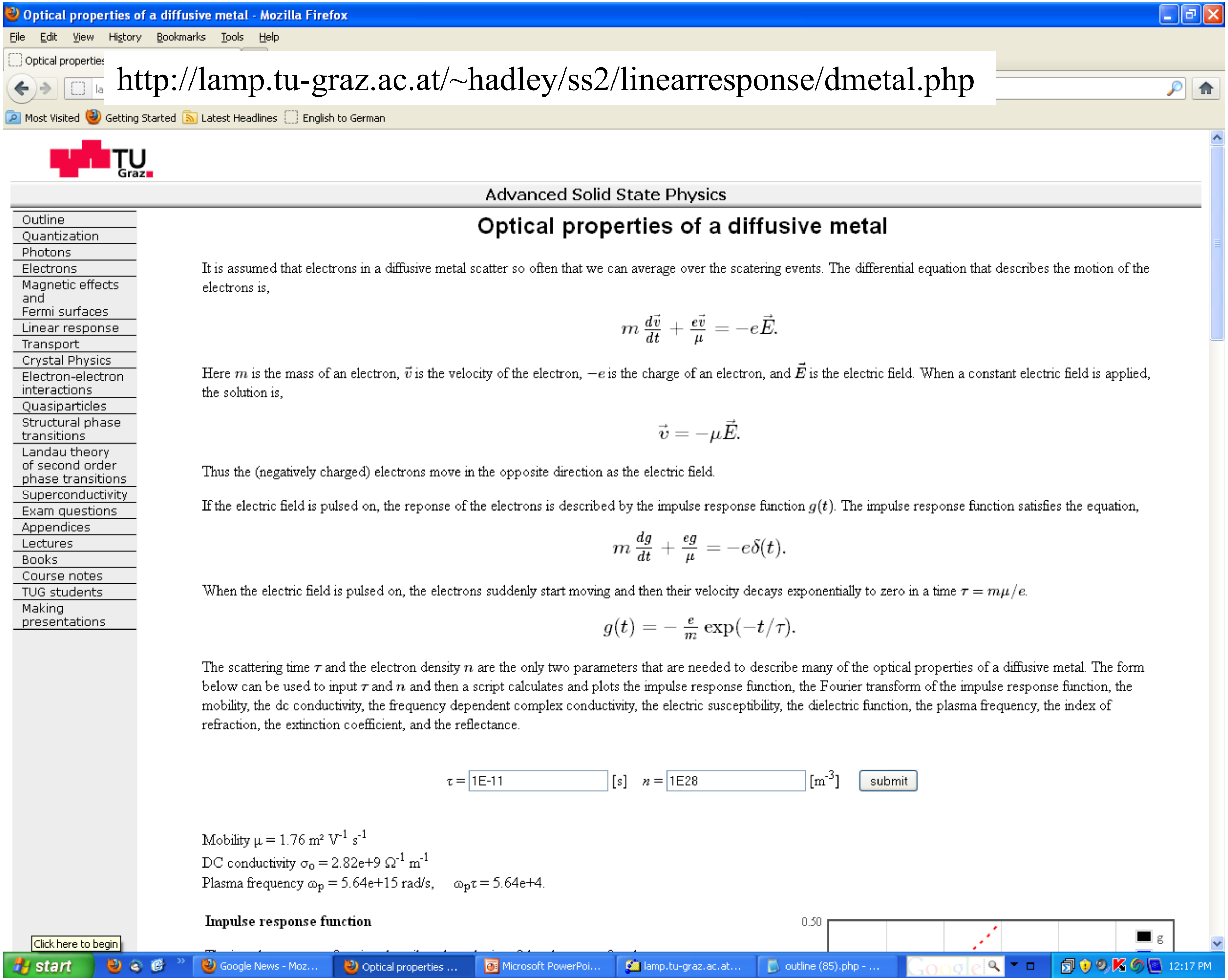
ITO

Conducting transparent contacts for LEDs and Solar cells

Windows that reflect infrared

Reflection of radio waves from ionosphere

$$\omega_p^2 \approx \frac{ne^2}{\varepsilon_0 m}$$



http://lamp.tu-graz.ac.at/~hadley/ss2/linearresponse/dmetal.php

Advanced Solid State Physics

Optical properties of a diffusive metal

It is assumed that electrons in a diffusive metal scatter so often that we can average over the scatering events. The differential equation that describes the motion of the electrons is,

$$m\frac{d\vec{v}}{dt}+\frac{e\vec{v}}{\mu}=-e\vec{E}.$$

Here m is the mass of an electron, $\vec{v}$ is the velocity of the electron, $-e$ is the charge of an electron, and $\vec{E}$ is the electric field. When a constant electric field is applied, the solution is,

$$\vec{v}=-\mu\vec{E}.$$

Thus the (negatively charged) electrons move in the opposite direction as the electric field.

If the electric field is pulsed on, the reponse of the electrons is described by the impulse response function $g(t)$. The impulse response function satisfies the equation,

$$m\frac{dg}{dt}+\frac{eg}{\mu}=-e\delta(t).$$

When the electric field is pulsed on, the electrons suddenly start moving and then their velocity decays exponentially to zero in a time $\tau=m\mu/e$.

$$g(t)=-\frac{e}{m}\exp(-t/\tau).$$

The scattering time τ and the electron density n are the only two parameters that are needed to describe many of the optical properties of a diffusive metal. The form below can be used to input τ and n and then a script calculates and plots the impulse response function, the Fourier transform of the impulse response function, the mobility, the dc conductivity, the frequency dependent complex conductivity, the electric susceptibility, the dielectric function, the plasma frequency, the index of refraction, the extinction coefficient, and the reflectance.

τ = 1E-11 [s] n = 1E28 [m^{-3}]

Mobility $\mu = 1.76$ m² V^{-1} s^{-1}

DC conductivity $\sigma_0 = 2.82e+9$ Ω^{-1} m^{-1}

Plasma frequency $\omega_p = 5.64e+15$ rad/s, $\omega_p\tau = 5.64e+4$.

Impulse response function

Print version

Advanced Solid State Physics

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- Superconductivity
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- Books
- Course notes
- TUG students
- Making presentations

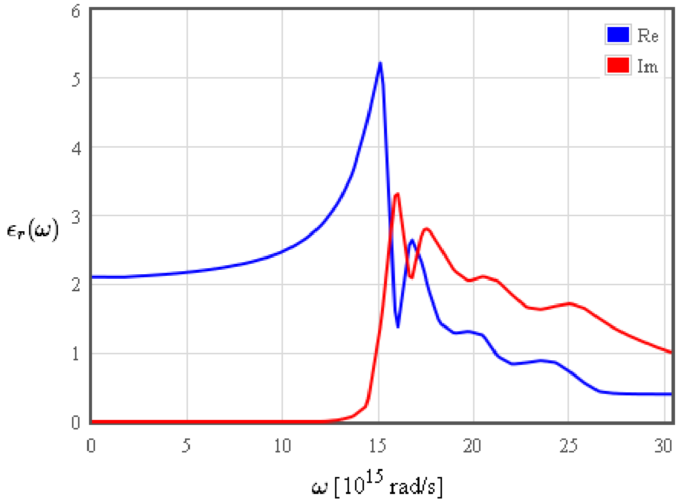
Cu | Si | SiO2 | diamond

The optical properties of SiO$_2$ (glass)

nanophotonics.csic.es

Dielectric function

The relative dielectric constant describes the relationship between the electric displacement $\vec{D}$ and the electric field $\vec{E}$, $\vec{D} = \epsilon_r\epsilon_0\vec{E} = \vec{P} + \epsilon_0\vec{E}$.

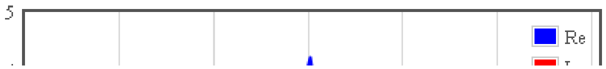


There are two conventions for dielectric function. Either it is assumed that the time dependence of $\vec{D}$, $\vec{P}$, and $\vec{E}$ is $\exp(-i\omega t)$ and the plot of the dielectric function looks as it is shown above, or it is assumed that the time dependence of $\vec{D}$, $\vec{P}$, and $\vec{E}$ is $\exp(i\omega t)$ and the imaginary part of the has the opposite sign as in the plot above. Here we will assume a time dependence of $\exp(-i\omega t)$.

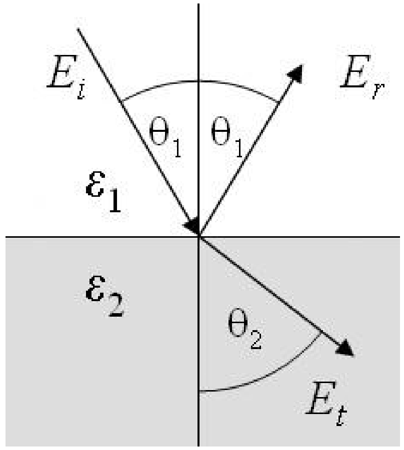
Electric susceptibility

The electric susceptibility χ_E describes the relationship between the polarization $\vec{P}$ and the electric field $\vec{E}$, $\vec{P} = \epsilon_0\chi_E\vec{E}$.

$$\chi_E = \epsilon_r - 1$$



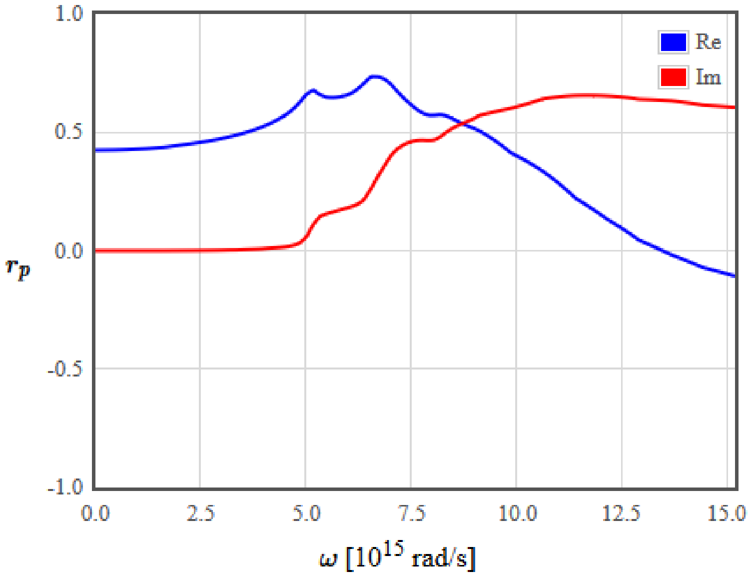
Ellipsometry

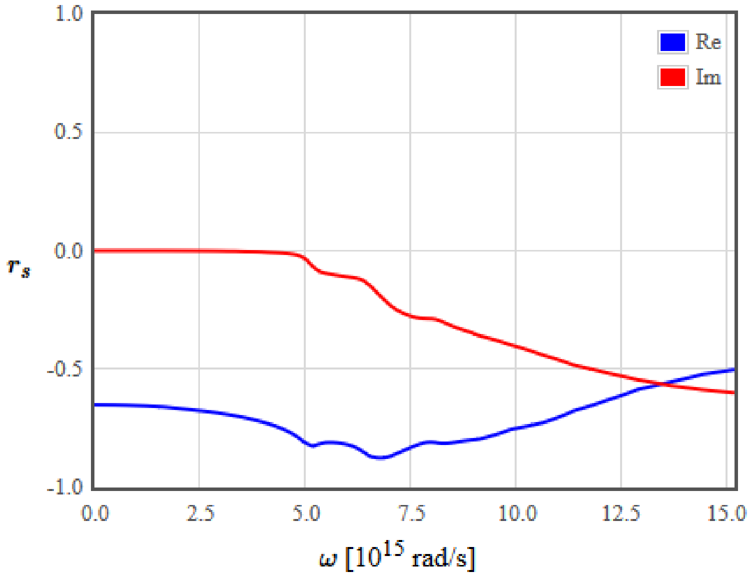


$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

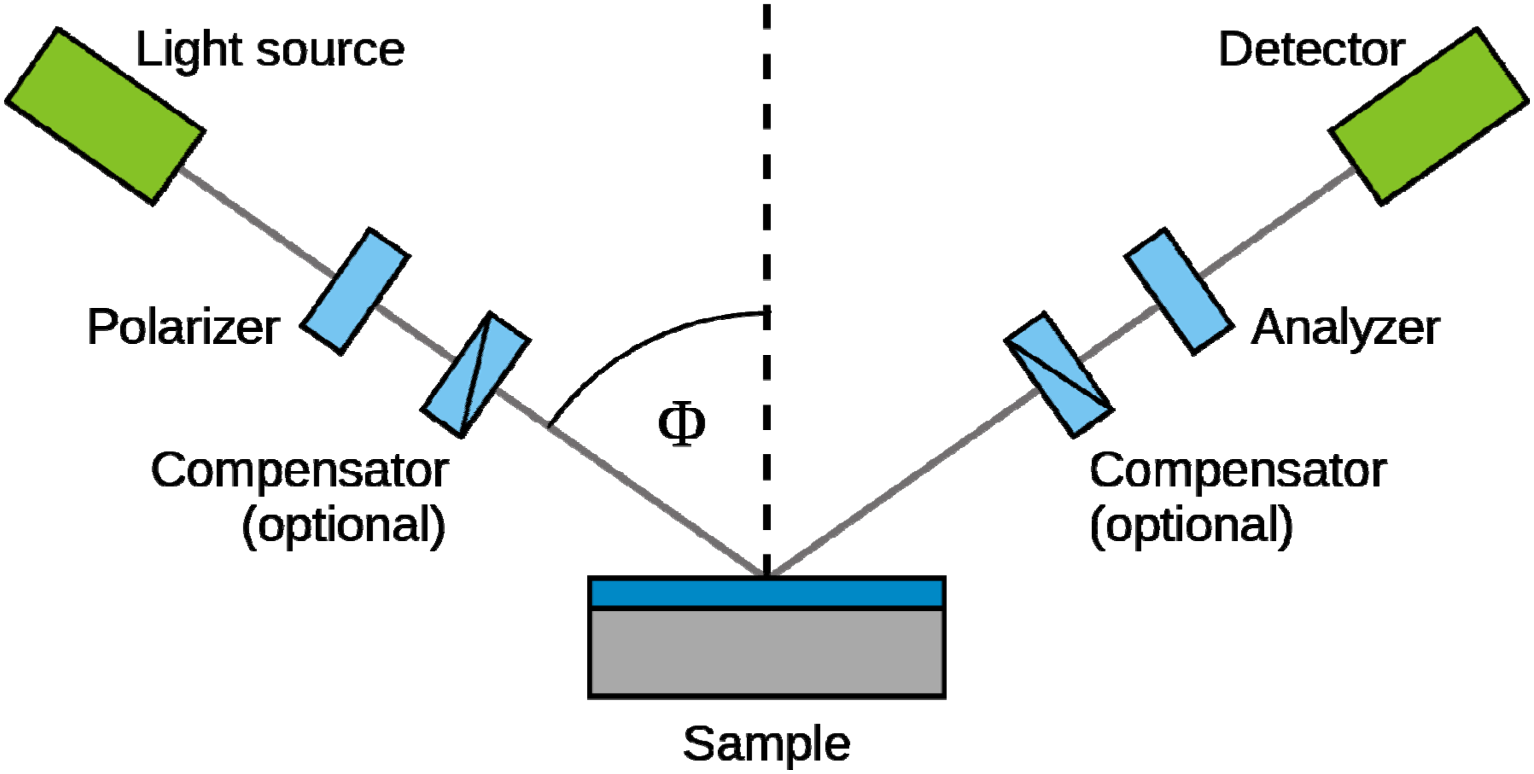
$$r_p = \frac{E_{rp}}{E_{ip}} = \frac{\sqrt{\epsilon_2}\cos\theta_1 - \sqrt{\epsilon_1}\cos\theta_2}{\sqrt{\epsilon_2}\cos\theta_1 + \sqrt{\epsilon_1}\cos\theta_2}$$

$$r_s = \frac{E_{sr}}{E_{si}} = \frac{\sqrt{\epsilon_2}\cos\theta_2 - \sqrt{\epsilon_1}\cos\theta_1}{\sqrt{\epsilon_1}\cos\theta_1 + \sqrt{\epsilon_2}\cos\theta_2}$$





Ellipsometry

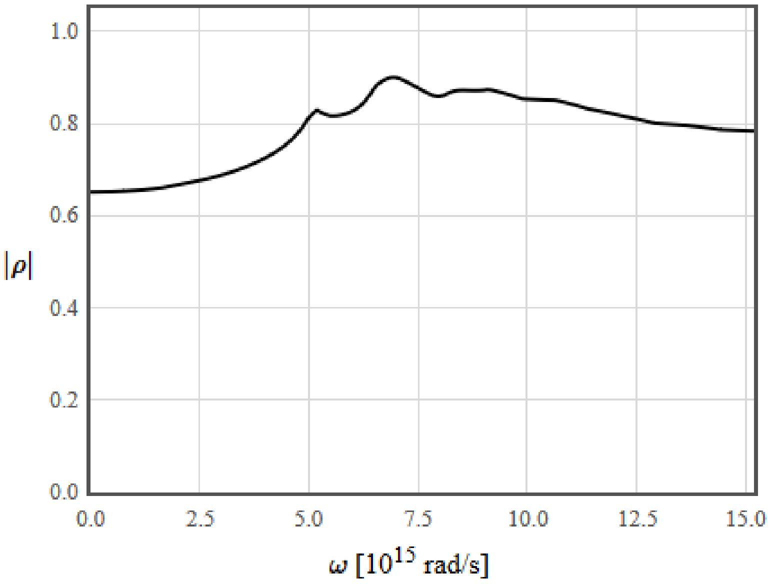


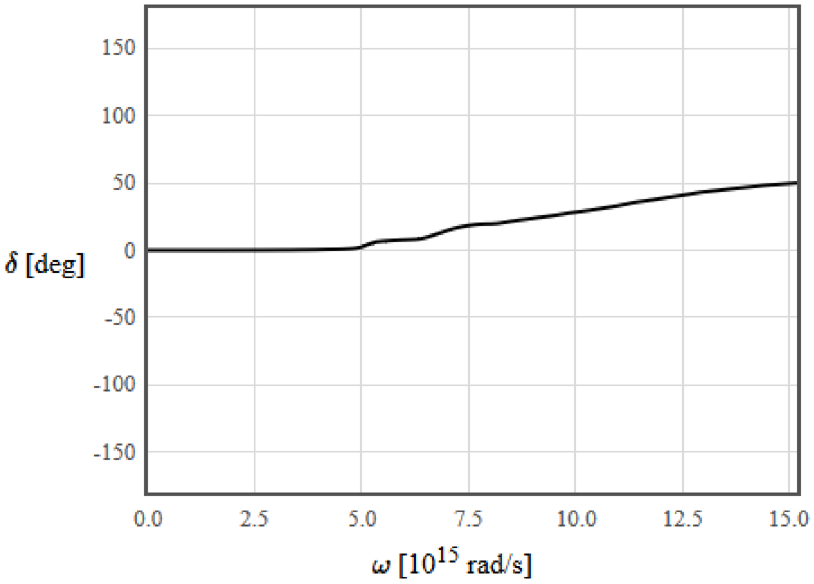
Ellipsometry measures the change of polarization upon reflection. The measured signal depends on the thickness and the dielectric constant.

http://en.wikipedia.org/wiki/Ellipsometry

Ellipsometry

$$\rho = \frac{r_p}{r_s} = |\rho| e^{i\delta}$$





The ratio of the two reflected polarizations is insensitive to instabilities of light source or atmospheric absorption.