Magnetic ordering

Ferromagnetism

\[ \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \]

Ferrimagnetism

\[ \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \]

Antiferromagnetism

\[ \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \]

Helimagnetism

All ordered magnetic states have excitations called magnons

\[ \varphi_2 = 2\varphi_1 \]

\[ \varphi_1 \]
Ferrimagnets

Magnetite Fe$_3$O$_4$
(Magnetstein)

Ferrites MO$_x$Fe$_{2-x}$O$_3$

M = Fe, Zn, Cd, Ni, Cu, Co, Mg

\[ \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \]

Two sublattices A and B.

Spinel crystal structure XY$_2$O$_4$

8 tetrahedral sites A (surrounded by 4 O) \[ 5\mu_B \uparrow \]

16 octahedral sites B (surrounded by 6 O) \[ 9\mu_B \downarrow \]

per unit cell
Ferrimagnets

Magnetite Fe₃O₄

Ferrites MO·Fe₂O₃

M = Fe, Zn, Cd, Ni, Cu, Co, Mg

Exchange integrals \( J_{AA}, J_{AB}, \) and \( J_{BB} \) are all negative (antiparallel preferred)

\[ |J_{AB}| > |J_{AA}|, |J_{BB}| \]
Mean field theory (Ferrimagnetism and Antiferromagnetism)

Heisenberg Hamiltonian

\[ H = -\sum_{i,j} J_{i,j} \vec{S}_i \cdot \vec{S}_j - g \mu_B B \sum_i \vec{S}_i \]

Mean field approximation

\[ \vec{B}_{MF,A} = \frac{1}{g \mu_B} \sum_{\delta} J_{i,AB} \langle \vec{S}_B \rangle + \frac{1}{g \mu_B} \sum_{\delta} J_{i,AA} \langle \vec{S}_A \rangle \]

\[ \vec{B}_{MF,B} = \frac{1}{g \mu_B} \sum_{\delta} J_{i,AB} \langle \vec{S}_A \rangle + \frac{1}{g \mu_B} \sum_{\delta} J_{i,BB} \langle \vec{S}_B \rangle \]

\[ \vec{M}_A = g \mu_B \frac{N}{V} \langle \vec{S}_A \rangle \]
\[ \vec{M}_B = g \mu_B \frac{N}{V} \langle \vec{S}_B \rangle \]
The spins can take on two energies. These energies are different on the A sites and B because the A spins see a different environment as the B spins.

\[ E_A = \pm \frac{1}{2} g \mu_B (B_{MF,A} + B_a) \quad E_B = \pm \frac{1}{2} g \mu_B (B_{MF,B} + B_a) \]

Calculate the average magnetization with Boltzmann factors:

\[ M_A = N \mu \tanh \left( \frac{\mu (B_{MF,A} + B_a)}{k_B T} \right) \quad M_B = N \mu \tanh \left( \frac{\mu (B_{MF,B} + B_a)}{k_B T} \right) \]

\[ M_A = M_{s,A} \tanh \left( \frac{\mu_0 \mu_{AB} M_A + \mu_0 \mu_{AA} M_A + \mu B_a}{k_B T} \right) \]

\[ M_B = M_{s,B} \tanh \left( \frac{\mu_0 \mu_{AB} M_A + \mu_0 \mu_{BB} M_B + \mu B_a}{k_B T} \right) \]
Ferrimagnetism

\[ \text{gauss} = 10^{-4} \text{T} \]
\[ \text{oersted} = \frac{10^{-4}}{4\pi \times 10^{-7}} \text{A/m} \]

Table 33.3
SELECTED FERRIMAGNETS, WITH CRITICAL TEMPERATURES \( T_c \) AND SATURATION MAGNETIZATION \( M_0 \)

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>( T_c ) (K)</th>
<th>( M_0 ) (gauss)(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Fe}_3\text{O}_4 ) (magnetite)</td>
<td>858</td>
<td>510</td>
</tr>
<tr>
<td>( \text{CoFe}_2\text{O}_4 )</td>
<td>793</td>
<td>475</td>
</tr>
<tr>
<td>( \text{NiFe}_2\text{O}_4 )</td>
<td>858</td>
<td>300</td>
</tr>
<tr>
<td>( \text{CuFe}_2\text{O}_4 )</td>
<td>728</td>
<td>160</td>
</tr>
<tr>
<td>( \text{MnFe}_2\text{O}_4 )</td>
<td>573</td>
<td>560</td>
</tr>
<tr>
<td>( \text{Y}_3\text{Fe}<em>5\text{O}</em>{12} ) (YIG)</td>
<td>560</td>
<td>195</td>
</tr>
</tbody>
</table>

\(^a\) At \( T = 0 \text{(K)} \).

Source: D. Gignoux, magnetic properties of Metallic systems

Kittel
Magnetization of a Magnetite Single Crystal Near the Curie Point

D. O. Smith†
Laboratory for Insulation Research, Massachusetts, Institute of Technology, Cambridge, Massachusetts
(Received January 20, 1956)

Fig. 2. Principle of the vibrating-coil magnetometer.

Fig. 9. $M_s/M_0$ vs $T$ in the [111] direction near the Curie point for single-crystal magnetite.
Antiferromagnetism

Negative exchange energy $J_{AB} < 0$.

At low temperatures, below the Neel temperature $T_N$, the spins are aligned antiparallel and the macroscopic magnetization is zero.

Spin ordering can be observed by neutron scattering.

At high temperature antiferromagnets become paramagnetic. The macroscopic magnetization is zero and the spins are disordered in zero field.

$$\chi \approx \frac{C}{T + \Theta}$$

Curie-Weiss temperature
Antiferromagnetism

Average spontaneous magnetization is zero at all temperatures.
### Table 2 Antiferromagnetic crystals

<table>
<thead>
<tr>
<th>Substance</th>
<th>Paramagnetic ion lattice</th>
<th>Transition temperature, $T_N$, in K</th>
<th>Curie-Weiss temperature, $\theta$, in K</th>
<th>$\frac{\theta}{T_N}$</th>
<th>$\frac{\chi(0)}{\chi(T_N)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MnO</td>
<td>fcc</td>
<td>116</td>
<td>610</td>
<td>5.3</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td>MnS</td>
<td>fcc</td>
<td>160</td>
<td>528</td>
<td>3.3</td>
<td>0.82</td>
</tr>
<tr>
<td>MnTe</td>
<td>hex. layer</td>
<td>307</td>
<td>690</td>
<td>2.25</td>
<td>0.76</td>
</tr>
<tr>
<td>MnF$_2$</td>
<td>bc tetr.</td>
<td>67</td>
<td>82</td>
<td>1.24</td>
<td>0.76</td>
</tr>
<tr>
<td>FeF$_2$</td>
<td>bc tetr.</td>
<td>79</td>
<td>117</td>
<td>1.48</td>
<td>0.72</td>
</tr>
<tr>
<td>FeCl$_2$</td>
<td>hex. layer</td>
<td>24</td>
<td>48</td>
<td>2.0</td>
<td>&lt;0.2</td>
</tr>
<tr>
<td>FeO</td>
<td>fcc</td>
<td>198</td>
<td>570</td>
<td>2.9</td>
<td>0.8</td>
</tr>
<tr>
<td>CoCl$_2$</td>
<td>hex. layer</td>
<td>25</td>
<td>38.1</td>
<td>1.53</td>
<td>1.14</td>
</tr>
<tr>
<td>CoO</td>
<td>fcc</td>
<td>291</td>
<td>330</td>
<td>1.14</td>
<td></td>
</tr>
<tr>
<td>NiCl$_2$</td>
<td>hex. layer</td>
<td>50</td>
<td>68.2</td>
<td>1.37</td>
<td></td>
</tr>
<tr>
<td>NiO</td>
<td>fcc</td>
<td>525</td>
<td>$\sim$2000</td>
<td>$\sim$4</td>
<td></td>
</tr>
<tr>
<td>Cr</td>
<td>bcc</td>
<td>308</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Paramagnetism**

$$\chi = \frac{C}{T}$$

Curie law

**Ferromagnetism**

$$\chi = \frac{C}{T - T_c}$$

Curie-Weiss law ($T > T_c$)

**Antiferromagnetism**

$$\chi = \frac{C}{T + \theta}$$

($T > T_N$)

from Kittel
Magnons are excitations of the ordered ferromagnetic state.
Magnons

Energy of the Heisenberg term involving spin $p$

$$-2JS_p \cdot \left( \vec{S}_{p+1} + \vec{S}_{p-1} \right)$$

The magnetic moment of spin $p$ is

$$\vec{\mu}_p = -g\mu_B \vec{S}_p$$

$$-\vec{\mu}_p \cdot \left( \frac{-2J}{g\mu_B} \right) \left( \vec{S}_{p+1} + \vec{S}_{p-1} \right)$$

This has the form $-\mu_p \cdot B_p$ where $B_p$ is

$$\vec{B}_p = \left( \frac{-2J}{g\mu_B} \right) \left( \vec{S}_{p+1} + \vec{S}_{p-1} \right)$$
Magnons

\[ \vec{\mu}_p = -g \mu_B \vec{S}_p \quad \vec{B}_p = \left( \frac{-2J}{g \mu_B} \right) (\vec{S}_{p+1} + \vec{S}_{p-1}) \]

The rate of change of angular momentum is the torque

\[ \hbar \frac{d\vec{S}_p}{dt} = \vec{\mu}_p \times \vec{B}_p = 2J \left( \vec{S}_p \times \vec{S}_{p+1} + \vec{S}_p \times \vec{S}_{p-1} \right) \]

If the amplitude of the deviations from perfect alignment along the z-axis are small:

\[ \hbar \frac{dS^x_p}{dt} = 2J |S| \left( S^{y}_{p+1} - 2S^y_p + S^y_{p-1} \right) \]
\[ \hbar \frac{dS^y_p}{dt} = 2J |S| \left( S^{x}_{p+1} - 2S^x_p + S^x_{p-1} \right) \]
\[ \hbar \frac{d\vec{S}^z_p}{dt} = 0 \]
These are coupled linear differential equations. The solutions have the form:

\[
\begin{pmatrix}
S^x_p \\
S^y_p
\end{pmatrix} =
\begin{pmatrix}
u^x_k \\
u^y_k
\end{pmatrix}
\exp\left[ i(kp \alpha - \omega t) \right]
\]

\[
-i\hbar \omega u^x_k e^{ikp a} = 2J |S|(e^{ik(p+1)a} + 2e^{ikp a} - e^{-ik(p-1)a})u^y_k
\]

\[
-i\hbar \omega u^y_k e^{ikp a} = -2J |S|(e^{ik(p+1)a} + 2e^{ikp a} - e^{-ik(p-1)a})u^x_k
\]

Cancel a factor of \(e^{ikp a}\).
Magnons

\[-i\hbar \omega u^x_k = 2J |S| \left(-e^{ika} + 2 - e^{-ika}\right) u^y_k\]
\[-i\hbar \omega u^y_k = -2J |S| \left(-e^{ika} + 2 - e^{-ika}\right) u^x_k\]

These equations will have solutions when,

\[
\begin{vmatrix}
  i\hbar \omega & 4J |S| (1 - \cos(ka)) \\
-4J |S| (1 - \cos(ka)) & i\hbar \omega
\end{vmatrix} = 0
\]

The dispersion relation is:

\[
\hbar \omega = 4J |S| (1 - \cos(ka))
\]
Magnon dispersion relation

\[ \hbar \omega = 4JS \left( 1 - \cos(ka) \right) \]

A phonon dispersion relation would be linear at the origin.
Magnon density of states

\[ \hbar \omega = 4JS \left(1 - \cos(ka)\right) \]

Mathematically this is the same problem as the tight binding model for electrons on a one-dimensional chain.

\[
D(E) = \frac{1}{at \sqrt{1 - \left(\frac{\varepsilon - E}{2t}\right)^2}} J^3 m^{-1}
\]

\[
E = \varepsilon - 2t \cos(k_x a)
\]
Density of states → Specific heat

The specific heat is the derivative of the internal energy with respect to the temperature:

$$c_v = \left( \frac{\partial u}{\partial T} \right)_{V,N}$$

This can be expressed in terms of an integral over the frequency $\omega$.

$$c_v = \frac{\partial}{\partial T} \int \tau(\omega) d\omega = \frac{\partial}{\partial T} \int \frac{\hbar \omega D(\omega)}{e^{\frac{\hbar \omega}{k_B T}} - 1} d\omega$$

The Leibniz integral rule can be used to bring the differentiation inside the integral. If the phonon density of states $D(\omega)$ is temperature independent, the result is,

$$c_v = \int \frac{\hbar \omega D(\omega)}{k_B T} \left( \frac{1}{e^{\frac{\hbar \omega}{k_B T}} - 1} \right) d\omega$$

Since only the Bose-Einstein factor depends on temperature, the differentiation can be performed analytically and the expression for the specific heat is,

$$c_v = \int \left( \frac{\hbar \omega}{T} \right)^2 \frac{D(\omega) e^{\frac{\hbar \omega}{k_B T}}}{k_B \cdot \left( e^{\frac{\hbar \omega}{k_B T}} - 1 \right)^2} d\omega$$

The form below uses this formula to calculate the temperature dependence of the specific heat from tabulated data for the density of states. The density of states data is input as two columns in the text box at the lower left. The first column is the angular-frequency $\omega$ in radians. The second column is the density of states. The units of the density of states depends on the dimensionality: a.m. for 1-d, m/s for 2-d, and m^2 for 3-d.

After the ‘DoS → cv(T)‘ button is pressed, the density of states is plotted on the left and $c_v(T)$ is plotted from temperature $T_{\text{min}}$ to temperature $T_{\text{max}}$ on the right. The data for the $c_v(T)$ plot also appear in tabular form at the lower right text box. The first column is the temperature in Kelvin and the second column is the specific heat in units of J m^{-2}, J K^{-1} m^{-3}, or J K^{-1} m^{-2} depending on the dimensionality.
The dispersion relation in one dimension:

\[ \hbar \omega = 4J |S| \left( 1 - \cos(ka) \right) \]

The dispersion relation for a cubic lattice in three dimensions:

\[ \hbar \omega = 2J |S| \left( z - \sum_{\delta} \cos(\bar{k} \cdot \bar{\delta}) \right) \]

The magnon contribution to thermodynamic properties can be calculated similar to the phonon contribution to the thermodynamic properties.
Magnons

\[ \hbar \omega = 2J |S| \left( z - \sum_\delta \cos(\vec{k} \cdot \vec{\delta}) \right) \]

Dispersion relation is mathematically equivalent to tight binding model for electrons.
Long wavelength / low temperature limit

Dispersion relation: \( \hbar \omega \approx 2JSk^2a^2 \)

The density of states: \( D(\omega) \propto \sqrt{\omega} \)

Magnons are bosons:

\[
\langle n_k \rangle = \frac{1}{\exp\left(\frac{\hbar \omega_k}{k_B T}\right) - 1}
\]

\[
u = \int_0^\infty \frac{\hbar \omega D(\omega)d\omega}{\exp\left(\frac{\hbar \omega_k}{k_B T}\right) - 1} \propto T^{5/2}
\]

\[c_v \propto T^{3/2}\]
Fig. 1. Constant-$E$ scan TAS-measured spin wave dispersion relation for various directions in a single crystal of Fe at 295 K. The dashed line corresponds to the Heisenberg model with $D = 281$ meV Å$^2$ and $\beta = 1.0$ Å$^2$ [68 S 3], see also [73 M 1].

Fig. 6b. Room-temperature spin wave dispersion curve for the [111] direction of $^{60}$Ni. ZB shows the position of the zone boundary [85 M 1]. The solid curve is from calculations [85 C 1, 83 C 1].
Neutron magnetic scattering

Neutrons can scatter inelastically from magnetic material and create or annihilate magnons

\[ \vec{k}_n = \vec{k}_n' + \vec{k}_{magnon} + \vec{G} \]

\[ \frac{\hbar^2 k'^2}{2m_n} \pm \hbar \omega_{magnon} = \frac{\hbar^2 k^2}{2m_n} + \frac{\hbar^2 G^2}{2m_{\text{crystal}}} \]
Antiferromagnet magnons

\[ \hbar \frac{dS_p^{Ax}}{dt} = 2J |S| \left( -S_p^{By} - 2S_p^{Ay} - S_{p-1}^{By} \right) \]

\[ \hbar \frac{dS_p^{Ay}}{dt} = -2J |S| \left( -S_p^{Bx} - 2S_p^{Ax} - S_{p-1}^{Bx} \right) \]

\[ \hbar \frac{dS_p^{Bx}}{dt} = 2J |S| \left( S_{p+1}^{Ay} + 2S_p^{By} + S_p^{Ay} \right) \]

\[ \hbar \frac{dS_p^{By}}{dt} = -2J |S| \left( S_{p+1}^{Ax} + 2S_p^{Bx} + S_p^{Ax} \right) \]

\[ \hbar \frac{dS_p^{Az}}{dt} = 0 \]

\[ \hbar \frac{dS_p^{Bz}}{dt} = 0 \]

\[ \left( \begin{array}{c} S_p^{Ax} \\ S_p^{Ay} \\ S_p^{Bx} \\ S_p^{By} \\ S_p^{Az} \end{array} \right) = \left( \begin{array}{c} u_k^{Ax} \\ u_k^{Ay} \\ u_k^{Bx} \\ u_k^{By} \end{array} \right) \exp \left[ i(2k\alpha - \omega t) \right] \]
Antiferromagnet magnons

\[ \hbar \omega = 4 |J| S |\sin(ka)| \]

Brillouin zone boundary is at \( k = \pi/2a \)
Antiferromagnet magnons

\[ \hbar \omega = 4 |J| S |\sin(ka)| \]

Mathematically equivalent to phonons in 1-d

\[ \omega = \sqrt{\frac{4C}{m}} \sin \left( \frac{ka}{2} \right) \]
Fig. 5.7 Schematic magnon and phonon dispersion curves. The magnon curve has been compressed by a factor of order 10 for illustrative purposes.

From: *Solid State Theory*, Harrison
Student project

Make a table of magnon properties like the table of phonon properties

<table>
<thead>
<tr>
<th>Magnons</th>
<th>1-D ferromagnetic magnon</th>
<th>1-D antiferromagnetic magnon</th>
<th>3-D low temperature limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equations of motion in mean field theory</td>
<td>$\begin{pmatrix} \gamma_x' \ \gamma_y' \end{pmatrix} = \begin{pmatrix} v_x \ v_y \end{pmatrix} \exp \left[ i \left( 2\pi z - \omega t \right) \right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eigenfunction solutions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dispersion relation</td>
<td>$\hbar \omega = 4 \sqrt{2} \left( 1 - \cos(\kappa z) \right)$</td>
<td>$\kappa$</td>
<td>Fe bcc</td>
</tr>
<tr>
<td></td>
<td>$\omega$ [rad/s]</td>
<td>$\delta$ [rad/s]</td>
<td>Ni fcc</td>
</tr>
<tr>
<td></td>
<td><img src="dispersion.png" alt="Dispersion Relation Graph" /></td>
<td><img src="dispersion.png" alt="Dispersion Relation Graph" /></td>
<td>Co hcp</td>
</tr>
<tr>
<td></td>
<td>1 student / column</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>