# Ballistic transport

$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{d\vec{v}}{dt}$$

$$\vec{v} = \frac{-e\vec{E}t}{m} + \vec{v}_0$$

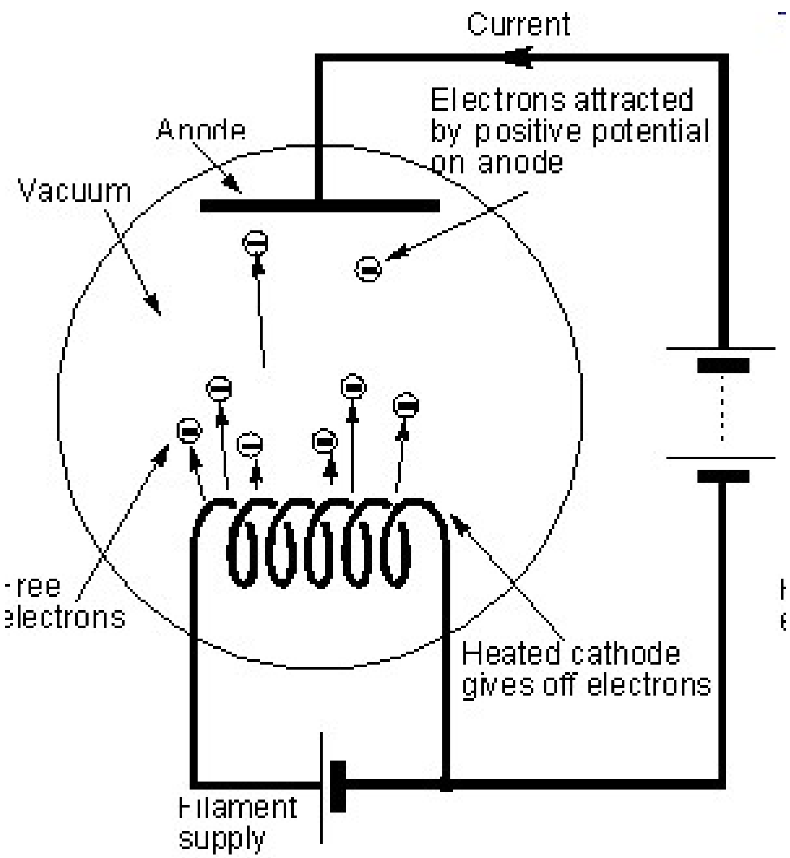
$$\vec{x} = \frac{-e\vec{E}t^2}{2m} + \vec{v}_0 t + \vec{x}_0$$

electrons in an electric field follow a parabola.

electrons in a magnetic field move in a spiral

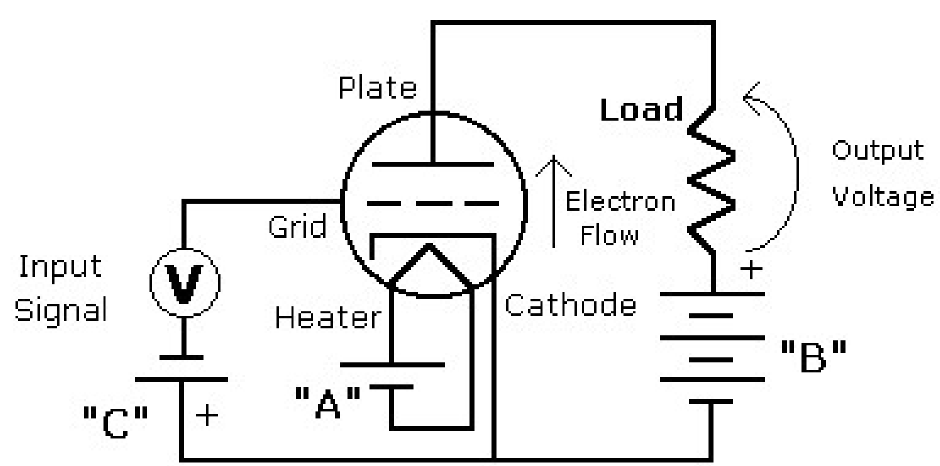
electrons crossed electric and magnetic fields spiral along the direction perpendicular to the electric and magnetic fields

# Vacuum diodes




diode

The Common-cathode Triode Amplifier



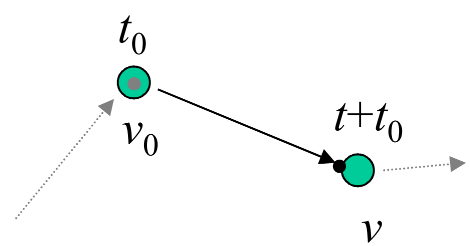

**Output voltage is proportional to plate current, which is controlled by grid voltage.**

# Diffusive transport

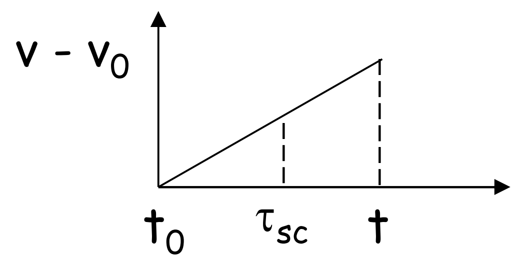
$$\vec{F} = -e\vec{E} = m^*\vec{a} = m^*\frac{d\vec{v}}{dt}$$

$$\vec{v} = \vec{v}_0 - \frac{e\vec{E}}{m^*}(t - t_0)$$

*time between two collisions*




$\langle v_0 \rangle = 0$ $\langle t - t_0 \rangle = \tau_{sc}$ < average time between scattering events




$$\vec{v}_d = \frac{-e\vec{E}\tau_{sc}}{m^*} = \frac{-e\vec{E}\ell}{m^* v_F}$$

drift velocity: $\vec{v}_d = -\mu\vec{E}$

# Matthiessen's rule

$$\frac{1}{\tau_{sc}} = \frac{1}{\tau_{sc,lattice}} + \frac{1}{\tau_{sc,impurity}}$$

mostly temperature independent

phonons, temperature dependent

$$\frac{1}{\mu} = \frac{1}{\mu_{lattice}} + \frac{1}{\mu_{impurity}}$$

# Ballistic transport in transistors

The mean free path ~100 nm > gate length ~ 20 nm

$v$ not proportional to $E$ ~~$\vec{v} = \mu \vec{E}$~~

$j$ not proportional to $E$ ~~$\vec{j} = \sigma \vec{E}$~~

nonlocal response

Electrons bend in a magnetic field like they do in vacuum.

# Magnetic field (diffusive regime)

$$\vec{F} = m\vec{a} = -e\vec{E} = m\frac{\vec{v}_d}{\tau_{\text{sc}}} \qquad -\frac{e\tau_{\text{sc}}}{m}\vec{E} = \vec{v}_d$$

$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v}\times\vec{B}\right) = m\frac{\vec{v}_d}{\tau_{\text{sc}}}$$

If $B$ is in the $z$-direction, the three components of the force are

$$-e\left(E_x + v_{dy}B_z\right) = m\frac{v_{dx}}{\tau_{\text{sc}}}$$

$$-e\left(E_y - v_{dx}B_z\right) = m\frac{v_{dy}}{\tau_{\text{sc}}}$$

$$-e\left(E_z\right) = m\frac{v_{dz}}{\tau_{\text{sc}}}$$

# Magnetic field (diffusive regime)

$$v_{d,x} = -\frac{eE_x\tau_{sc}}{m} - \frac{eB_z}{m}\tau_{sc}v_{d,y}$$

$$v_{d,y} = -\frac{eE_y\tau_{sc}}{m} + \frac{eB_z}{m}\tau_{sc}v_{d,x}$$

$$v_{d,z} = -\frac{eE_z\tau_{sc}}{m}$$

If $E_y = 0$,

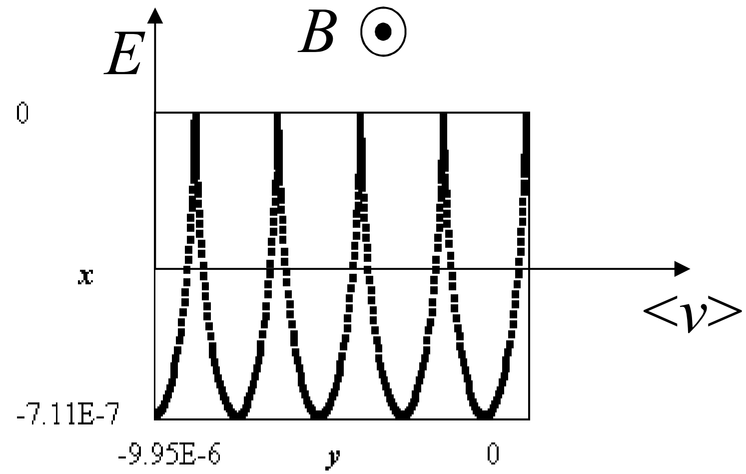
$$v_{d,y} = -\frac{eB_z}{m}\tau_{sc}v_{d,x}$$

$$\tan\theta_H = -\frac{eB_z}{m}\tau_{sc}$$

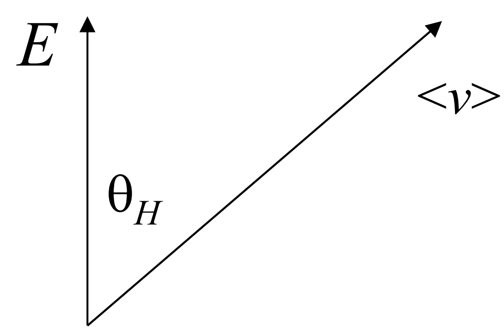
# Crossed *E* and *B* fields

Ballistic transport

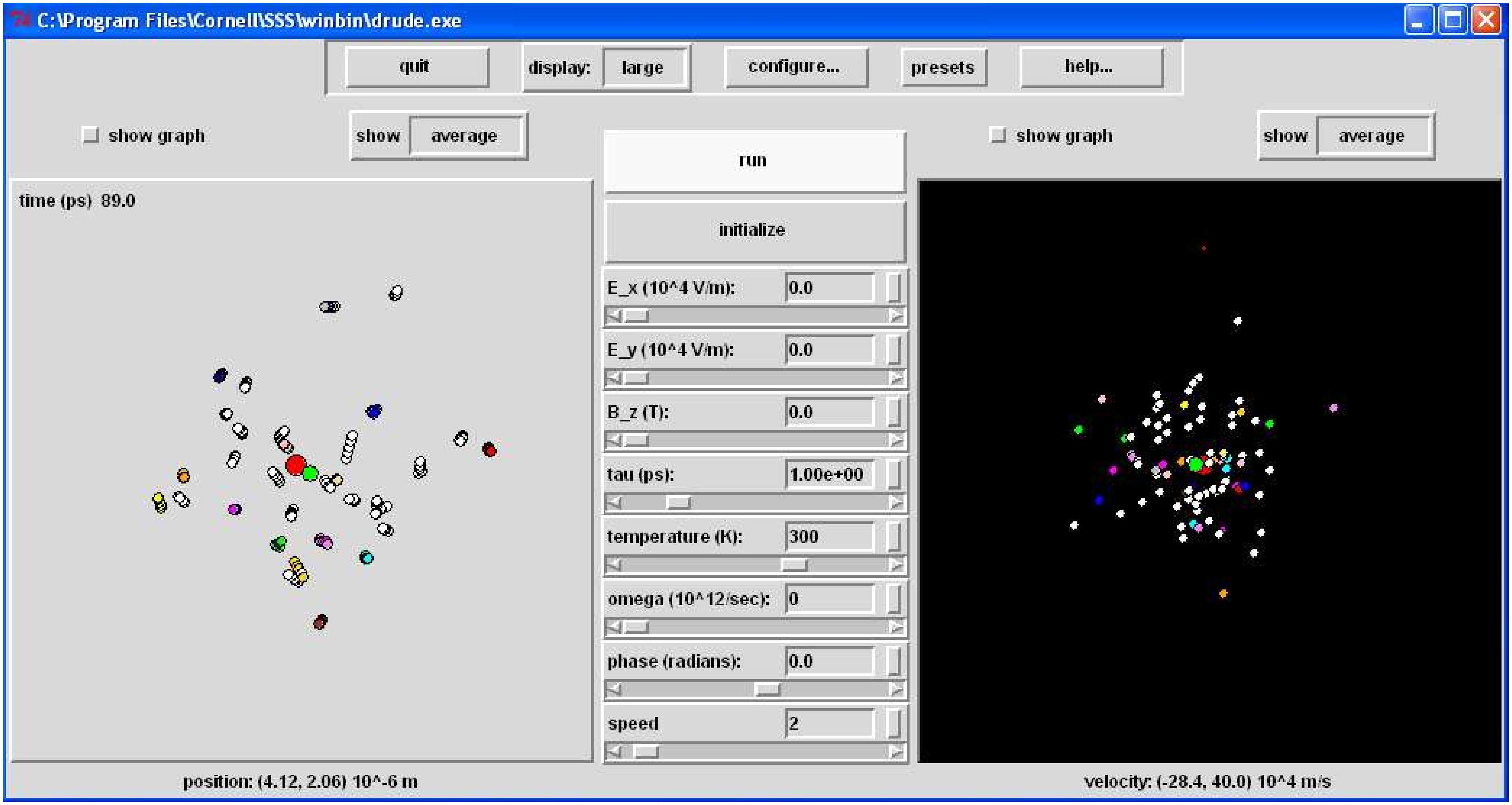
$$\vec{F} = m\vec{a} = -e\left(\vec{E} + \vec{v} \times \vec{B}\right)$$



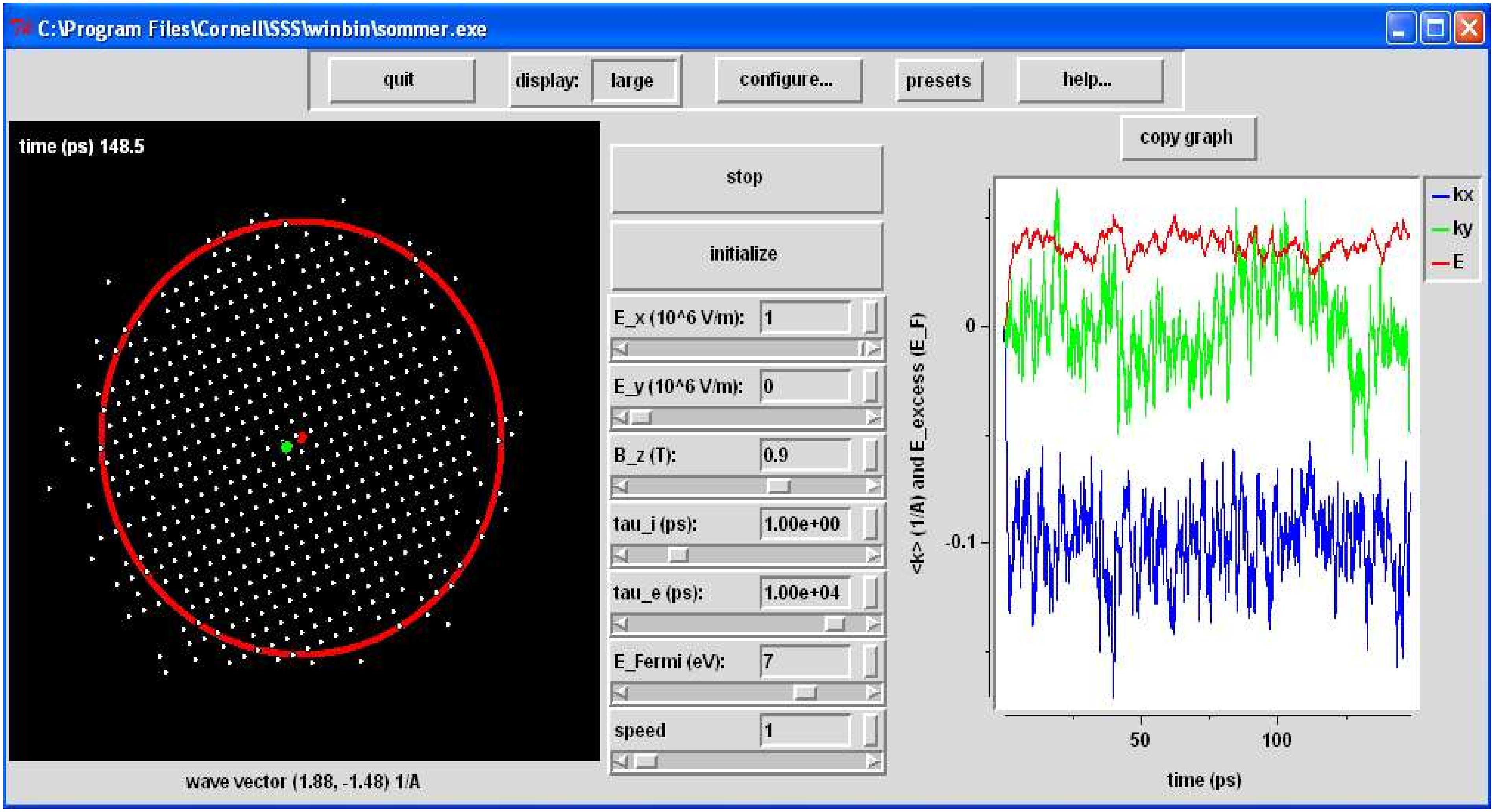

Diffusive transport



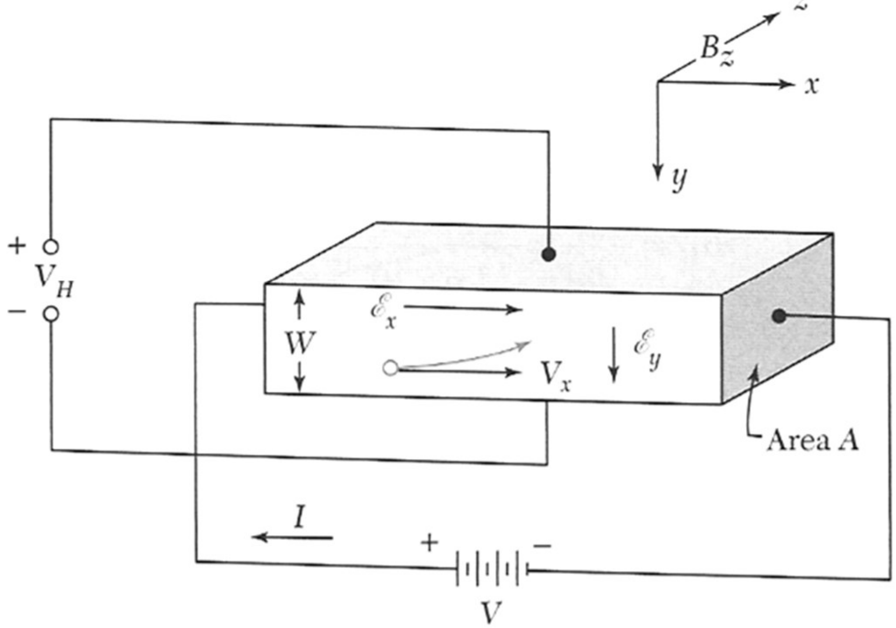

Hall angle: $\theta_H = \tan^{-1}\left(-\frac{eB_z\tau_{sc}}{m}\right)$




If no forces are applied, the electrons diffuse.
The average velocity moves against an electric field.
In just a magnetic field, the average velocity is zero.
In an electric and magnetic field, the electrons move in a straight line at the Hall angle.


C:\Program Files\Cornell\SSS\winbin\sommer.exe
quit
display:
large
configure...
presets
help...
time (ps) 148.5
stop
initialize
E_x (10^6 V/m):
1
E_y (10^6 V/m):
0
B_z (T):
0.9
tau_i (ps):
1.00e+00
tau_e (ps):
1.00e+04
E_Fermi (eV):
7
speed
1
wave vector (1.88, -1.48) 1/A
copy graph
kx
ky
E
0
-0.1
<k> (1/A) and E_excess (E_F)
50
100
time (ps)

# The Hall Effect (diffusive regime)




$$v_{d,x} = -\frac{eE_x\tau_{sc}}{m} - \frac{eB_z}{m}\tau_{sc}v_{d,y}$$

$$v_{d,y} = -\frac{eE_y\tau_{sc}}{m} + \frac{eB_z}{m}\tau_{sc}v_{d,x}$$

$$v_{d,z} = -\frac{eE_z\tau_{sc}}{m}$$

If $v_{d,y} = 0$,

$E_y = v_{d,x}B_z = V_H/W = R_H j_x B_z$ $V_H$ = Hall voltage, $R_H$ = Hall Constant

$v_{d,x} = - j_x/ne$

$$R_H = E_y/j_x B_z = - 1/ne$$

| Metal | Method | Experimental $R_H$, in $10^{-24}$ CGS units | Assumed carriers per atom | Calculated $-1/nec$, in $10^{-24}$ CGS units |
|---|---|---|---|---|
| Li | conv. | −1.89 | 1 electron | −1.48 |
| Na | helicon | −2.619 | 1 electron | −2.603 |
| | conv. | −2.3 | | |
| K | helicon | −4.946 | 1 electron | −4.944 |
| | conv. | −4.7 | | |
| Rb | conv. | −5.6 | 1 electron | −6.04 |
| Cu | conv. | −0.6 | 1 electron | −0.82 |
| Ag | conv. | −1.0 | 1 electron | −1.19 |
| Au | conv. | −0.8 | 1 electron | −1.18 |
| Be | conv. | +2.7 | — | — |
| Mg | conv. | −0.92 | — | — |
| Al | helicon | +1.136 | 1 hole | +1.135 |
| In | helicon | +1.774 | 1 hole | +1.780 |
| As | conv. | +50. | — | — |
| Sb | conv. | −22. | — | — |
| Bi | conv. | −6000. | — | — |

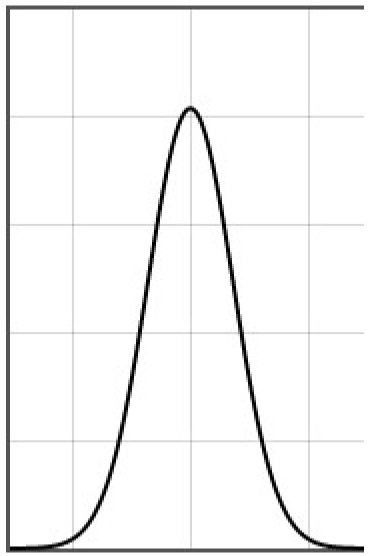
# Diffusion equation/ heat equation

$$\frac{dn}{dt} = -D\nabla^2 n$$

Diffusion constant

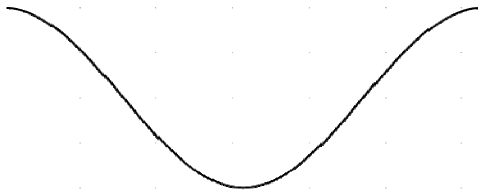
Fick's law $\vec{j} = -D\nabla n$

Continuity equation $\frac{dn}{dt} = \nabla \cdot \vec{j}$



$$n = \frac{1}{\sqrt{4\pi D t}} \exp\left(\frac{-r^2}{4Dt}\right)$$

# Einstein relation



$$n(x) = A\exp\left(\frac{-U_{pot}(x)}{k_B T}\right)$$ Boltzmann factor

$$\vec{F} = -\nabla U_{pot} = -e\vec{E}$$

In equilibrium, drift = diffusion

$$en\mu\vec{E} + eD\nabla n = 0$$

$$\nabla n = -\frac{1}{k_B T} A\exp\left(\frac{-U_{pot}}{k_B T}\right)\nabla U_{pot} = -\frac{n}{k_B T}\nabla U_{pot} = \frac{-en\vec{E}}{k_B T}$$

$$en\mu\vec{E} - e^2 D\frac{n\vec{E}}{k_B T} = 0$$

$$\boxed{D = \frac{\mu k_B T}{e}}$$

Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen, A. Einstein (1905).

# Thermal conductivity

$$\vec{j}_U = \bar{E}\vec{j}$$

Average particle energy

$$u = \bar{E}n$$

internal energy density

$$\vec{j}_U = -\bar{E}D\nabla n = -D\nabla u$$

$$\vec{j}_U = -D\frac{du}{dT}\nabla T = -Dc_v\nabla T$$

$$\vec{j}_U = -K\nabla T$$

Thermal conductivity

$$K = Dc_v$$

$$K \to 0 \quad \text{as} \quad T \to 0$$

# Wiedemann - Franz law

$$\frac{K}{\sigma} = \frac{Dc_v}{ne\mu}$$

Einstein relation: $D = \frac{\mu k_B T}{e}$

Dulong - Petit: $c_v = 3nk_B$

$$\frac{K}{\sigma} = \frac{3k_B^2}{e^2}T$$

Wiedemann Franz law $L = \frac{K_{el}}{\sigma T} = 2.22 \times 10^{-8}$ W Ω / K $^2$

Lorentz number

# Lorenz number

$$L = \frac{K_{el}}{\sigma T} = 2.22 \times 10^{-8} \qquad \mathrm{W}\,\Omega / \mathrm{K}^2$$

**Table 5 Experimental Lorenz numbers**

| | $L \times 10^8$ watt-ohm/deg$^2$ | | | $L \times 10^8$ watt-ohm/deg$^2$ | |
|---|---|---|---|---|---|
| Metal | 0°C | 100°C | Metal | 0°C | 100°C |
| Ag | 2.31 | 2.37 | Pb | 2.47 | 2.56 |
| Au | 2.35 | 2.40 | Pt | 2.51 | 2.60 |
| Cd | 2.42 | 2.43 | Su | 2.52 | 2.49 |
| Cu | 2.23 | 2.33 | W | 3.04 | 3.20 |
| Mo | 2.61 | 2.79 | Zn | 2.31 | 2.33 |

At low temperatures the classical predictions for the thermal and electrical conductivities are too high but their ratio is correct. Only the electrons within $k_B T$ of the Fermi surface contribute.

# Crystal Physics

Crystal physics explains what effects the symmetries of the crystal have on observable quantities.

An Introduction to Crystal Physics Ervin Hartmann
http://ww1.iucr.org/comm/cteach/pamphlets/18/index.html

International Tables for Crystallography
http://it.iucr.org/

Kittel chapter 3: elastic strain

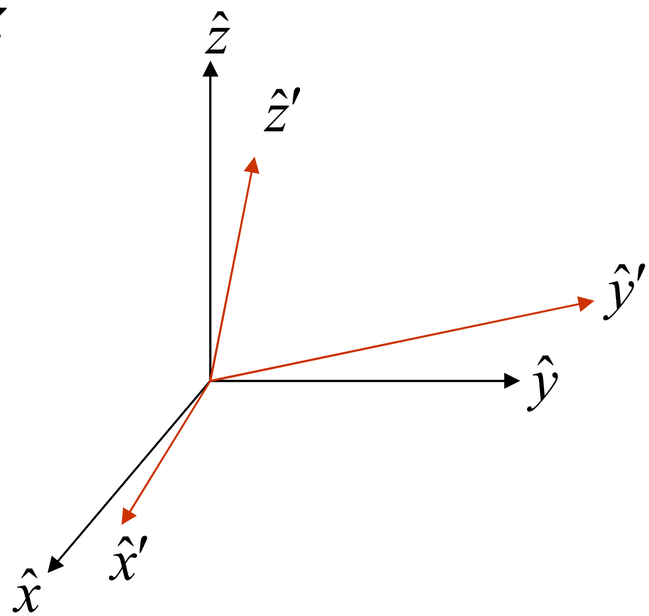
# Strain

A distortion of a material is described by the strain matrix

$$x' = (1+\varepsilon_{xx})\hat{x} + \varepsilon_{xy}\hat{y} + \varepsilon_{xz}\hat{z}$$

$$y' = \varepsilon_{yx}\hat{x} + (1+\varepsilon_{yy})\hat{y} + \varepsilon_{yz}\hat{z}$$

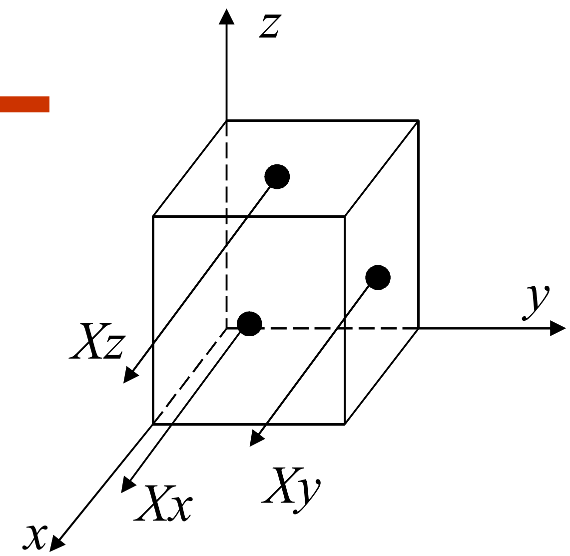
$$z' = \varepsilon_{zx}\hat{x} + \varepsilon_{zy}\hat{y} + (1+\varepsilon_{zz})\hat{z}$$



# Stress

9 forces describe the stress

*Xx, Xy, Xz, Yx, Yy, Yz, Zx, Zy, Zz*




*Xx* is a force applied in the *x*-direction to the plane normal to *x*

*Xy* is a sheer force applied in the *x*-direction to the plane normal to *y*

stress tensor:

$$\sigma = \begin{bmatrix} \frac{X_x}{A_x} & \frac{X_y}{A_y} & \frac{X_z}{A_z} \\ \frac{Y_x}{A_x} & \frac{Y_y}{A_y} & \frac{Y_z}{A_z} \\ \frac{Z_x}{A_x} & \frac{Z_y}{A_y} & \frac{Z_z}{A_z} \end{bmatrix}$$

Stress is force/$m^2$

# Stress and Strain

$$\varepsilon_{ij} = s_{ijkl}\sigma_{kl}$$

The stress - strain relationship is described by a rank 4 stiffness tensor. The inverse of the stiffness tensor is the compliance tensor.

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$$

Einstein convention: sum over repeated indices.

$$\varepsilon_{xx} = s_{xxxx}\sigma_{xx} + s_{xxxy}\sigma_{xy} + s_{xxxz}\sigma_{xz} + s_{xxyx}\sigma_{yx} + s_{xxyy}\sigma_{yy}$$
$$+ s_{xxyz}\sigma_{yz} + s_{xxzx}\sigma_{zx} + s_{xxzy}\sigma_{zy} + s_{xxzz}\sigma_{zz}$$

# Statistical Physics

Microcannonical Ensemble: Internal energy is expressed in terms of extrinsic quantities $U(S, M, P, \varepsilon, N, V)$.

$$dU = TdS + \sigma_{ij} d\varepsilon_{ij} + E_k dP_K + H_l dM_l$$

$$dU = \frac{\partial U}{\partial S} dS + \frac{\partial U}{\partial \varepsilon_{ij}} d\varepsilon_{ij} + \frac{\partial U}{\partial P_k} dP_K + \frac{\partial U}{\partial M_l} dM_l$$

The normal modes must be solved for in the presence of electric and magnetic fields (Advanced Solid State Physics course).

# Statistical Physics

Microcannonical Ensemble: Internal energy is expressed in terms of extrinsic quantities $U(S, M, P, \varepsilon, N, V)$. $\varepsilon_{ij} \Rightarrow V\varepsilon_{ij}$

$$dU = \frac{\partial U}{\partial S} dS + \frac{\partial U}{\partial \varepsilon_{ij}} d\varepsilon_{ij} + \frac{\partial U}{\partial P_k} dP_K + \frac{\partial U}{\partial M_l} dM_l$$

$$dU = TdS + \sigma_{ij} d\varepsilon_{ij} + E_k dP_K + H_l dM_l$$

Cannonical ensemble: At constant temperature, make a Legendre transformation to the Helmholtz free energy.

$F = U - TS$

$F(V, T, N, M, P, \varepsilon)$

Make a Legendre transformation to the Gibbs potential $G(T, H, E, \sigma)$

$$G = U - TS - \sigma_{ij}\varepsilon_{ij} - E_k P_K - H_l M_l$$

# Gibbs free energy

$$G = U - TS - \sigma_{ij}\varepsilon_{ij} - E_k P_K - H_l M_l$$

$$dG = dU - TdS - SdT - \sigma_{ij} d\varepsilon_{ij} - \varepsilon_{ij} d\sigma_{ij} - E_k dP_k - P_k dE_k - H_l dM_l - M_l dH_l$$

$$dU = TdS + \sigma_{ij} d\varepsilon_{ij} + E_k dP_K + H_l dM_l$$

$$dG = -SdT - \varepsilon_{ij} d\sigma_{ij} - P_k dE_k - M_l dH_l$$

total derivative: $dG = \left(\frac{\partial G}{\partial T}\right) dT + \left(\frac{\partial G}{\partial \sigma_{ij}}\right) d\sigma_{ij} + \left(\frac{\partial G}{\partial E_k}\right) dE_k + \left(\frac{\partial G}{\partial H_l}\right) dH_l$

$$\left(\frac{\partial G}{\partial \sigma_{ij}}\right) = -\varepsilon_{ij} \qquad \left(\frac{\partial G}{\partial E_k}\right) = -P_k$$

$$\left(\frac{\partial G}{\partial H_l}\right) = -M_l \qquad \left(\frac{\partial G}{\partial T}\right) = -S$$

$$d\varepsilon_{ij} = \underbrace{\left(\frac{\partial \varepsilon_{ij}}{\partial \sigma_{kl}}\right)}_{1} d\sigma_{kl} + \underbrace{\left(\frac{\partial \varepsilon_{ij}}{\partial E_k}\right)}_{2} dE_k + \underbrace{\left(\frac{\partial \varepsilon_{ij}}{\partial H_l}\right)}_{3} dH_l + \underbrace{\left(\frac{\partial \varepsilon_{ij}}{\partial T}\right)}_{4} dT$$

$$dP_i = \underbrace{\left(\frac{\partial P_i}{\partial \sigma_{kl}}\right)}_{5} d\sigma_{kl} + \underbrace{\left(\frac{\partial P_i}{\partial E_k}\right)}_{6} dE_k + \underbrace{\left(\frac{\partial P_i}{\partial H_l}\right)}_{7} dH_l + \underbrace{\left(\frac{\partial P_i}{\partial T}\right)}_{8} dT$$

$$dM_i = \underbrace{\left(\frac{\partial M_i}{\partial \sigma_{kl}}\right)}_{9} d\sigma_{kl} + \underbrace{\left(\frac{\partial M_i}{\partial E_k}\right)}_{10} dE_k + \underbrace{\left(\frac{\partial M_i}{\partial H_l}\right)}_{11} dH_l + \underbrace{\left(\frac{\partial M_i}{\partial T}\right)}_{12} dT$$

$$dS = \underbrace{\left(\frac{\partial S}{\partial \sigma_{kl}}\right)}_{13} d\sigma_{kl} + \underbrace{\left(\frac{\partial S}{\partial E_k}\right)}_{14} dE_k + \underbrace{\left(\frac{\partial S}{\partial H_l}\right)}_{15} dH_l + \underbrace{\left(\frac{\partial S}{\partial T}\right)}_{16} dT$$

1. Elastic deformation.
2. Reciprocal (or converse) piezo-electric effect.
3. Reciprocal (or converse) piezo-magnetic effect.
4. Thermal dilatation.
5. Piezo-electric effect.
6. Electric polarization.
7. Magneto-electric polarization.
8. Pyroelectricity.
9. Piezo-magnetic effect.
10. Reciprocal (or converse) magneto-electric polarization.
11. Magnetic polarization.
12. Pyromagnetism.
13. Piezo-caloric effect.
14. Electro-caloric effect.
15. Magneto-caloric effect.
16. Heat transmission.