Uniaxial superconducting transmission lines

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Abstract. A model has been developed to study the propagation of electromagnetic waves in a planar structure consisting of an arbitrary number of layers. Each layer can either be a lossy dielectric, a normal conductor or a superconductor. Uniaxial materials, which have a conductivity in one direction that is different from the conductivity in the other two directions, can be included. The phase velocity and attenuation constants are obtained through a scattering matrix approach of the stacked structure. This model was used to study a parallel plate transmission line composed of uniaxial superconductors, such as YBCO. Superconducting layers thinner than the penetration depth were studied as well as layers on the order of the penetration depth. Using thin superconducting layers and thin dielectric layers results in slow waves where the phase velocity is lower than it would be in an unbounded dielectric. The effects of anisotropy were also investigated. The anisotropy of the superconductor has a profound influence on the propagation characteristics if the optic axis is along the propagation direction, as it would be for an *a*-axis grown YBCO-film. However, the uniaxial model gives essentially the same results as an isotropic model when the optic axis is perpendicular to the propagation direction as it would be for *c*-axis grown YBCO-films.

Using a program that calculates the propagation of electromagnetic waves in layered structures, we have investigated a parallel plate transmission line composed of uniaxial superconductors. The structure that was considered consists of five layers: a substrate, a lower superconductor, a dielectric spacer, an upper superconductor, and a superstrate. This model is useful for analyzing structures where the superconductors are on the order of or thinner than the penetration depth. In this regime, a large part of the electromagnetic power propagates through the superconductors and the fields penetrate into the sub- and superstrate. One consequence of this is that the waves propagate much slower than they would in a structure consisting of thick superconducting and thick dielectric layers. If thicker superconducting layers are used, the model produces essentially the same results as quasi-TEM models [1][2][3][4].

We focus on the parameters applicable to devices made from YBCO. To model the superconductor, we use a complex conductivity, $\sigma_{sc} = \sigma_n - \frac{j}{\omega}\sigma_s$. Here both the superconducting matrix σ_s and the normal matrix σ_n were fit to experiments on YBCO thin films [8], where σ_n accounts for all the losses in the superconducting film. The superconducting component of the conductivity matrix is related to the magnetic penetration depth by, $\sigma_s = (\mu_0 \lambda_L^2)^{-1}$. For YBCO, the penetration depth is about 7 times as high in the c-direction than a-b-direction [6][7].

To check the program, we compared the results with the quasi-TEM formula of Swithart [5],

$$N_{eff} = \{\varepsilon_r \left[1 + (\lambda_1/t)\coth(d_1/\lambda_1) + (\lambda_2/t)\coth(d_2/\lambda_2)\right]\}^{-\frac{1}{2}}, \tag{1}$$

where t is the thickness and ε_r the dielectric constant of the dielectric layer and d_n and λ_n are the thickness and London penetration depth of superconducting layer n. This formula assumes a slowly varying E-field and does not take losses or the refractive index of the sub- or superstrate into account. Nevertheless, the quasi-TEM formula gives the same results as the five layer full wave analysis to within 0.5 % for superconducting layers as thin as 5 nm. The right hand side of Fig. 1 shows effective refraction index and the attenuation as a function of the penetration depth for a five layer configuration consisting of two 300 nm YBCO c-axis films separated by 200 nm of MgO (loss tangent $\tan\delta = 10^{-3}$) on a $LaAlO_3$ substrate ($\tan\delta = 10^{-6}$). The slow wave characteristics are apparent from the large value of the effective refractive index.
Experimentally one varies the penetration depth by adjusting the temperature. This has the undesirable side effects that the temperature dependence of the dielectric constant and the thermal expansion of the structure are included in the measurement. These side effects can be minimized by using the thin superconducting layers. The propagation of waves in a parallel plate configuration of thin superconducting layers is more sensitive to variations in the penetration depth than the same configuration using thick layers making the thin layer configuration a more sensitive probe of the penetration depth. Unfortunately the parallel plate waveguide exhibits a very low impedance (similar to a capacitor), so a lot of the microwave power is reflected.

The full wave analysis is necessary to take the uniaxiallity of the superconductors into account. The films can either be oriented with the c-axis normal to the substrate and the direction of poor conductivity perpendicular to the propagation direction, or a a-axis orientation with the direction of poor conductivity parallel to the propagation direction. In either case, Maxwell's equations decouple into TE- and TM-waves. For thin layers and low frequencies, $t\omega\sqrt{\varepsilon_r}/c_0 \ll 2\pi$, where c_0 is the speed of light in vacuum and ω is the radial frequency, only the TM_0-mode will be guided by the structure and single-mode solutions can be calculated. In this way we obtain for the H_y-component of the electromagnetic field the following differential equation

$$\left\{\nabla_t^2 + \omega^2\varepsilon_0\varepsilon_r\mu_0 - j\omega\mu_0\sigma_{sc}\right\} H_y = 0, \tag{2}$$

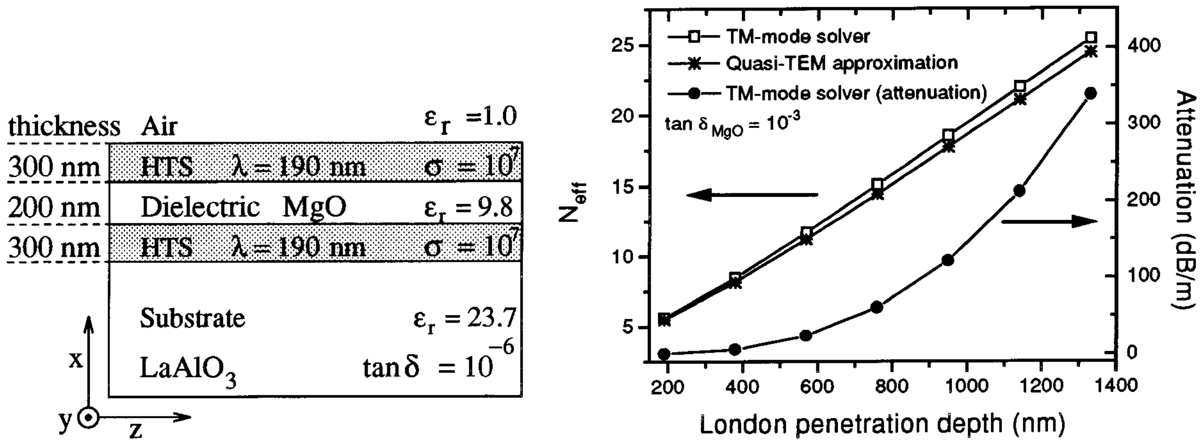

Figure 1. The configuration and TM $(1\,GHz)$ versus quasi-TEM solutions

where $\nabla_t^2 = \partial_x^2 - k^2$. Equation (2) holds for all layers and can be coupled using the continuity of the field components. To avoid instabilities in the computer program, due to losses, the scattering matrix (e.g. [9]) is used to calculate the source-free solution of the structure. In [9] details can be found about the implementation for uniaxial layers. With a complex zero-searching routine, e.g. Muller's method, source-free solutions can be found in the complex plane for some value of the wavenumber $k = j\alpha + \beta$. The phase constant, β, determines the effective refractive index, $N_{eff} = \beta c_0/\omega$. The constant α characterizes the attenuation of the TM-wave.

To investigate the influence of uniaxiallity on the propagation characteristics, a thin five layer configuration was studied for both *a*-axis and *c*-axis oriented films. In the quasi-TEM approximation the currents flow along the direction of propagation. Thus the currents flow along the *c*-axis for an *a*-axis oriented film and in the *a*-*b*-plane for a *c*-axis oriented film. The full wave analysis shows that the currents do flow primarily in the *a*-*b*-plane for *c*-axis oriented films, but that current has components both along the planes and perpendicular to the planes for *a*-axis films. This is due to much higher (super)conductivity in the *a*-*b*-plane. Consequently the quasi-TEM approximation works very well for *c*-axis oriented films. The deviations in propagation characteristics of a uniaxial superconductor compared to an isotropic superconductor and the quasi-TEM formula are illustrated in Fig. 2. Similar results were obtained using three uniaxial layer configurations. Especially with thin layers one should be able to measure the anisotropy of the superconductor.

In conclusion, the propagation characteristics of a five layer superconducting parallel plate transmission line were investigated. The anisotropy of YBCO has the greatest influence on the propagation for *a*-axis oriented films. Slow wave characteristics in very thin layers were found. The five layer full wave analysis did not yield significantly different results from the simple, lossless, quasi-TEM formula of Swithart.

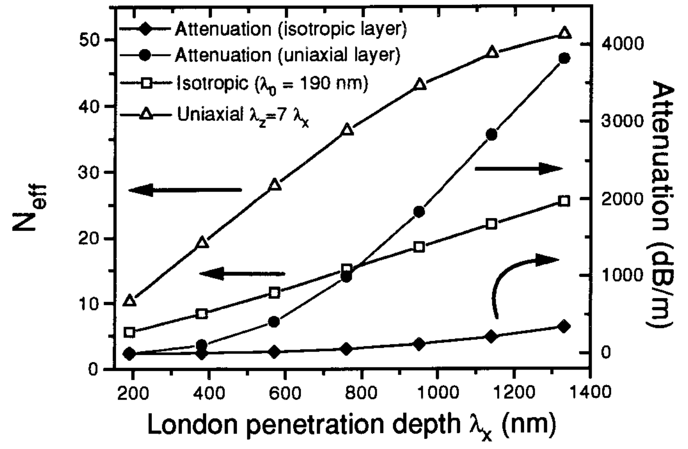

Figure 2. Uniaxial versus isotropic ($1\ GHz$ and $\tan\delta_{MgO} = 10^{-3}$)

Acknowledgement

Discussions with and support from H. Blok are gratefully acknowledged.

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