# Dynamics of 1-D parallel arrays of underdamped Josephson junctions


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Parallel arrays of underdamped $Al - Al_2O_3 - Al$ junctions have been fabricated using a shadow evaporation technique. The critical current is periodic in the applied magnetic field implying that the junctions are uniform. The current-voltage characteristics are hysteretic with several different voltage states observed over a range of bias currents and applied fields. Voltage plateaus are observed which are reminiscent of Fiske steps in long Josephson junctions however the discrete nature of the arrays does not allow for the standard Fiske step analysis. We have performed linear stability analysis on the equations that describe the dynamics of the array. The stability analysis shows that periodic solutions to the equations naturally decompose into one center of mass variable that must be described by a non-linear equation and a set of transverse variables that describe the fluctuations around the center of mass motion. The plateaus in the voltage are related to the normal modes of the linearized transverse variables.


Underdamped Josephson junction arrays exhibit richer dynamics than their overdamped counterparts. Typically the current-voltage (I-V) characteristics of underdamped arrays are hysteretic with multiple possible voltage states for a given bias current. Each voltage state corresponds to a distinct dynamic state. Two dynamic states of underdamped arrays currently of interest are the ballistic transport vortices [1] and the quantum interference of vortices [2]. Here we examine the dynamics of the simplest array of underdamped junctions, a one-dimensional parallel array. We show that for any bias current and field, the dynamics of this array can be reduced to that of a particle moving in a periodic potential.

We have fabricated 1-D parallel arrays of $N$ high-quality underdamped $Al - Al_2O_3 - Al$ junctions using a shadow evaporation technique. The numbers of junctions in the arrays are N=9, 27 or 81. The area enclosed by two junctions is (10x10 or 10x30 $\mu m$). In zero field the current-voltage characteristic of the arrays is equivalent to a single junction I-V curve. The junctions have a normal resistance of $R_N = 1.7k\Omega$, a critical current of $I_c = 0.18\mu A$ and a capacitance of $C \approx 25$ fF. This means that the junctions are underdamped with a McCumber parameter, $\beta_c \approx 24$. The critical current diffraction pattern is symmetric and periodic in the frustration, $f = \Phi_e/\Phi_o$, implying that the junctions are uniform across the array. The Josephson penetration length, $\Lambda_J = \sqrt{L_J/L_0}$, is about 14 cells.

When applying a field perpendicular to the array voltage plateaus appear within the gap corresponding to different distinct dynamic states of the array. The voltage plateaus are caused by the interaction of the Josephson oscillations with normal modes of the inductor-capacitor network formed by the junction capacitances and a combination of the geometrical inductances and the nonlinear Josephson inductances.

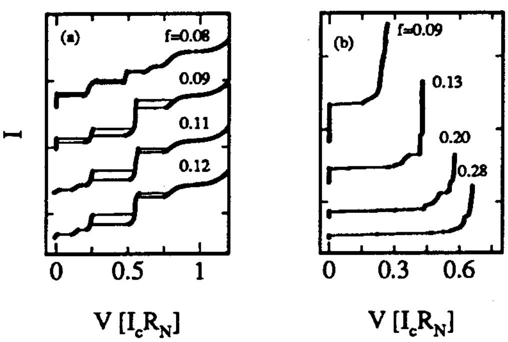


Figure 1. (a) Measured I-V curves showing the various dynamic states at different frustrations for a short array, $N = 9$ and (b) a long array, $N = 81$. The curves have an offset in the current for clarity.

Figure 1a shows several I-V curves for different values of the field for a small array, $N = 9$. Several voltage plateaus are apparent in the figure. The voltage plateaus are qualitatively similar to Fiske steps [3] that are observed in long Josephson junctions. Fiske steps are caused by the interaction of Josephson oscillations with a standing wave resonance in a long junction. Whenever an integer or half-integer times the wavelength equals the length of the junction, there is a standing wave resonance. These resonances are equally spaced in frequency. For a discrete array of underdamped junctions the resonances that cause the voltage plateaus are the normal modes of the inductor-capacitor network. In an array of $N$ junctions there are $N-1$ resonances and they are not

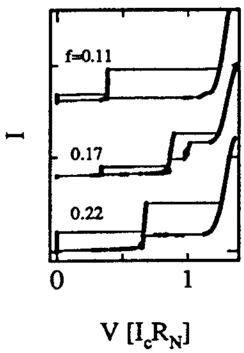


Figure 2. Simulated I-V curves for a short array ($N = 9$) for different values of the frustration. We used $\beta_c = 24$ and $L_o = 0.1$.

equally spaced in frequency.

For long arrays the behaviour of the array is identical to that of a long junction. When the length of the array is increased to $N = 81$, the number of resonant modes of the inductor-capacitor network increases to 80. The voltage plateaus become closely spaced and merge into one broad plateau whose position is very sensitive to the applied magnetic field (see Fig. 1b). In long Josephson junctions this broad plateau is known as the Eck peak [3].

To study the voltage plateaus in greater detail we have simulated the dynamics of the array using a non-linear RCSJ model that includes the non-linear conductance of the tunnel junctions and an inductance matrix that describes the mutual inductance between the cells that make up the array. The normalized equations are,

$$\Phi_i = \phi_i - \phi_{i+1} = \Phi_e - \sum_j L_{ij} I_j \qquad (1)$$

$$I_b + I_i - I_{i-1} = \beta_c \frac{d^2\phi_i}{dt^2} + G_N(V_i)\frac{d\phi_i}{dt} + \sin\phi_i \qquad (2)$$

Here $\phi_i$ is the gauge invariant phase. The bias-current, $I_b$, and loop-currents, $I_i$, are measured in units of the critical current, $I_c$; flux, $\Phi_i$, in units of the flux-quantum, $\Phi_o/2\pi$; voltage in units of $I_c R_N$, inductance in units of the Josephson inductance, $L_J = \Phi_o/2\pi I_c$, and time in units of $2\pi I_c R_N/\Phi_o$. The McCumber parameter $\beta_c = 2\pi I_c R_N^2 C/\Phi_o$ measures the damping of the junctions. The non-linear conductance $G_N(V)$ is defined by the subgap resistance $G_N(V) = R_N/R_{sg} \ll 1$ for voltages smaller than the gap voltage and unity elsewhere. The I-V curves generated by these simulations are shown in Fig. 2. The simulated I-V characteristics exhibit the same voltage plateaus as the experiment. The voltage at which the plateaus appear and the width of the plateaus is sensitive to the bias current and the applied field. To study these solutions in more detail we have performed linear stability analysis that describes the evolution of small perturbations to the numerically determined solutions. These linear equations are,

$$\tilde{\Phi}_i = \tilde{\phi}_i - \tilde{\phi}_{i+1} = -\sum_j L_{ij}\tilde{I}_j \qquad (3)$$

$$\tilde{I}_i - \tilde{I}_{i-1} = \beta_c \frac{d^2\tilde{\phi}_i}{dt^2} + \tilde{G}_N(V_i)\frac{d\tilde{\phi}_i}{dt} + \tilde{\phi}_i \cos\phi_i \qquad (4)$$

where $\tilde{G}_N(V_i) = G_N(V_i) + V_i(\partial G_N(V_i)/\partial V_i)$. The eigenfunctions of the linear equations that describe the perturbations are the modes that interact with the Josephson oscillations to cause the voltage plateaus. The eigenvalues correspond to the resonant frequencies of the modes.

The periodic dynamics of the array correspond to a closed one dimensional trajectory in phase space. One important insight gained from performing the linear stability analysis is that the set of coupled non-linear equations that describe the dynamics of the array naturally decompose into two components. One is a non-linear equation which describes the motion along the one-dimensional trajectory. This is equivalent to the motion of a single particle in a periodic potential. This potential can be constructed and by doing so one gains a better intuitive feeling for the dynamics by considering the motion of this particle than by trying to consider the set of $N$ coupled non-linear differential equations. The other component is a set of linear equations that describe the evolution of perturbations away from the one-dimensional trajectory.

## Acknowledgements

We would like to thank T.P. Orlando and H.S.J van der Zant for stimulating discussions and support. This work was supported by the Dutch Foundation for Fundamental Research on Matter (FOM).

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