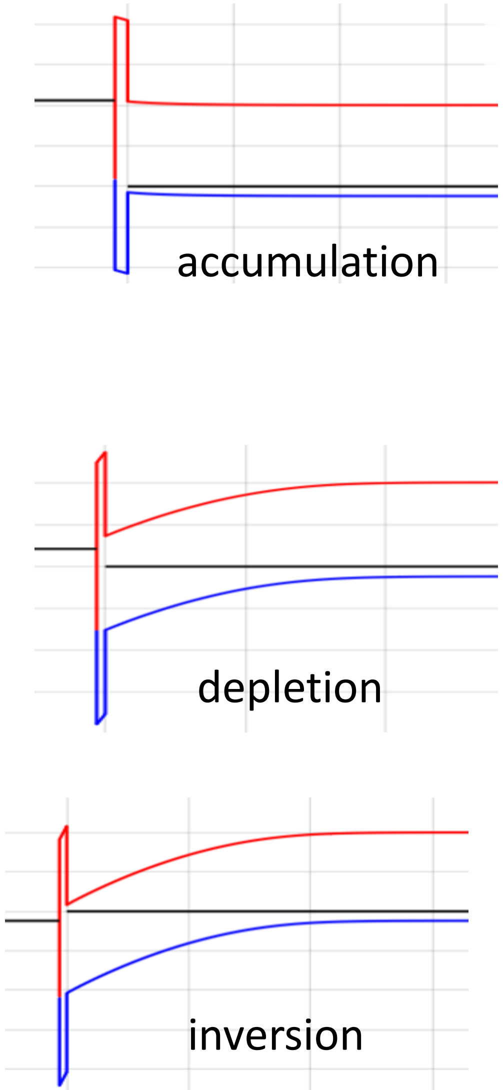
10

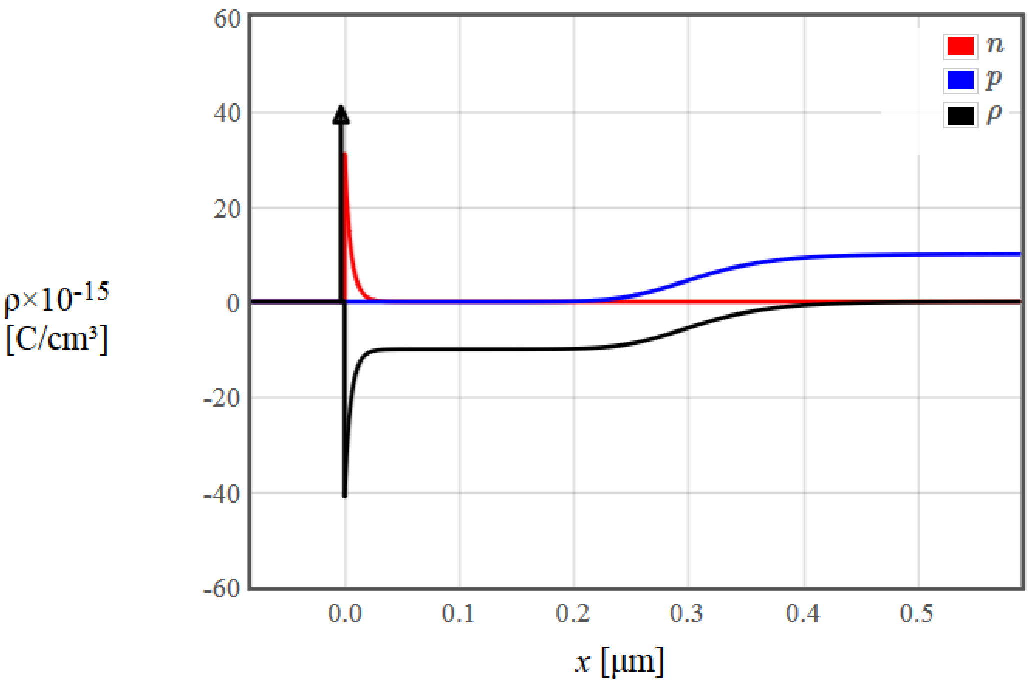
MOSFETs: Gradual Channel Approximation

December 2025

Gradual channel approximation

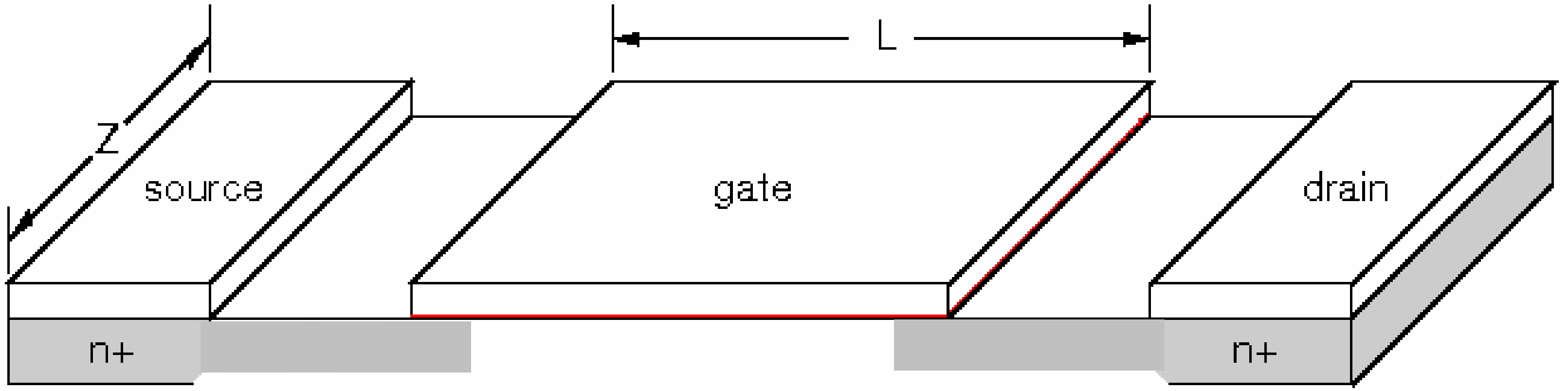


$$Q_{\text{mobile}} = \begin{cases} 0, & \text{for } V_G - V_B < V_T \\ -C_{\text{ox}}(V_G - V_B - V_T) & \text{for } V_G - V_B > V_T \end{cases}$$



http://lampz.tugraz.at/~hadley/psd/L10/gradualchannelapprox.php

Gradual channel approximation



$j = -nev_d = ne\mu_n E_y$ Ohm's law

$I = Zt\,j = Zt\,ne\mu_n E_y = Z\,e\mu_n n_s E_y$

$n_s = n\,t$ is the sheet charge at the interface

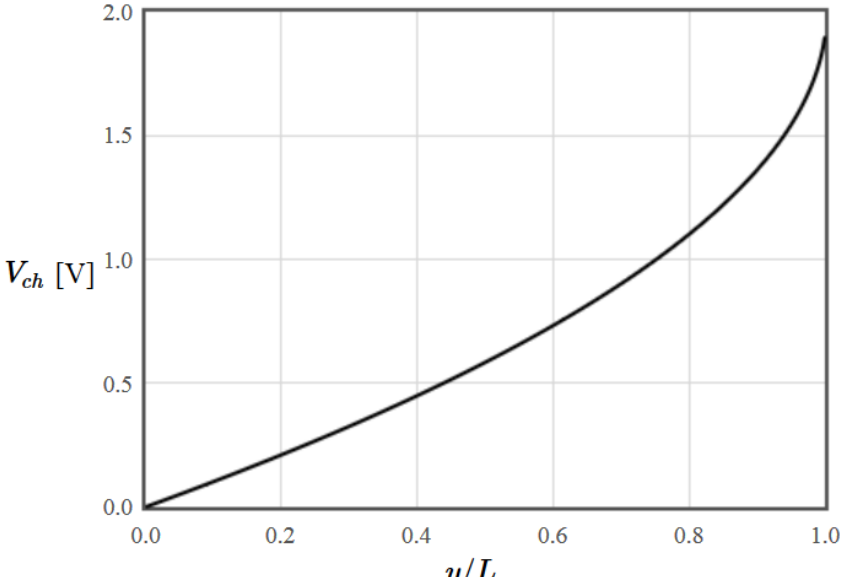
$$n_s(y) = -\frac{Q}{e} = \frac{C_{ox}(V_G - V_{ch}(y) - V_T)}{e}$$

$$I_D = -Z\mu_n C_{ox}(V_G - V_{ch}(y) - V_T)\frac{dV_{ch}}{dy}$$

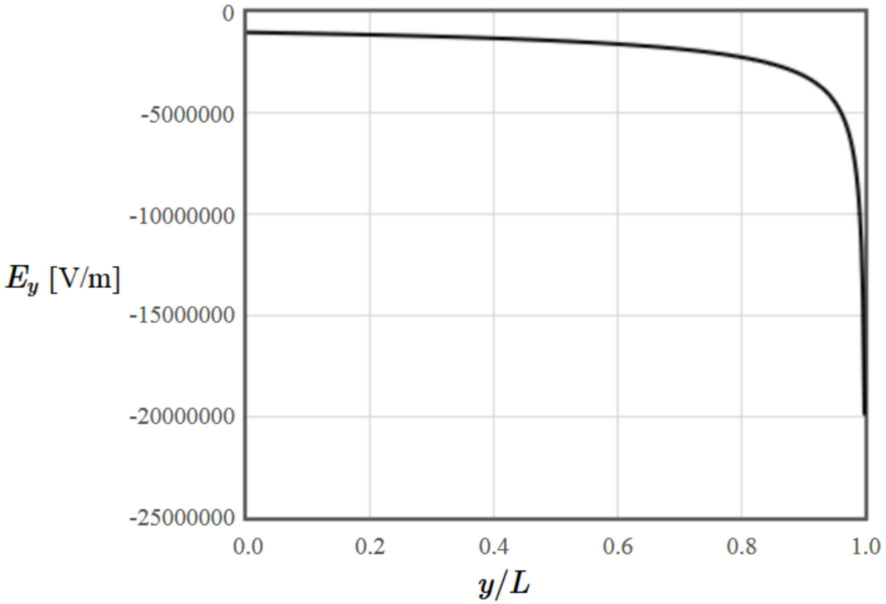
differential equation for V_{ch}

Gradual channel approximation

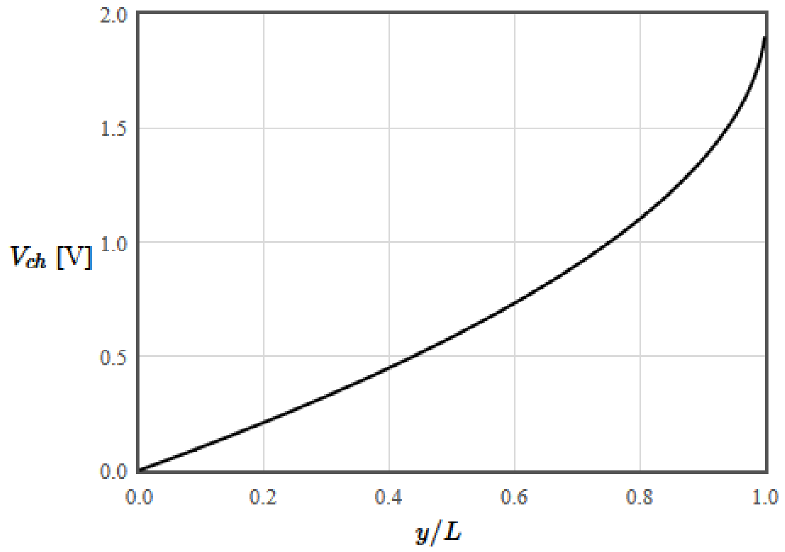
$$V_{ch}(y) = V_G - V_T - \sqrt{(V_G - V_T)^2 - \frac{2I_D y}{Z\mu_n C_{ox}}}$$



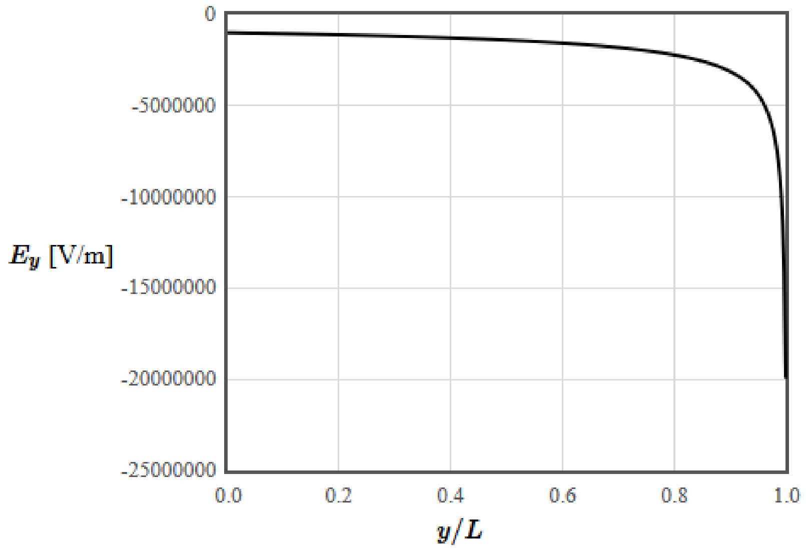
$$E_y = -\frac{dV_{ch}}{dy} = -\frac{I_D}{Z\mu_n C_{ox}\sqrt{(V_G - V_T)^2 - \frac{2I_D y}{Z\mu_n C_{ox}}}}$$

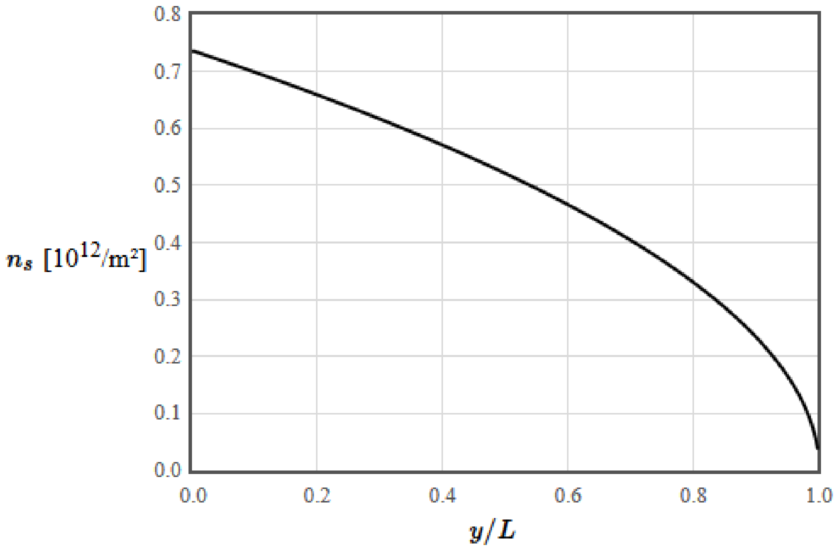


Gradual channel approximation



$Z =$ 1E-5 m
$L =$ 1E-6 m
$\mu_n =$ 1500 cm²/Vs
$\epsilon_r =$ 4
$t_{ox} =$ 3E-9 m
$V_D =$ 1.9 V
$V_G =$ 3 V
$V_T =$ 1 V
Replot





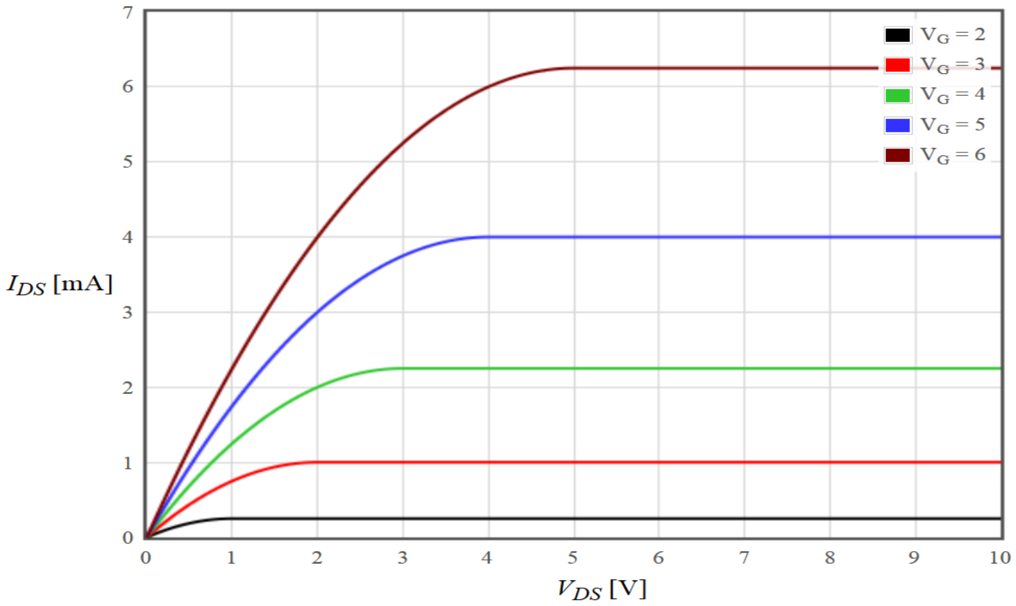
http://lampz.tugraz.at/~hadley/psd/L10/gradualchannelapprox.php

Gradual channel approximation

$$\int_0^L I_D \, \mathrm{d}y = \int_0^{V_D} Z\mu_n C_{ox}(V_G - V_{ch}(y) - V_T)\, \mathrm{d}V$$

$$\boxed{I_D = \frac{Z}{L}\mu_n C_{ox}\left[(V_G - V_T)V_D - \frac{V_D^2}{2}\right]}$$

valid in the linear regime (until pinch-off occurs at the drain)



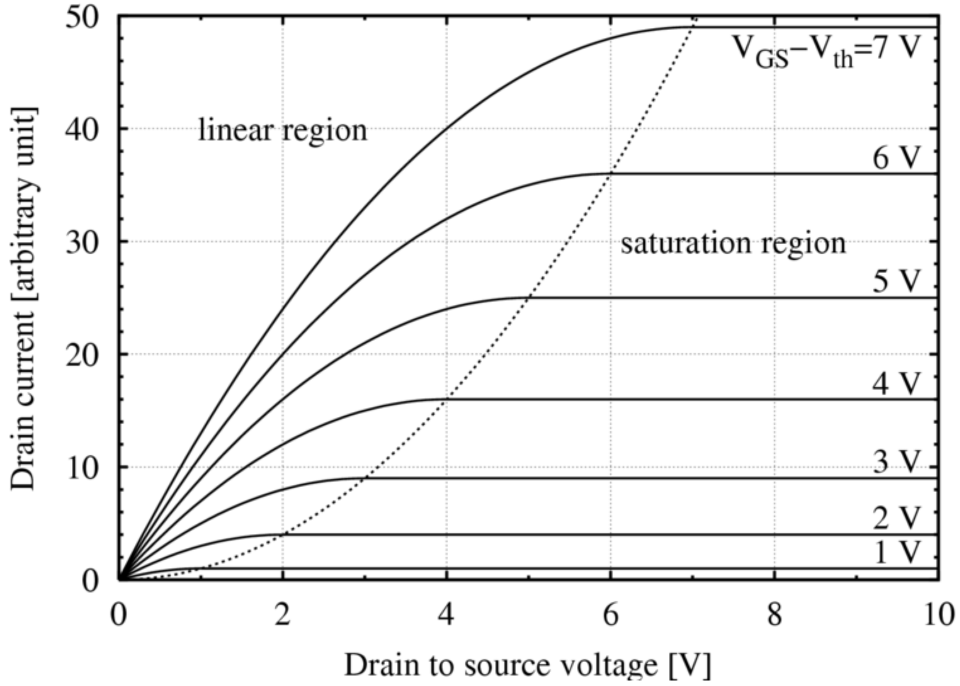
MOSFET – saturation voltage

$$I = \frac{Z}{L}\mu_n C_{ox}\left[(V_G - V_T)V_D - \frac{V_D^2}{2}\right]$$

At pinch-off, $dI_{ds}/dV_{ds} = 0$

$$\frac{dI}{dV_D} = \frac{Z}{L}\mu_n C_{ox}[(V_G - V_T) - V_D] = 0$$

$$V_{sat} = (V_G - V_T)$$



A MOSFET in saturation is a voltage controlled current source.

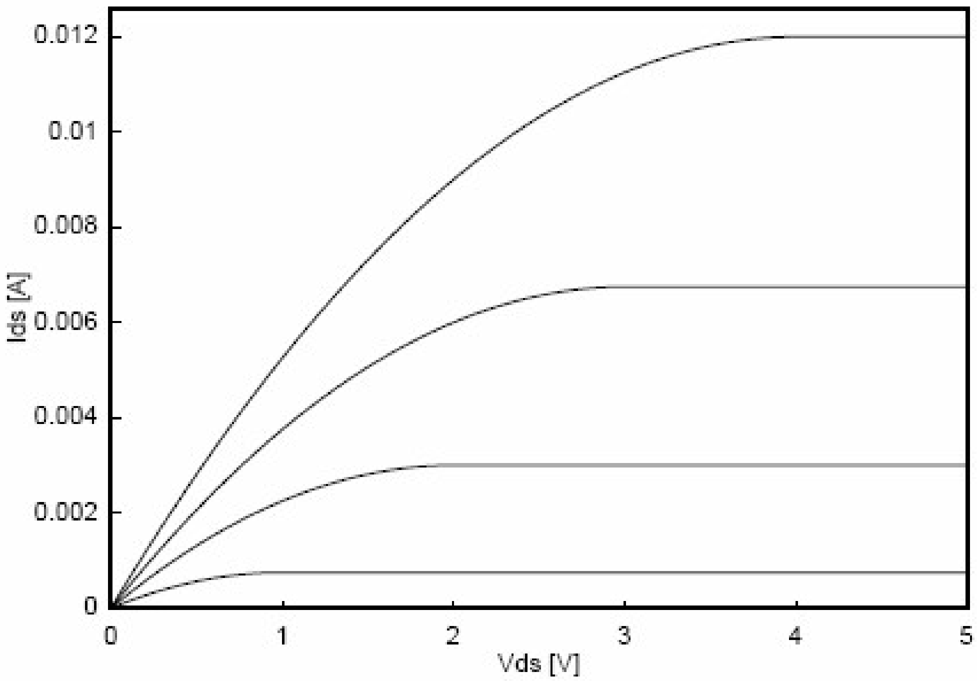
MOSFET – saturation current

Use the saturation voltage at pinch-off to determine the saturation current

$$V_{sat} = (V_G - V_T)$$

$$I = \frac{Z}{L}\mu_n C_{ox}\left[(V_G - V_T)V_D - \frac{V_D^2}{2}\right]$$

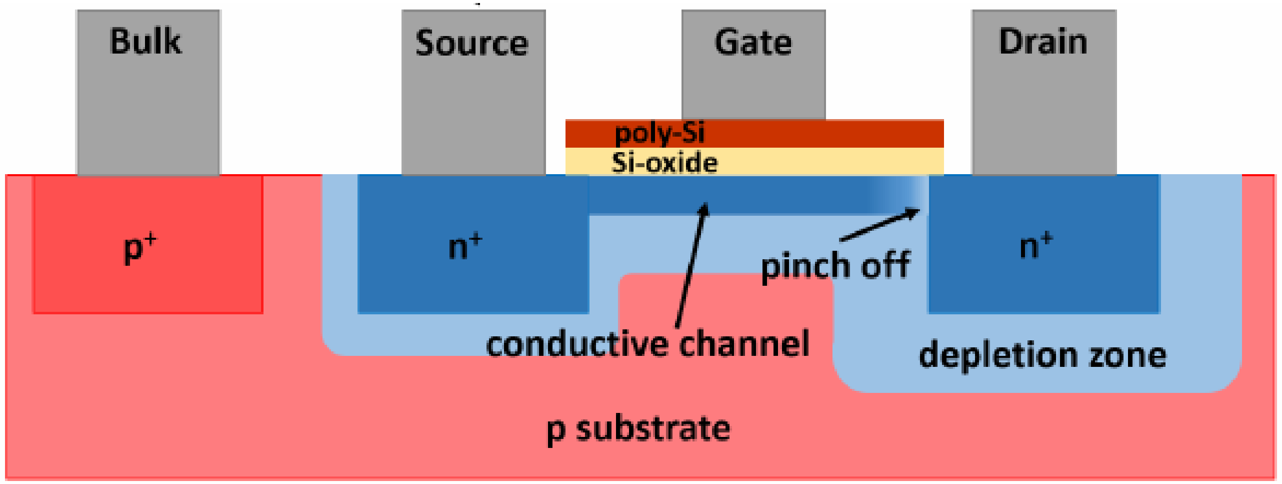
$$I_{sat} = \frac{Z}{2L}\mu_n C_{ox}(V_G - V_T)^2$$

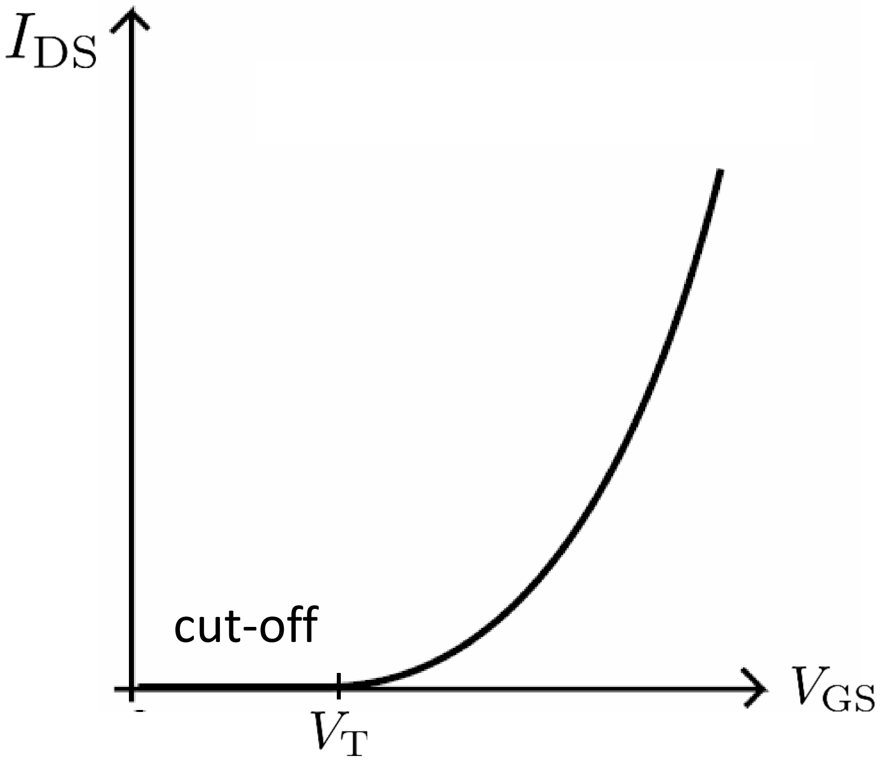


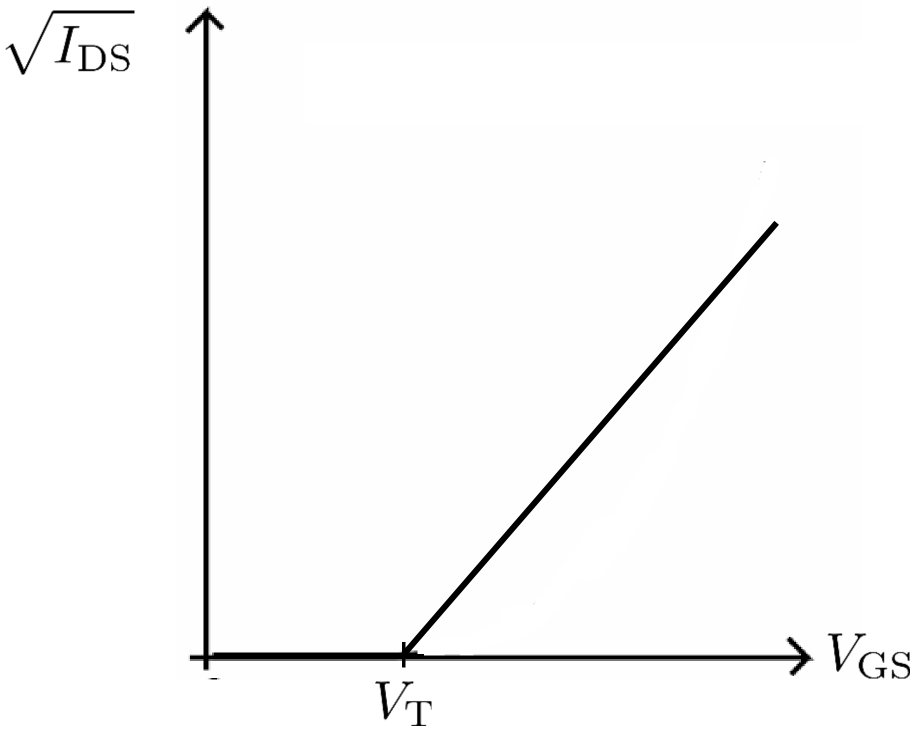
MOSFET – saturation regime

$$I_{sat} = \frac{Z}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2$$

$$V_{DS} \geq V_{DSsat} = V_G - V_T$$







MOSFET – linear regime

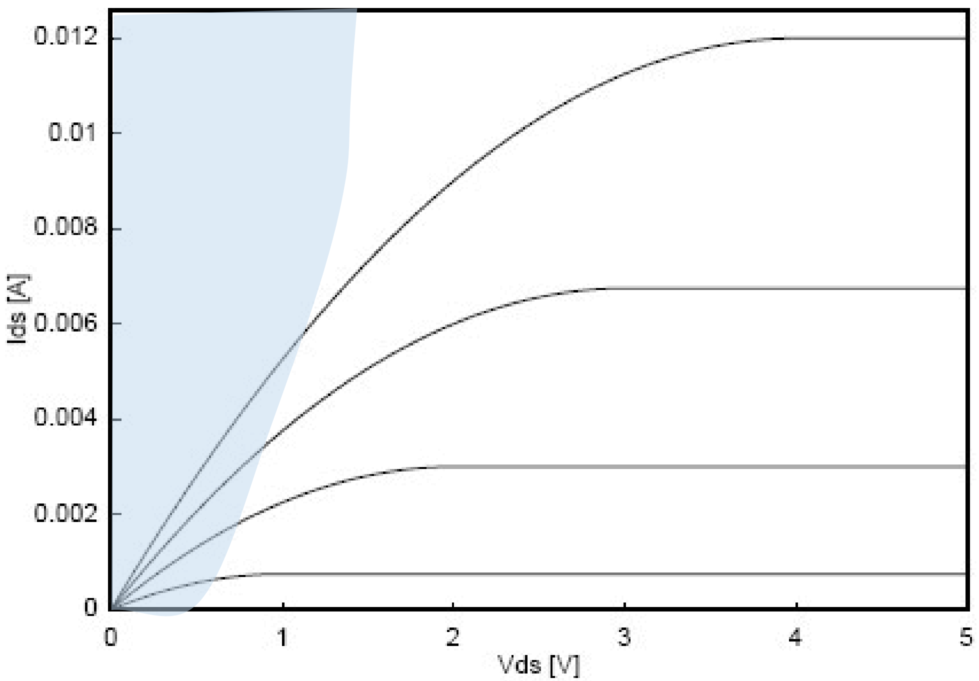
Channel conductance in the linear regime. For small V_D

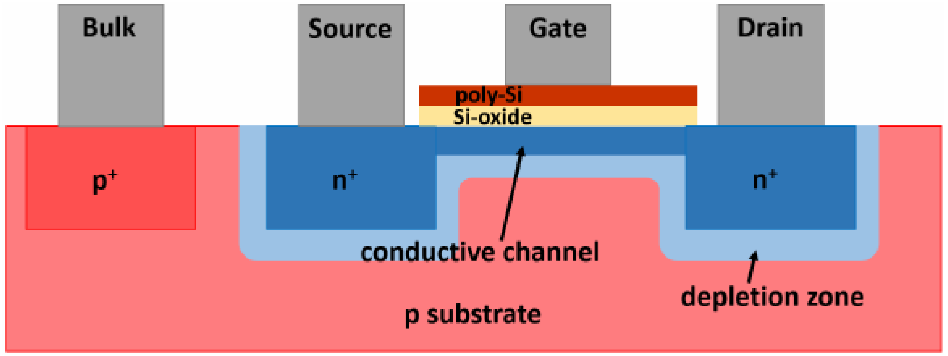
$$I \approx \frac{Z}{L}\mu_n C_{ox}[(V_G - V_T)V_D]$$

$$g_D = \frac{dI_D}{dV_D} = \frac{Z}{L}\mu_n C_{ox}(V_G - V_T)$$

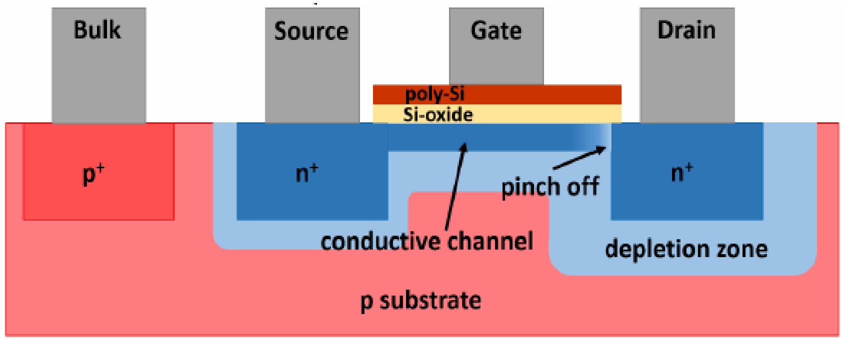
Transconductance

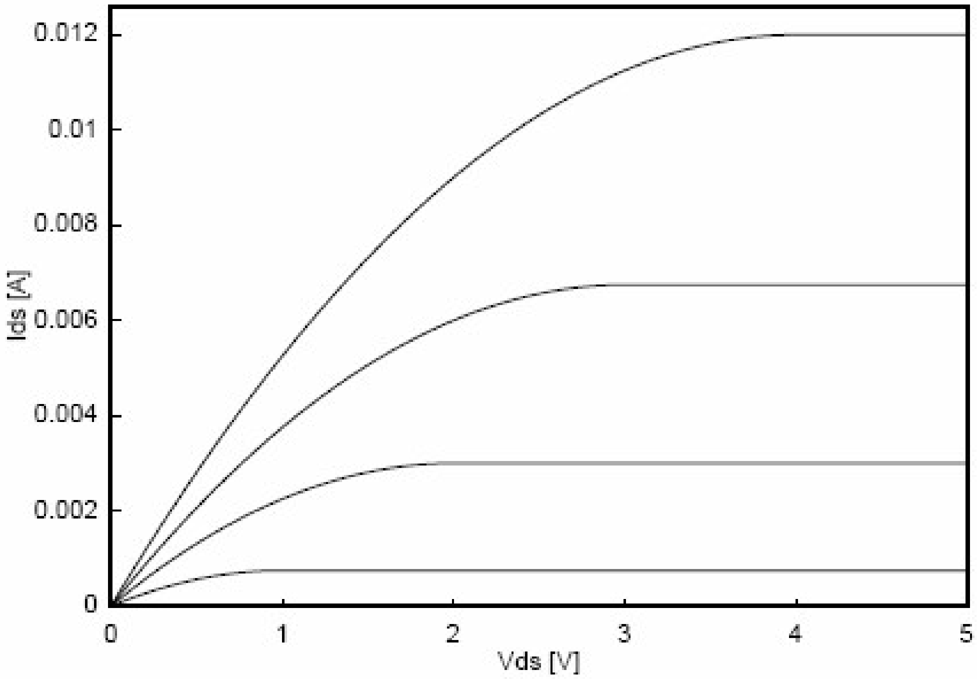
$$g_m = \frac{dI_D}{dV_G} = \frac{Z}{L}\mu_n C_{ox} V_D$$





MOSFET – saturation regime





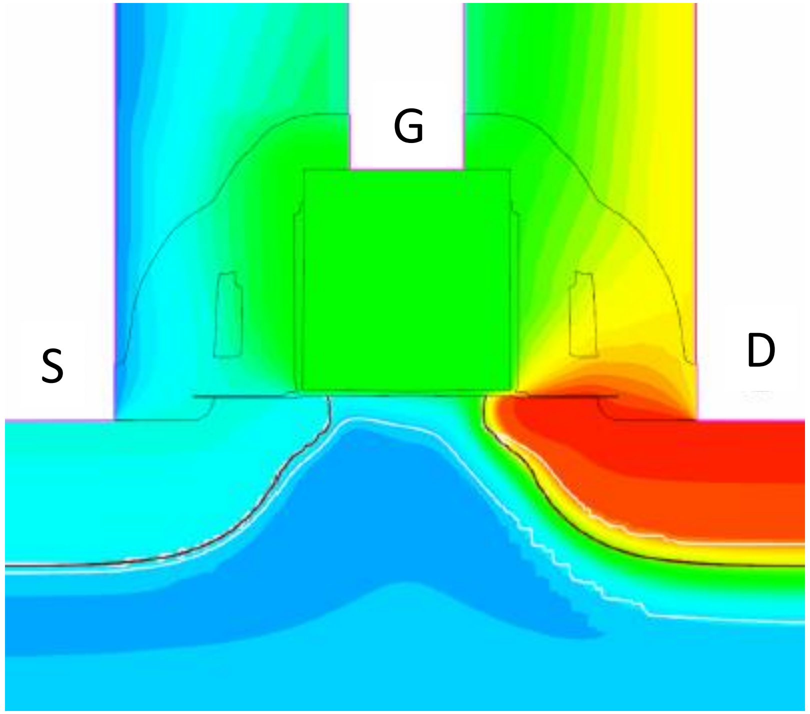
$$I_{sat} = \frac{Z}{2L} \mu_n C_{ox} \left(V_G - V_T\right)^2$$

Transconductance

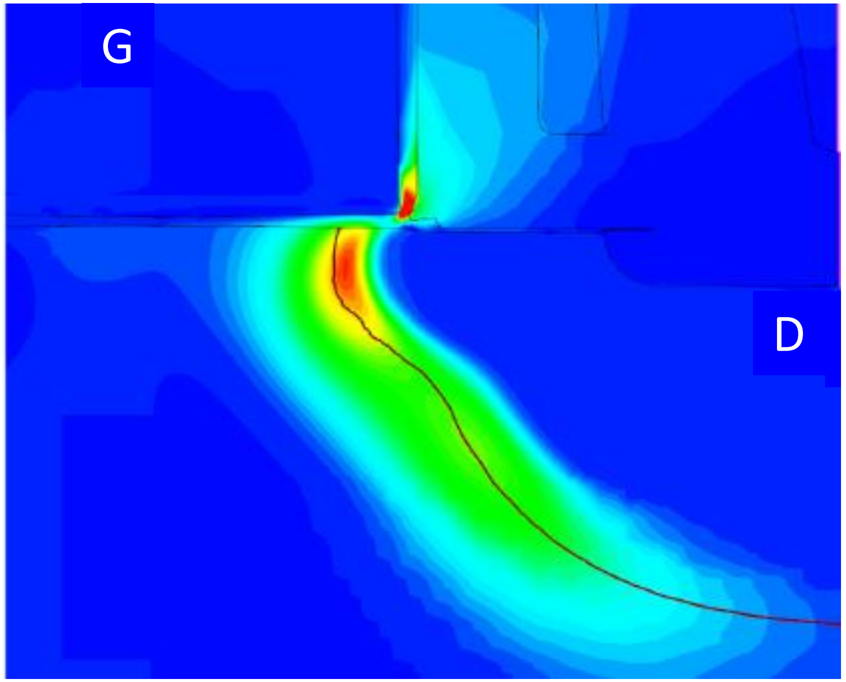
$$g_m = \frac{dI_D}{dV_G} = \frac{Z}{L} \mu_n C_{ox} \left(V_G - V_T\right)$$

A MOSFET in the saturation regime acts like a voltage controlled current source.

MOSFET – saturation



Potential

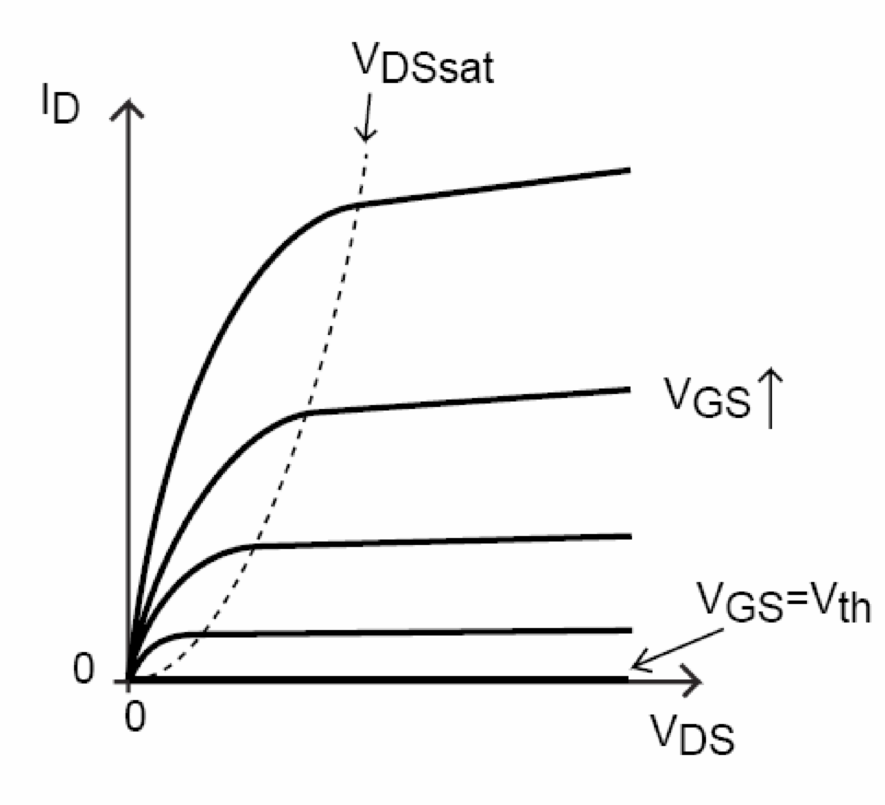


Electric field strength

Alexander Schiffmann, Master Thesis (2016)

MOSFET – saturation regime

$$I_{sat} = \frac{Z}{2L}\mu_n C_{ox}(V_G - V_T)^2\big(1 - \lambda(V_D - V_{sat})\big)$$



Experimentally: channel length modulation

$$\lambda \propto \frac{1}{L}$$

high frequencies

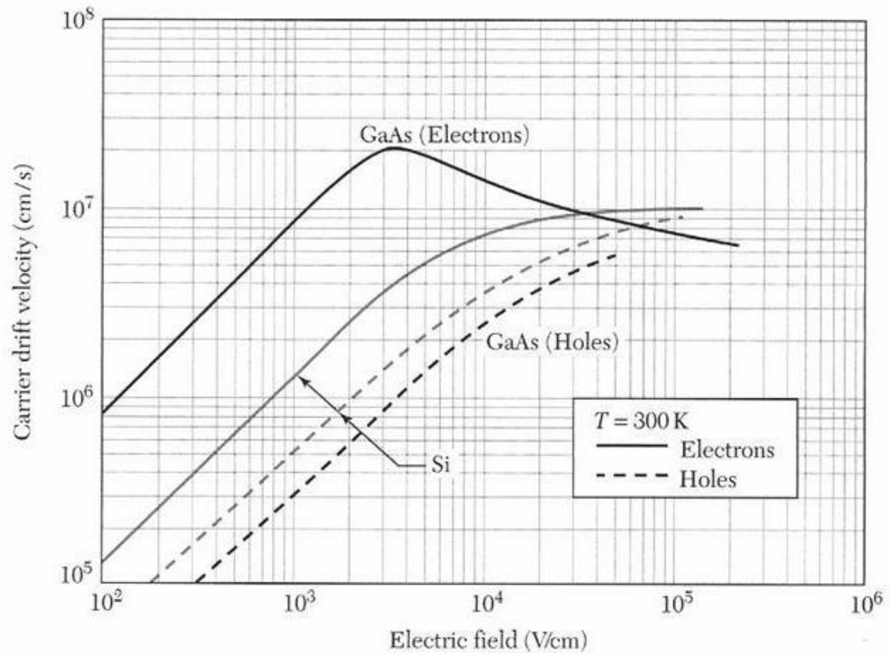
$$\tilde{\imath}_{in} = 2\pi f C_G \tilde{v}_G \qquad \tilde{\imath}_{out} = g_m \tilde{v}_G$$

$$\tilde{\imath}_{in} < \tilde{\imath}_{out}$$

$$f < \frac{g_m}{2\pi C_G} \propto \frac{1}{L^2} = f_T$$

For large E, Ohm's law ($j = ne\mu E$) is not valid.
The electron velocity saturates.
For velocity saturation:

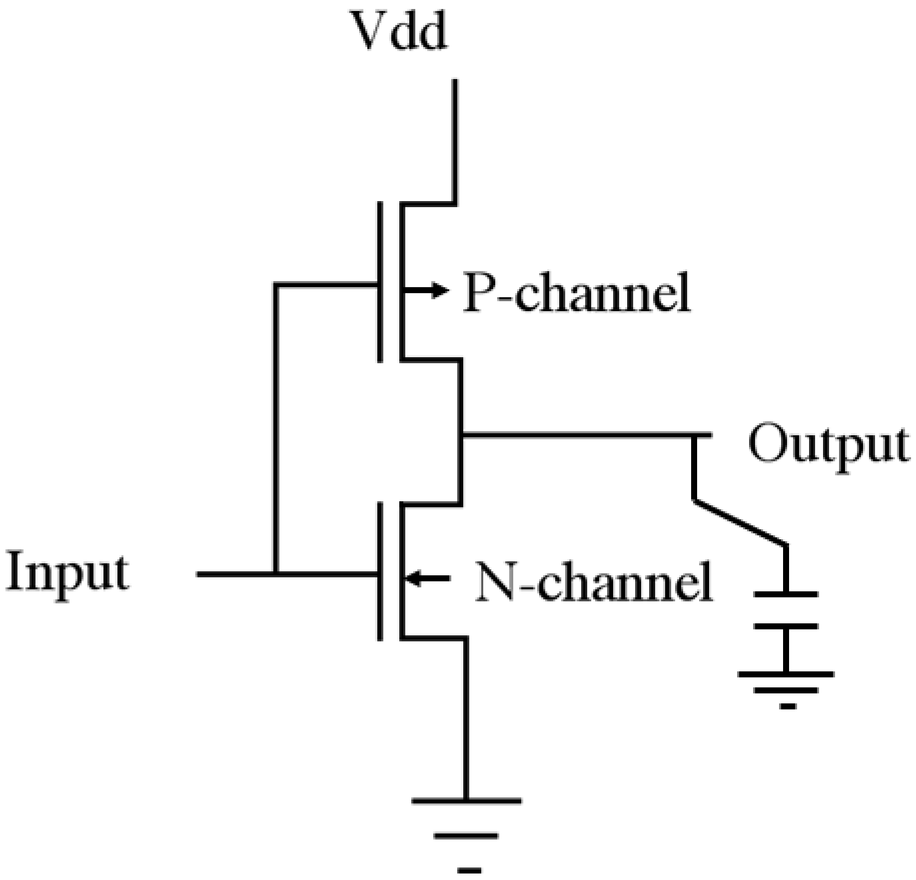
$$f_T \approx \frac{v_s}{L}$$



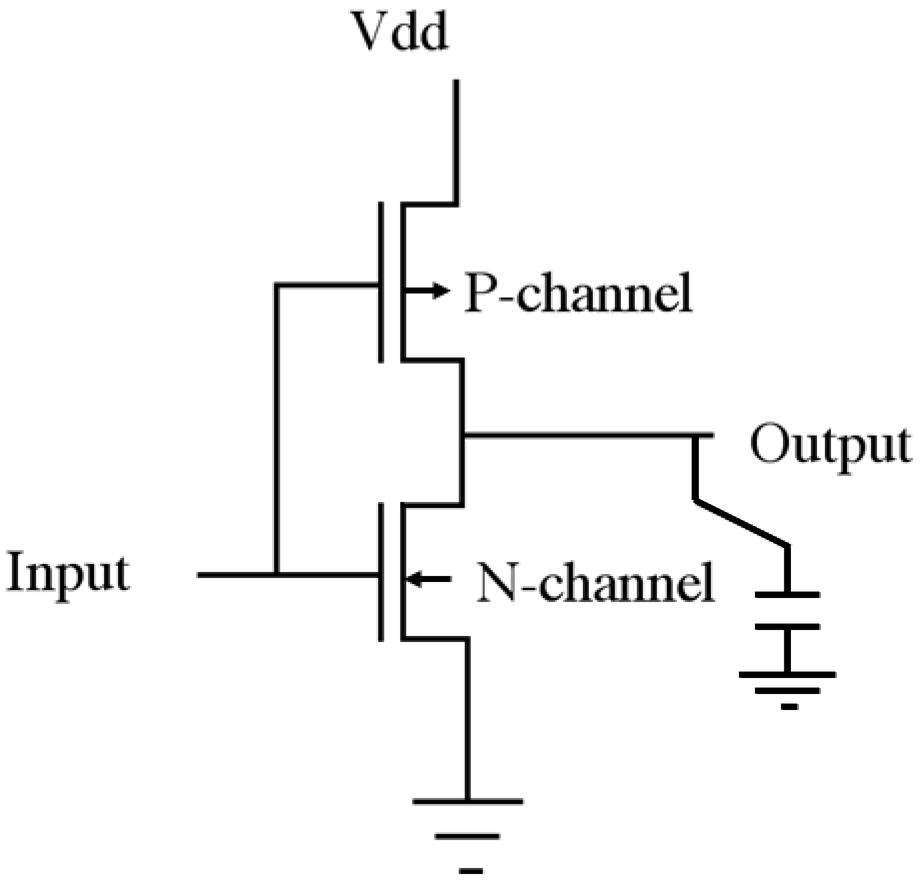
CMOS inverter

Dissipates little power except when switching

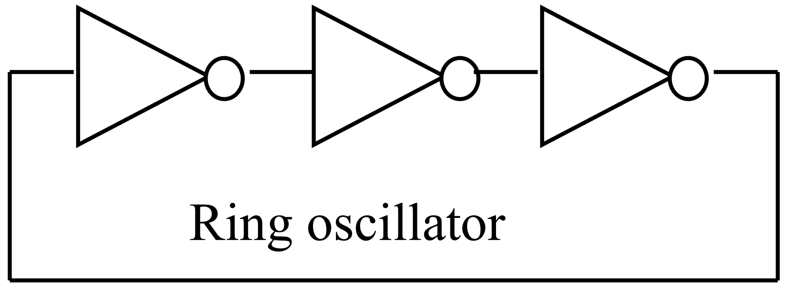
$$E = QV_{dd} = CV_{dd}^2$$



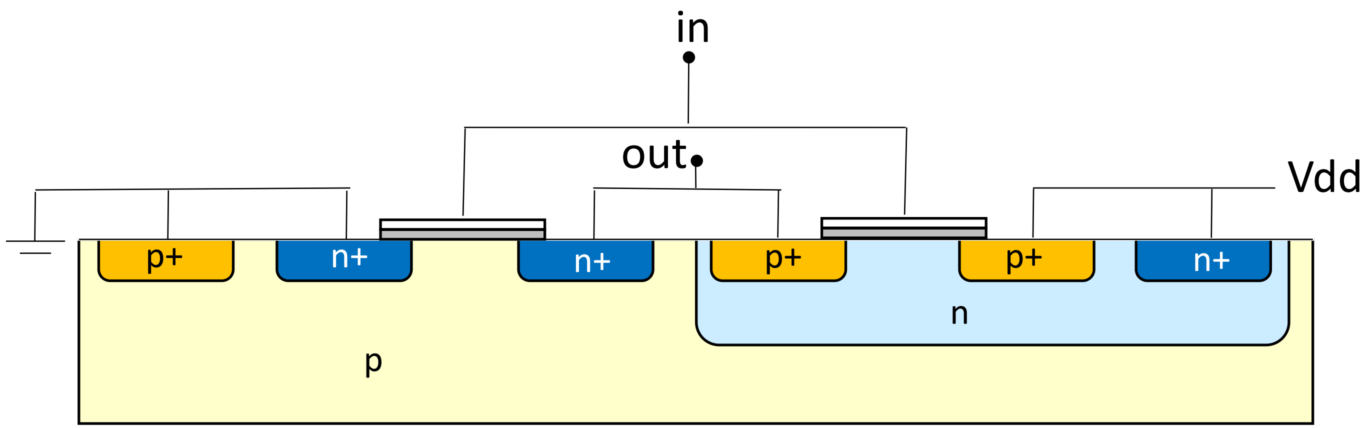
Gate delay

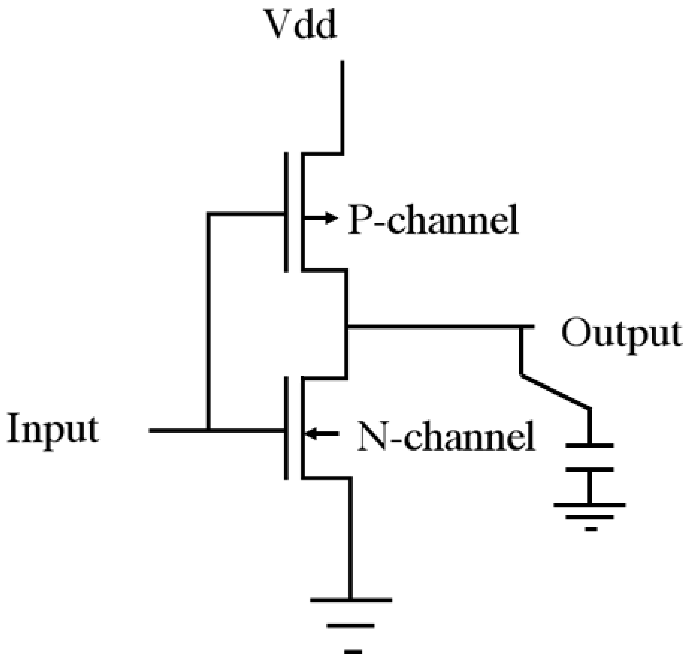


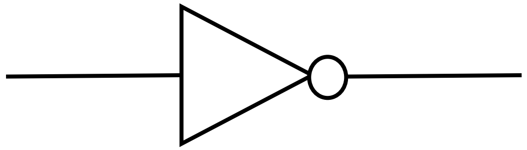
Gate delay is limited by $C_{gate}V_{dd}/I$.



CMOS inverter

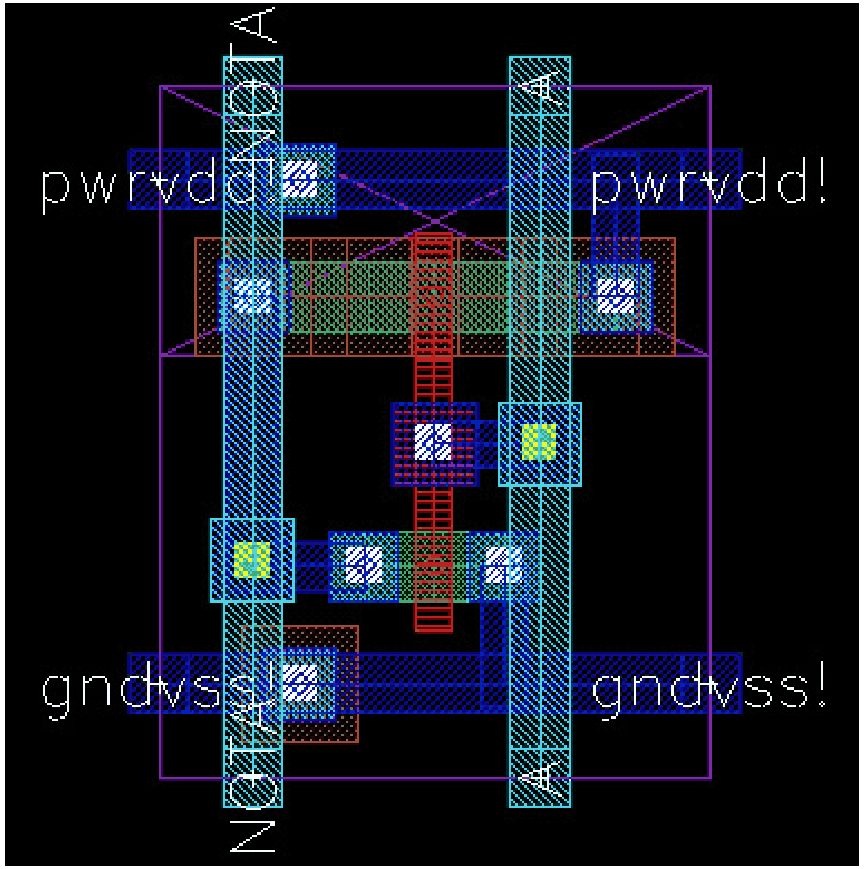


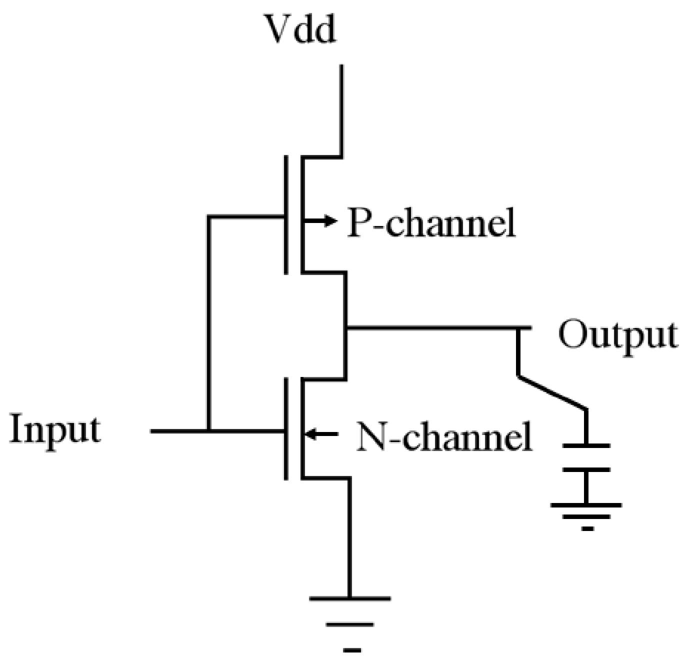


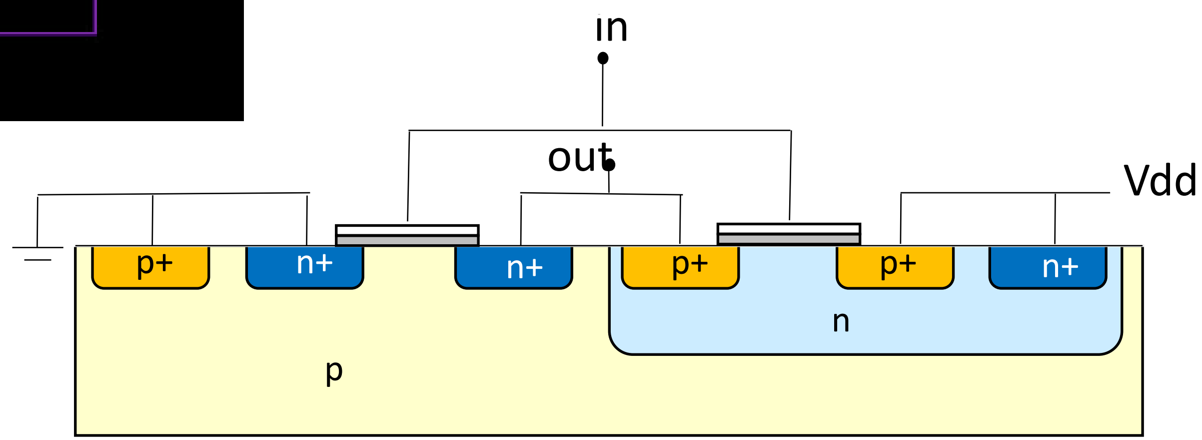


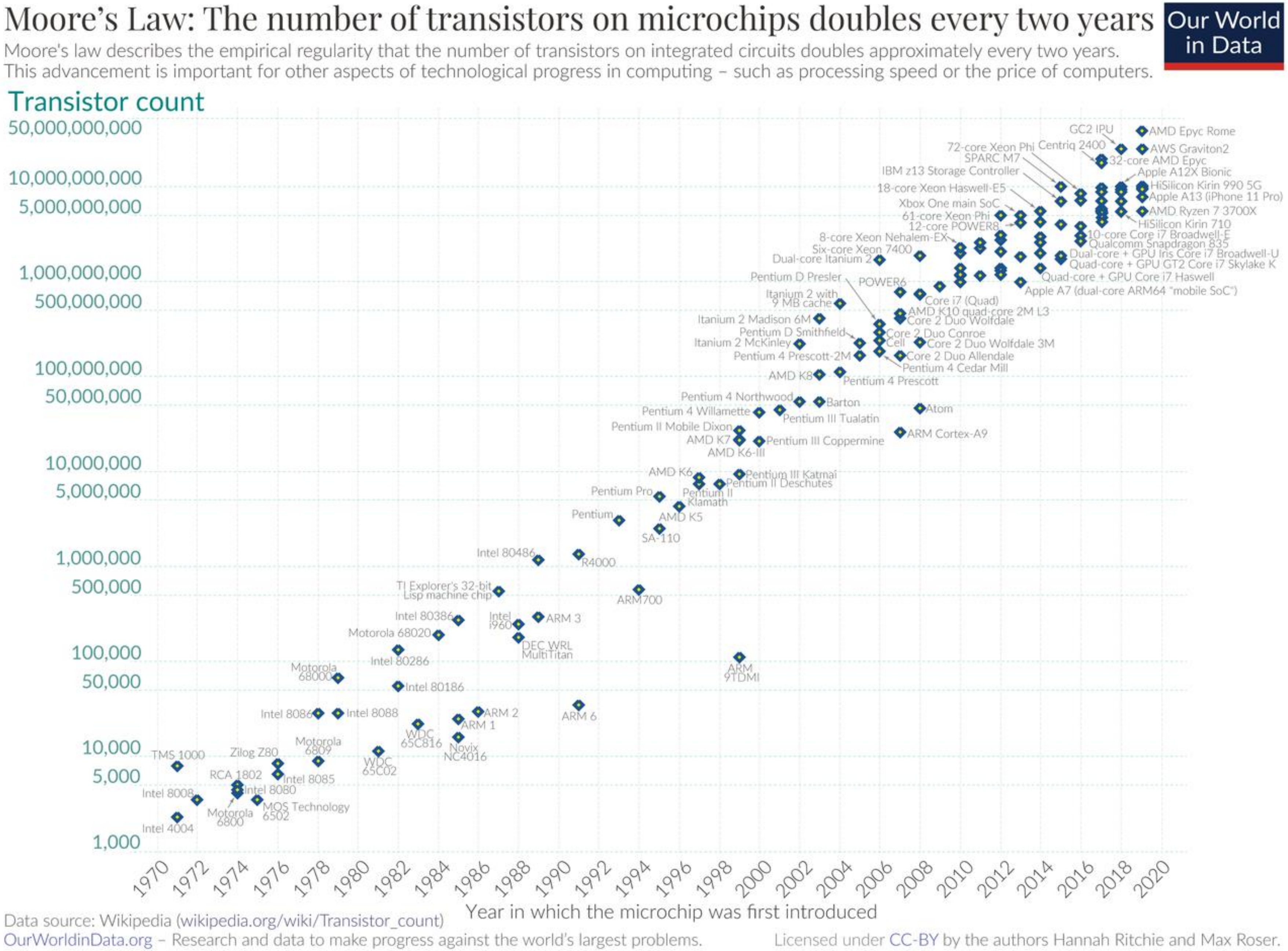
inverter

CMOS inverter



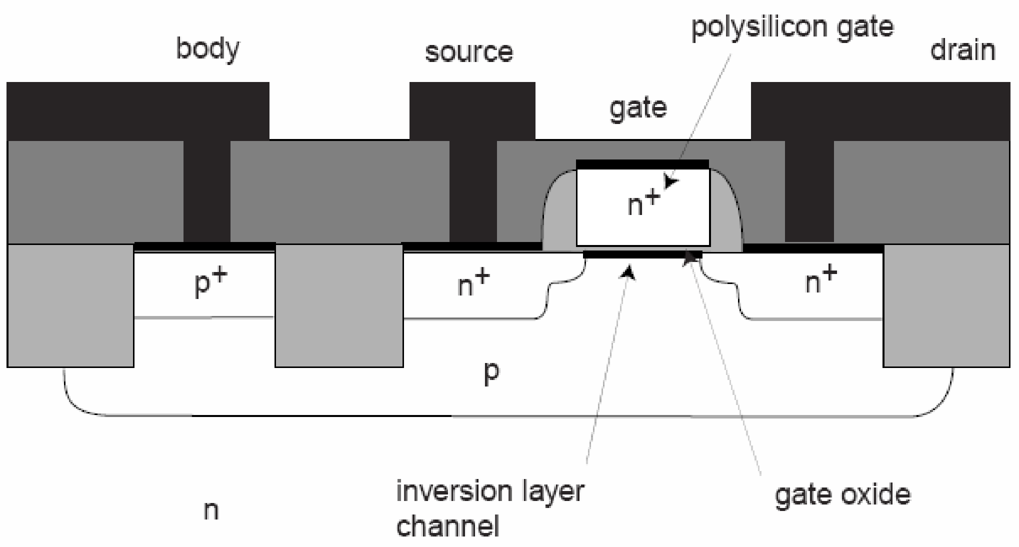






By Max Roser, Hannah Ritchie - https://ourworldindata.org/uploads/2020/11/Transistor-Count-over-time.png, CC BY 4.0, https://commons.wikimedia.org/w/index.php?curid=98219918

Constant E-field Scaling



Gate length L, transistor width Z, oxide thickness t_{ox} are scaled down.

V_{ds}, V_{gs}, and V_T are reduced to keep the electric field constant.

Power density remains constant.

$L \sim 45\ t_{ox}$

1975 - 1990: "Days of happy scaling"

Constant E-field Scaling

$$I_{sat} = \frac{Z}{2L}\mu_n \frac{\varepsilon_{ox}}{t_{ox}}(V_G - V_T)^2$$

$$L \Rightarrow sL, \qquad Z \Rightarrow sZ, \qquad t_{ox} \Rightarrow st_{ox}, \qquad V_T \Rightarrow sV_T$$

$$I_{sat} \Rightarrow sI_{sat}$$ ← I_{sat} gets smaller

$$g_m = \frac{dI_D}{dV_G} = \frac{Z}{L}\mu_n \frac{\varepsilon_{ox}}{t_{ox}}(V_G - V_T)$$ ← Transconductance stays the same.

Power per transistor decreases like L^2. Power per unit area remains constant.

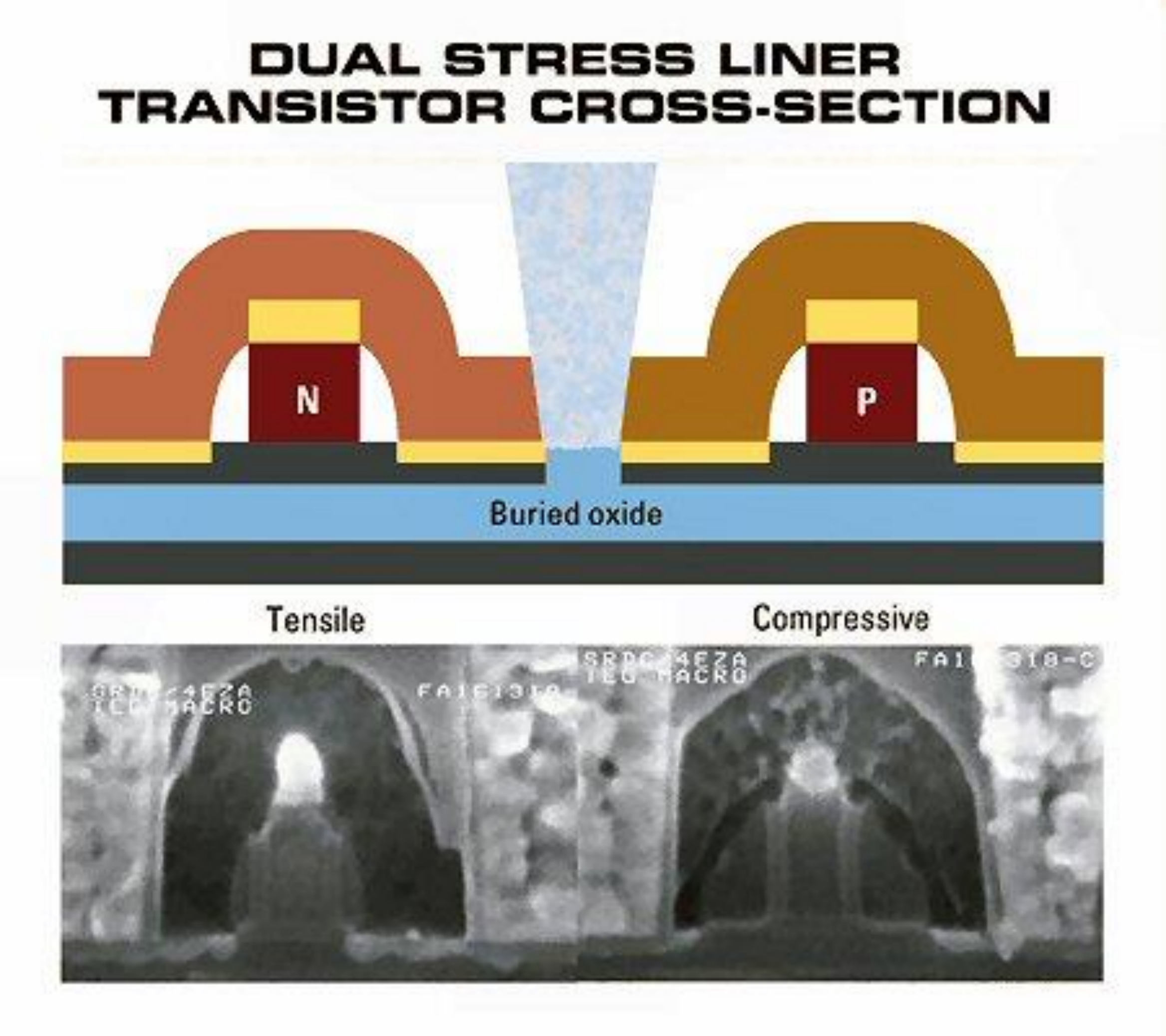
The heat dissipation problem

Microprocessors are hot ~ 100 C
Hotter operation will cause dopants to diffuse

When more transistors are put on a chip they must dissipate less power.

Power per transistor decreases like L^2.

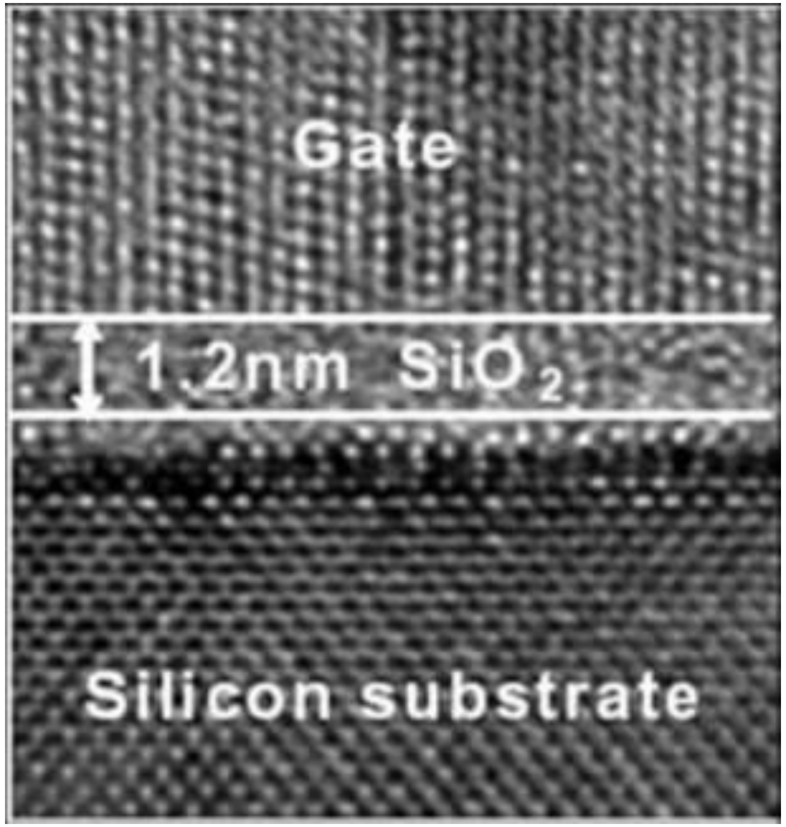
Dual stress liners



Tensile silicon nitride film over the NMOS and a compressive silicon nitride film over the PMOS improves the mobility.

http://www.xbitlabs.com/articles/cpu/display/athlon64-venice_2.html

Gate dielectric

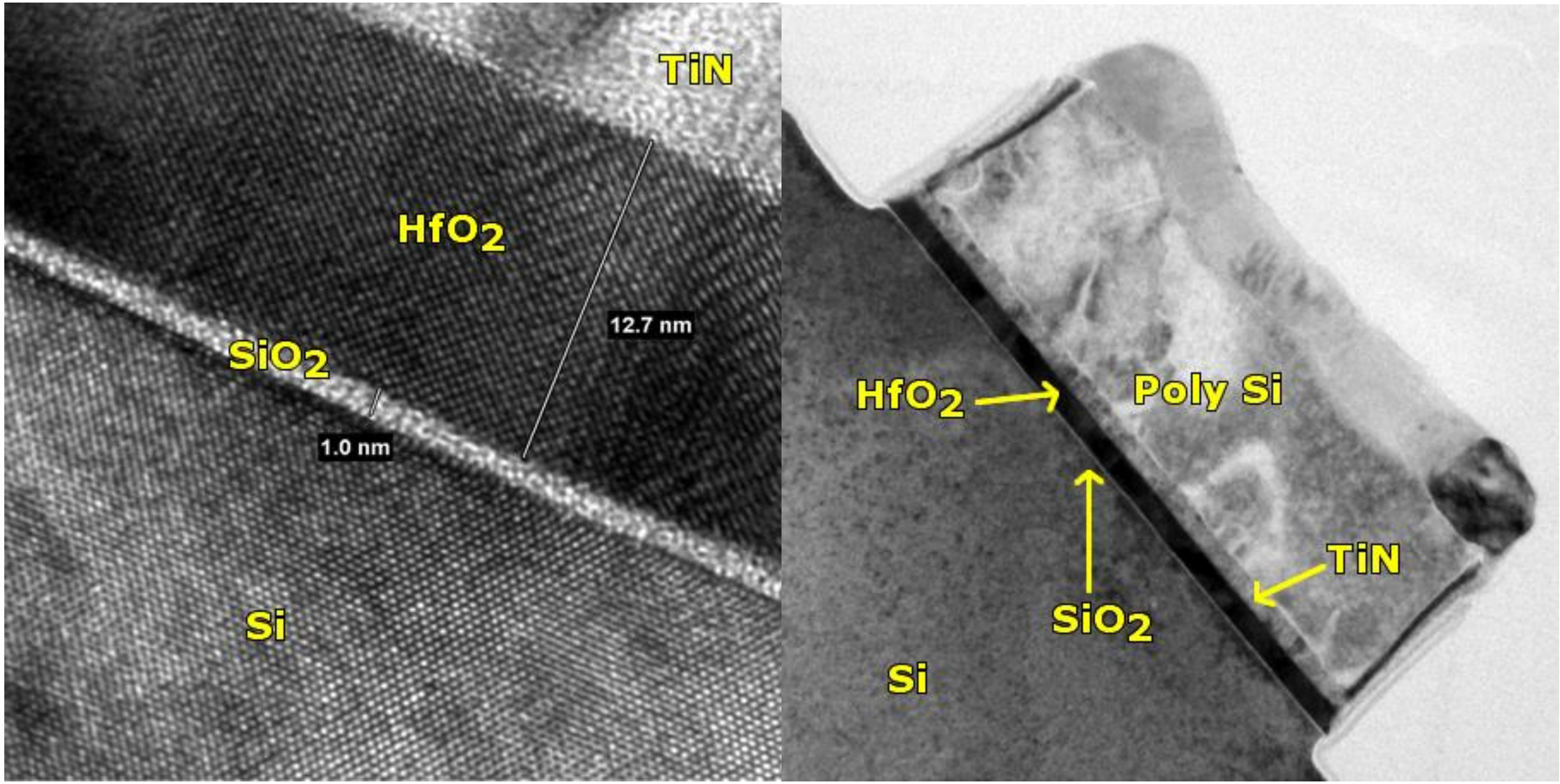


Thinner than 1 nm:
electrons tunnel

Large dielectric constant desirable
$\varepsilon_r(SiO_2) \sim 4$

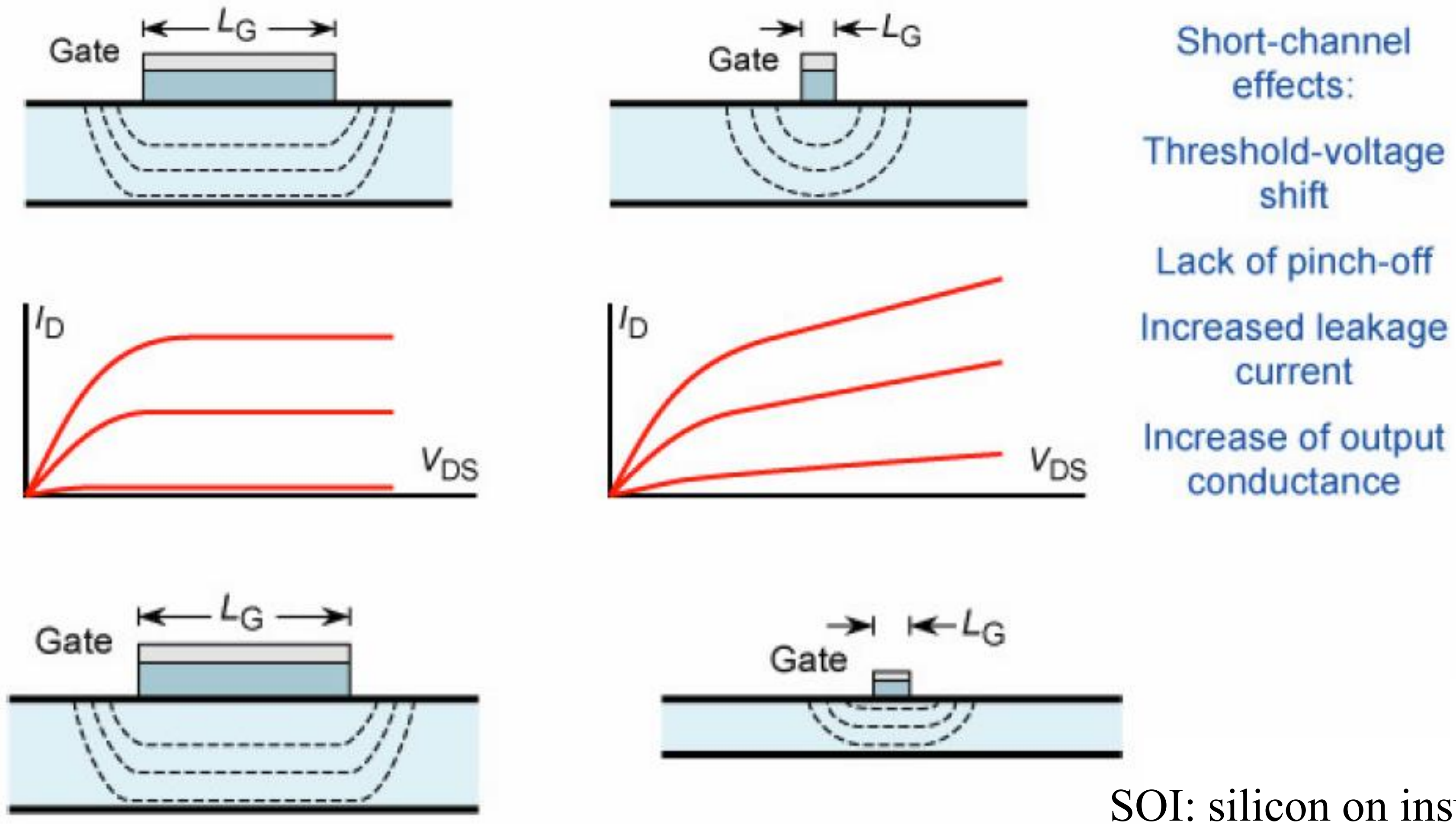
$\varepsilon_r(Si_3N_4) \sim 7$

High-k dielectrics



http://nano.boisestate.edu/research-areas/gate-oxide-studies/

Short channel effects



SOI: silicon on insulator

CMOS SOI (Semiconductor-on-insulator)

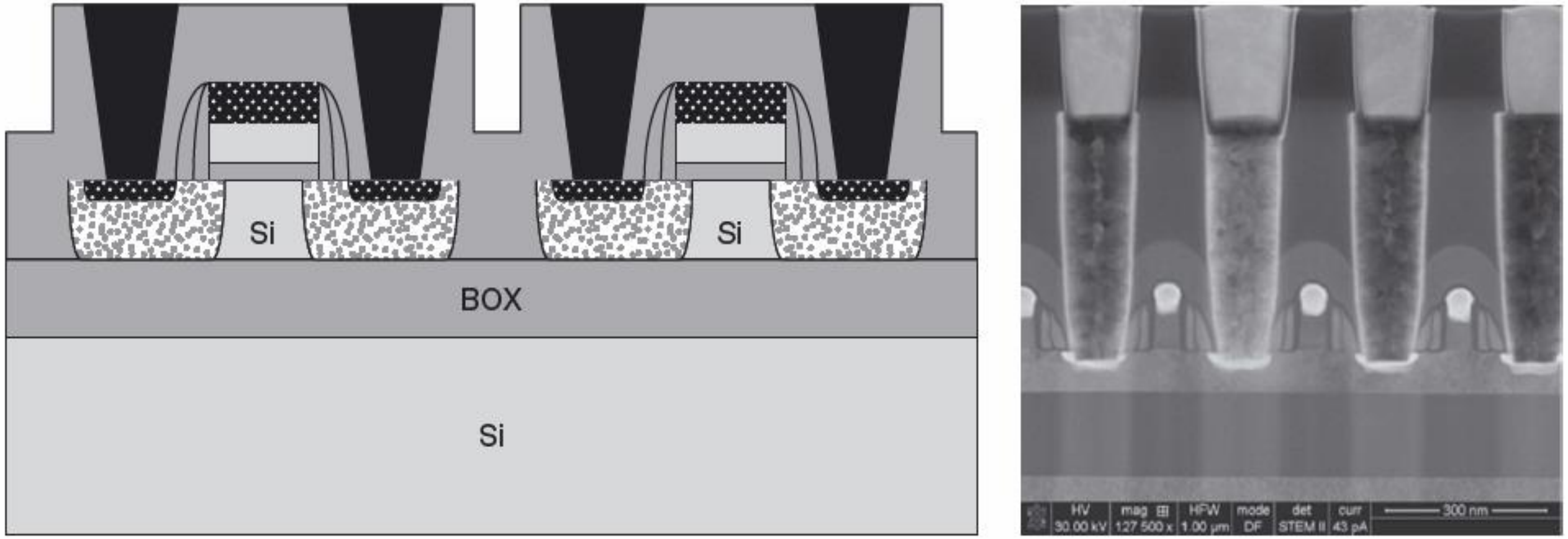
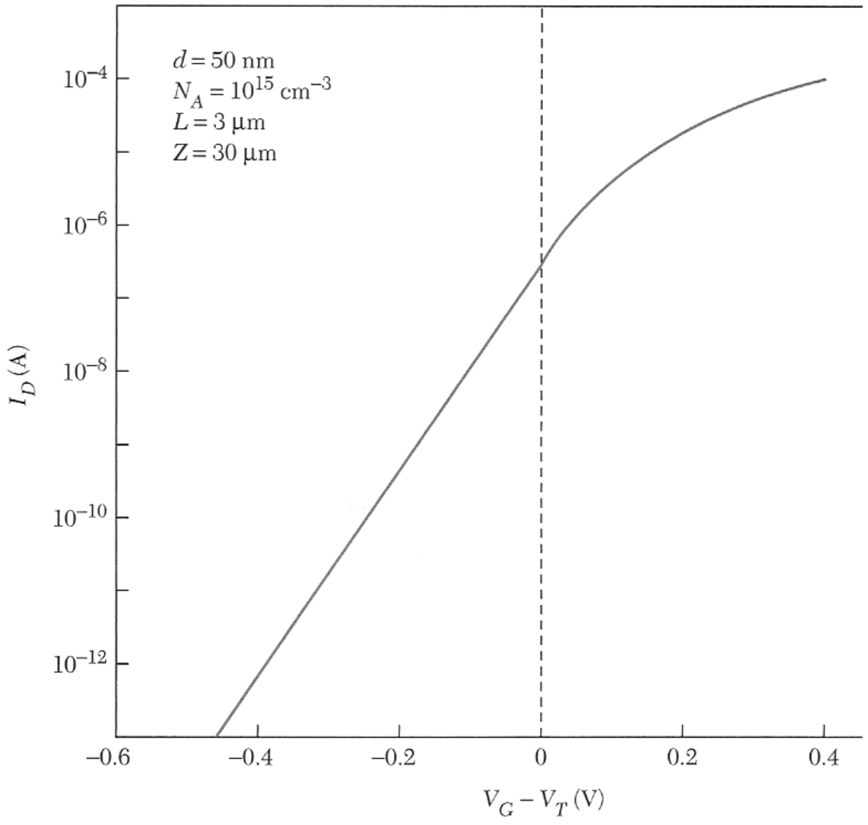


Figure 26.16 SOI MOSFET with first-level metal, schematic and TEM. Courtesy Brandon Van Leer, FEI Company4

Fransila

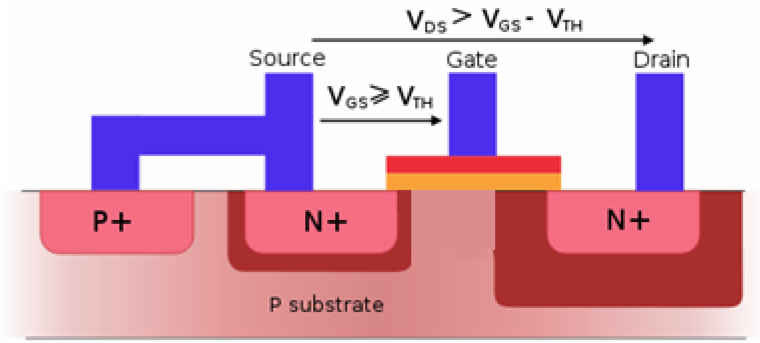
Subthreshold current

For $V_G < V_T$ the transistor should switch off but there is a diffusion current. The current is not really off until ~ 0.5 V below the threshold voltage.



Weak inversion

$$I_D \propto \exp\left(\frac{e(V_G - V_T)}{k_B T}\right)$$

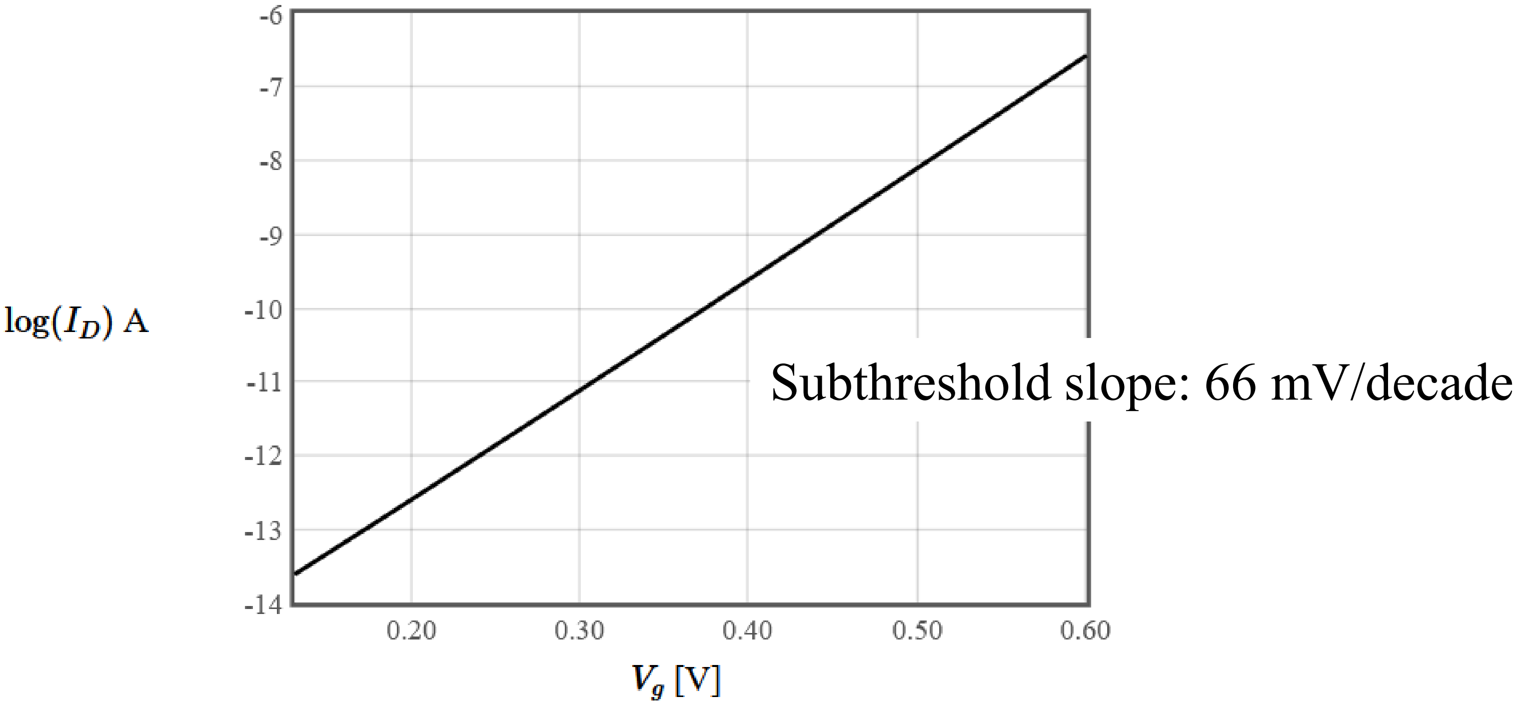


Subthreshold slope: 70-100 mV/decade

Subthreshold current

$$x_p = -\frac{\epsilon_s}{\epsilon_{ox}} t_{ox} + \sqrt{\left(\frac{\epsilon_s}{\epsilon_{ox}} t_{ox}\right)^2 + \frac{2\epsilon_s}{eN_A}(V_g - V_{fb})}$$

$$I_D \approx \frac{k_B T \mu W n_{2d}}{L} = \frac{n_i^2 \mu \epsilon_s k_B^2 T^2 W}{e^2 N_A^2 x_p L} \exp\left(\frac{e^2 N_A x_p^2}{2\epsilon_s k_B T}\right)$$



lampz.tugraz.at/~hadley/psd/L10/moscap_da.php