22. Wellen

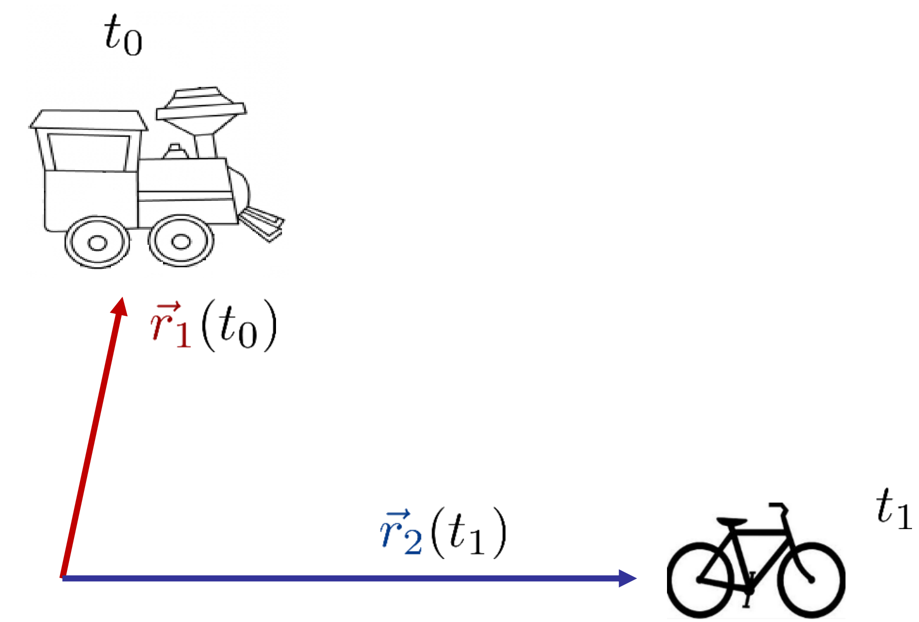
13. Jan. 2020

Überlagerung von Wellen

- Reflektion und Transmission von Wellen an Grenzflächen
- Ausprägen und Eigenschaften stehender Wellen
- Überlagerung der Bewegung von Beobachter und Wellenquelle

Dopplereffekt

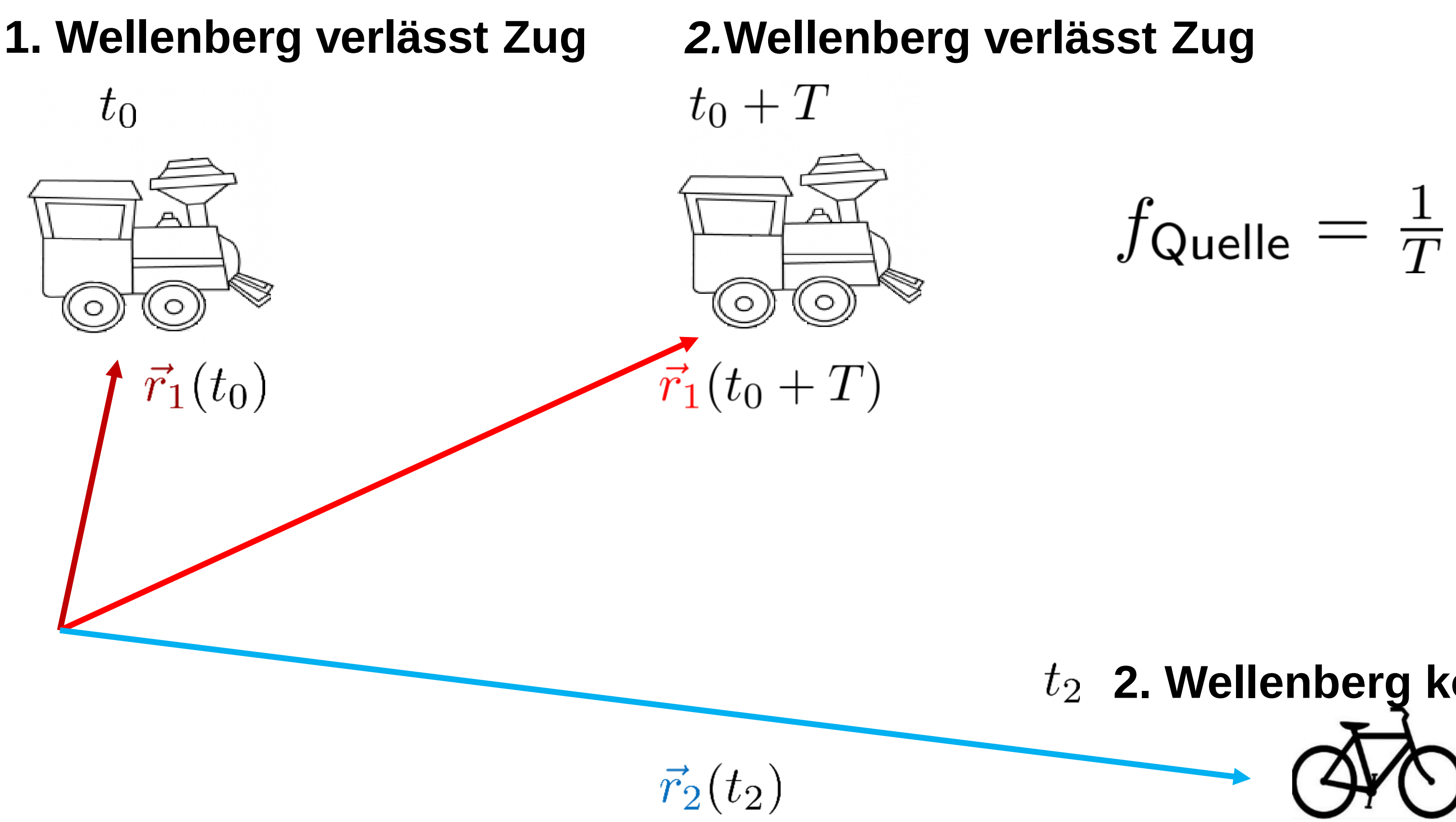
1. Wellenberg verlässt Zug



1. Wellenberg kommt an

$$|\vec{r}_1(t_0) - \vec{r}_2(t_1)| = c(t_1 - t_0)$$

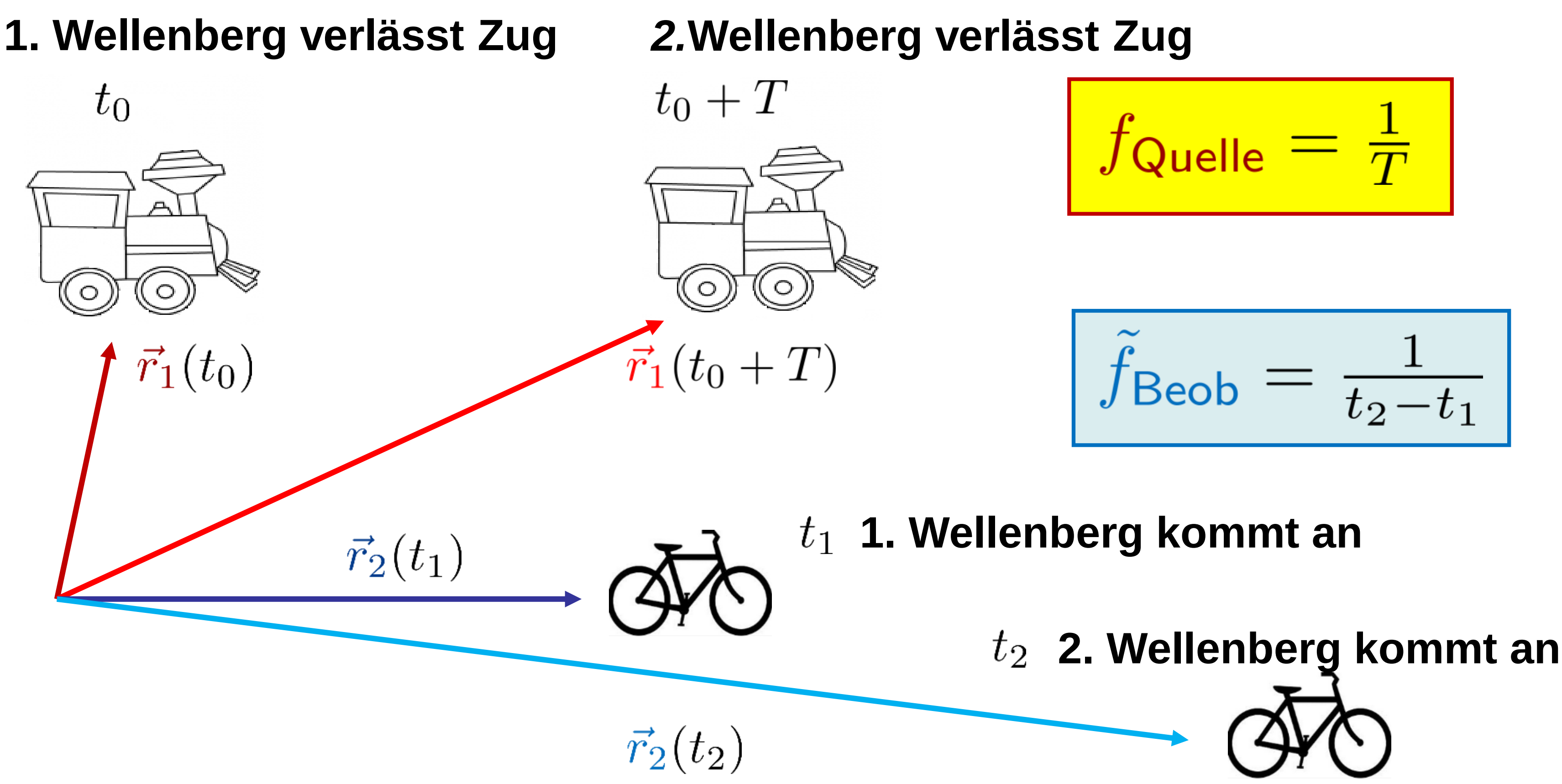
Dopplereffekt



$$f_{\text{Quelle}} = \frac{1}{T}$$

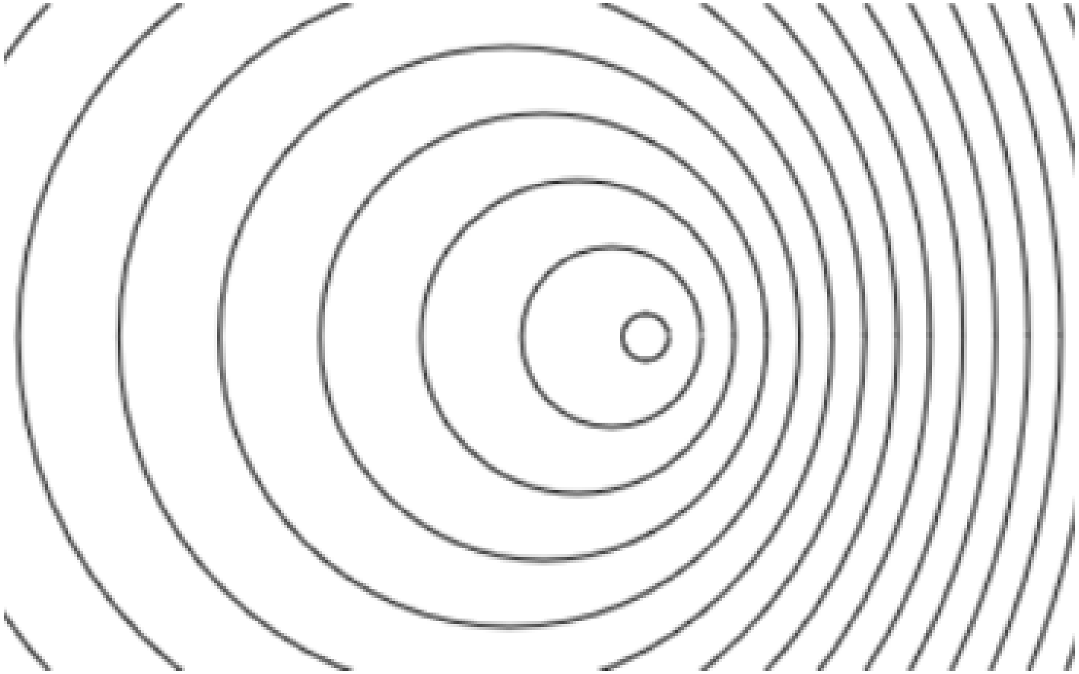
$$|\vec{r}_1(t_0 + T) - \vec{r}_2(t_2)| = c(t_2 - t_0 - T)$$

Dopplereffekt



Beobachter nimmt eine modifizierte Frequenz wahr

Dopplereffekt



$$f_{\text{Quelle}} = \frac{1}{T}$$

$$\tilde{f}_{\text{Beob}} = \frac{1}{t_2 - t_1}$$

modifizierte Frequenz durch Eintreffzeiten benachbarter Wellenberge bestimmt

$$|\vec{r}_1(t_0) - \vec{r}_2(t_1)| = c(t_1 - t_0)$$

$$|\vec{r}_1(t_0 + T) - \vec{r}_2(t_2)| = c(t_2 - t_0 - T)$$

Dopplereffekt - Spezialfälle

Quelle	Beobachter	beobachtete Frequenz	
•	← •	$f_B = f_Q \left(1 + \frac{v_B}{c}\right)$	(5.205)
•	• →	$f_B = f_Q \left(1 - \frac{v_B}{c}\right)$	(5.206)
• →	•	$f_B = \frac{f_Q}{1 - \frac{v_Q}{c}}$	(5.207)
← •	•	$f_B = \frac{f_Q}{1 + \frac{v_Q}{c}}$	(5.208)
• →	← •	$f_B = f_Q \frac{c + v_B}{c - v_Q}$	(5.209)
← •	• →	$f_B = f_Q \frac{c - v_B}{c + v_Q}$	(5.210)
← •	← •	$f_B = f_Q \frac{c + v_B}{c + v_Q}$	(5.211)
• →	• →	$f_B = f_Q \frac{c - v_B}{c - v_Q}$	(5.212)

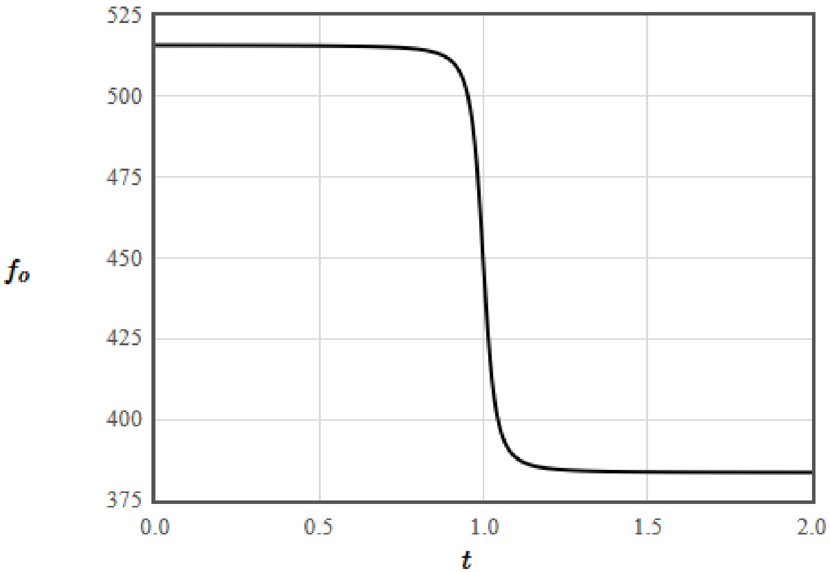
Hering

Dopplereffekt

$$|\vec{r}_o(t_1) - \vec{r}_s(t_0)| = c(t_1 - t_0),$$

$$|\vec{r}_o(t_2) - \vec{r}_s(t_0 + T)| = c(t_2 - t_0 - T),$$

$$f_o = \frac{1}{t_2 - t_1}.$$



$\vec{r}_s(t) =$ 50*t-50 $\hat{x}+$ 0 $\hat{y}+$ 0 $\hat{z}$ [m].

$\vec{r}_o(t) =$ 0 $\hat{x}+$ 2 $\hat{y}+$ 0 $\hat{z}$ [m].

$f_s =$ 440 [Hz] $c =$ 340 [m/s]

Plot f_o from $t =$ 0 to $t =$ 2.

At time $t = 0$ s, the observer hears a frequency of 515.8 Hz.

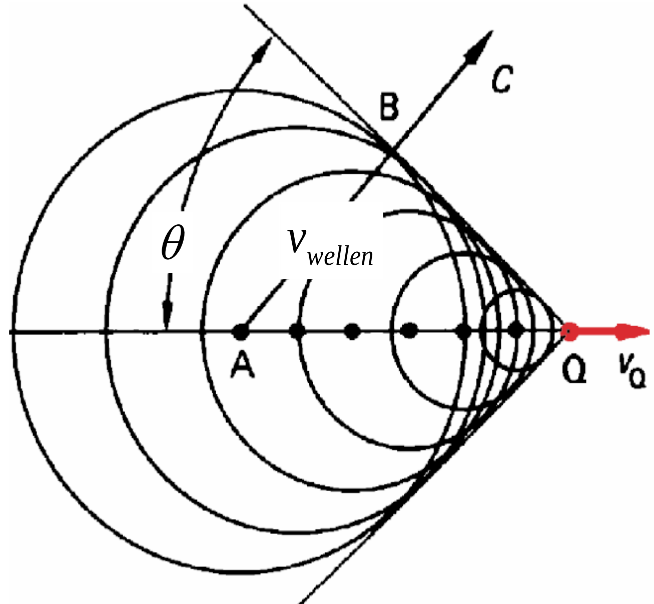
At time $t = 2$ s, the observer hears a frequency of 383.6 Hz.

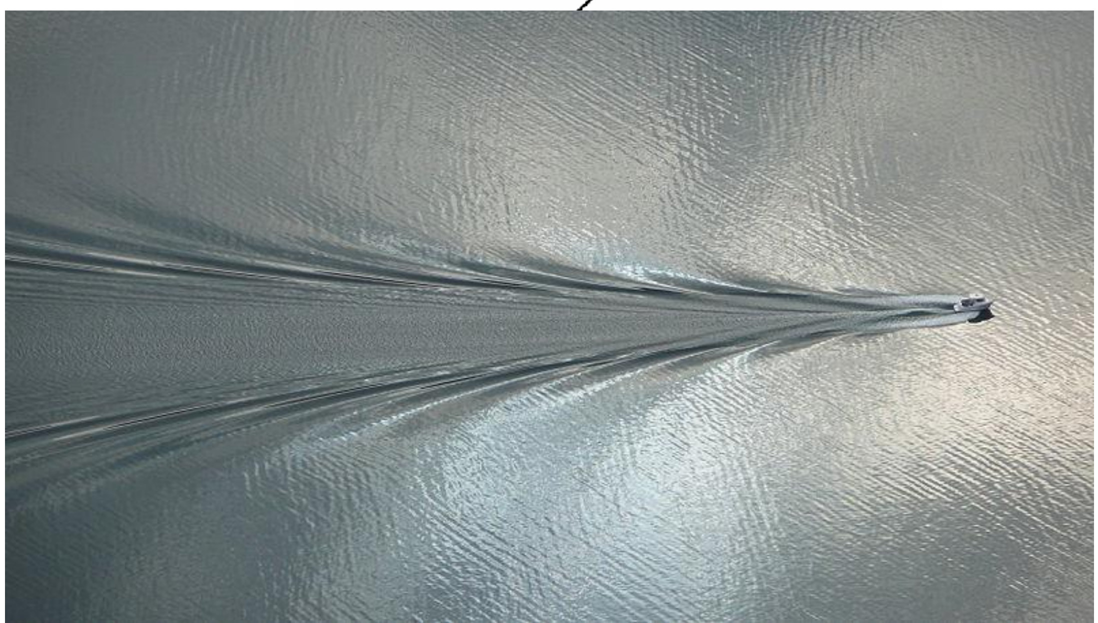
http://lampx.tugraz.at/~hadley/physikm/apps/doppler.de.php

Überschallgeschwindigkeit

Hering

$$\sin\theta = \frac{v_{wellen}}{v_{flugzeug}}$$



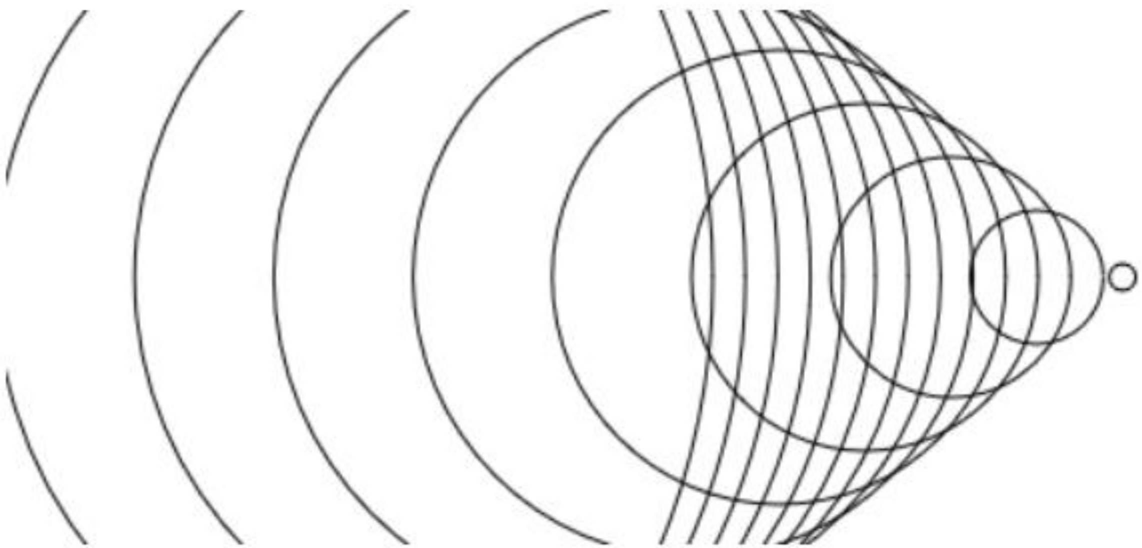


http://en.wikipedia.org/wiki/File:Fjord.surface.wave.boat.jpeg

$$\text{Mach-Zahl} = \frac{v}{v_{wellen}}$$

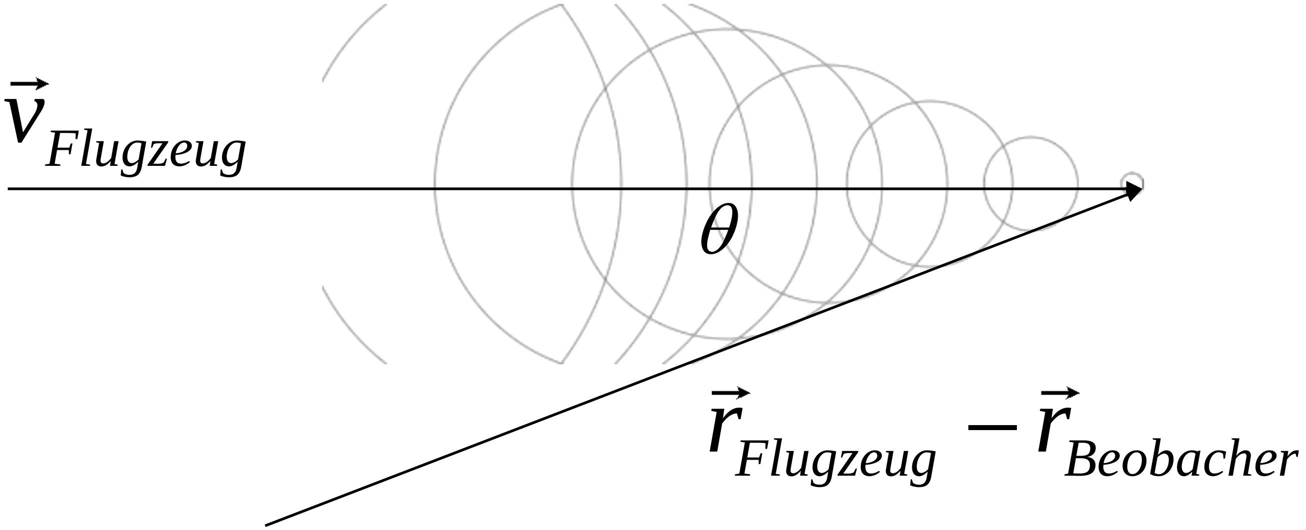
Ernst Mach

Bewegte Wellenquelle



http://lamp.tu-graz.ac.at/~hadley/physikm/apps/shockwave.en.php

Überschallgeschwindigkeit



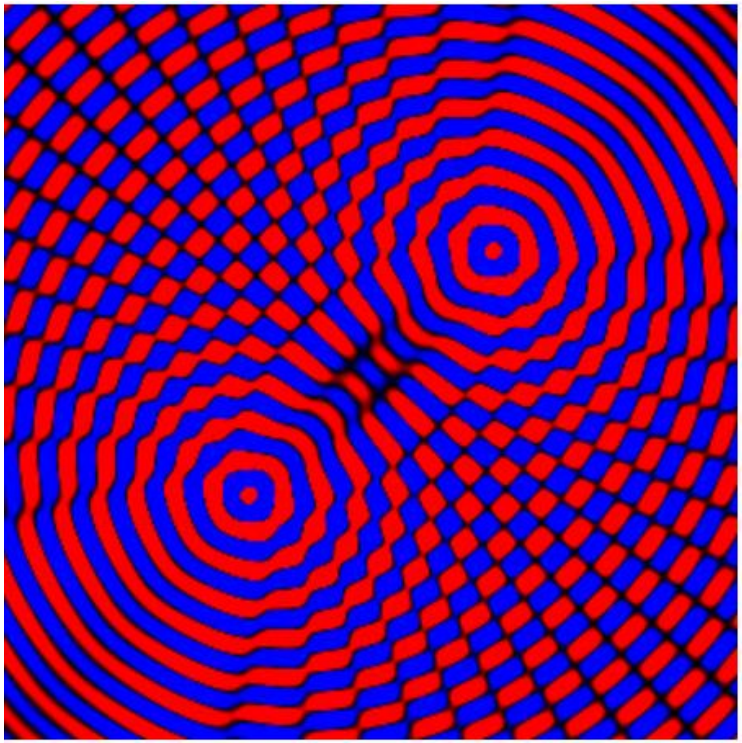
$$\sin\theta = \frac{\left|\vec{v}_{wellen}\right|}{\left|\vec{v}_{flugzeug}\right|}$$

$$\cos\theta = \frac{\vec{v}_{Flugzeug} \cdot \left(\vec{r}_{Flugzeug} - \vec{r}_{Beobacher}\right)}{\left|\vec{v}_{flugzeug}\right|\left|\vec{r}_{Flugzeug} - \vec{r}_{Beobacher}\right|}$$

Überlagerung von Wellen

- Überlagerung von mehreren Wellen
  - Interferenz
  - Beugung

Interferenz zweier Oberflächenwellen



$A_1 =$	0.1	[cm²]	$A_2 =$	0.1	[cm²]
$x_1 =$	2	[cm]	$x_2 =$	4	[cm]
$y_1 =$	2	[cm]	$y_2 =$	4	[cm]
$\phi_1 =$	0	[rad]	$\phi_2 =$	0	[rad]
$\lambda =$	0.3	[cm]	$T =$	0.5	[s]

plot at $t =$ 0 [s].

t - T/10 | t + T/10

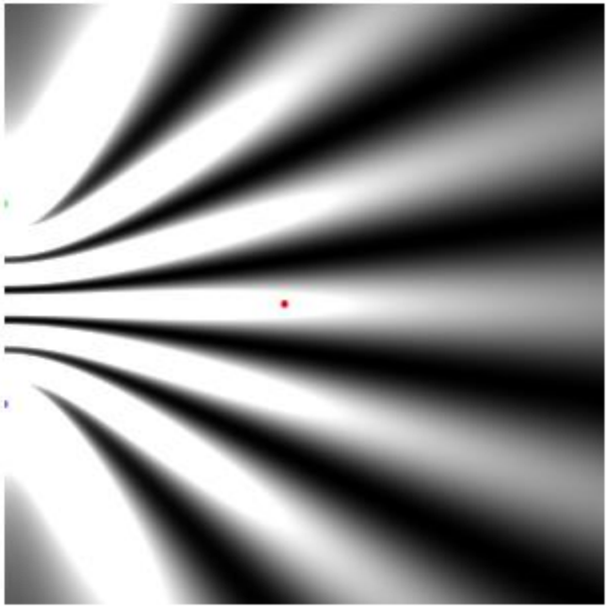
$$c = \frac{\lambda}{T}$$

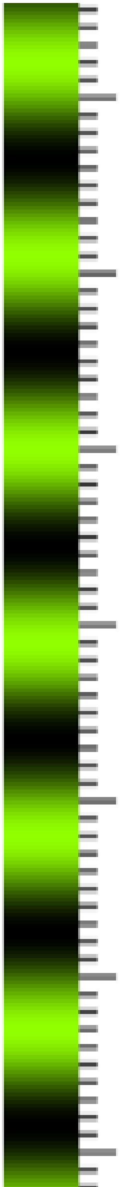
$$z(r,t) = \frac{A_1}{\sqrt{|r-r_1|}}\cos\left(k|r-r_1| - \omega t + \varphi_1\right) + \frac{A_2}{\sqrt{|r-r_2|}}\cos\left(k|r-r_2| - \omega t + \varphi_2\right)$$

Interferenz zweier Oberflächenwellen

$$z(r,t) = \frac{A_1}{\sqrt{|r - r_1|}} \cos\left(k|r - r_1| - \omega t + \varphi_1\right) + \frac{A_2}{\sqrt{|r - r_2|}} \cos\left(k|r - r_2| - \omega t + \varphi_2\right)$$

Doppelspalt





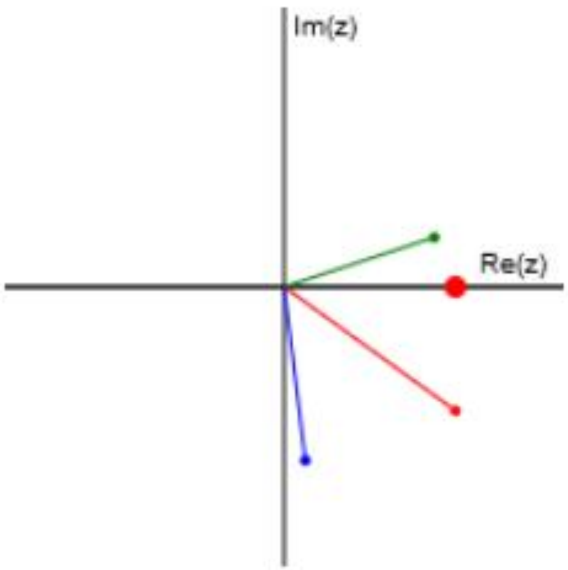
Gesamtamplitude A

$$A = \sum_{j=1}^{N} \frac{A_j}{r_j} e^{i(k|\vec{r}-\vec{r}_j|+\phi_j)}$$

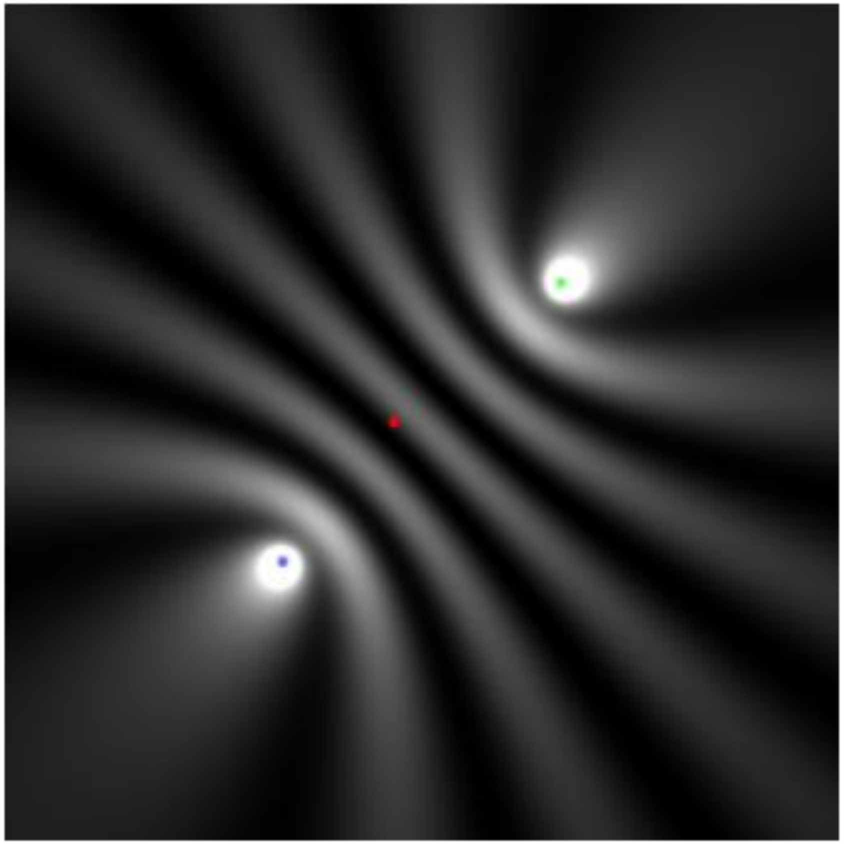
Intensität

$$I \propto A^* A$$

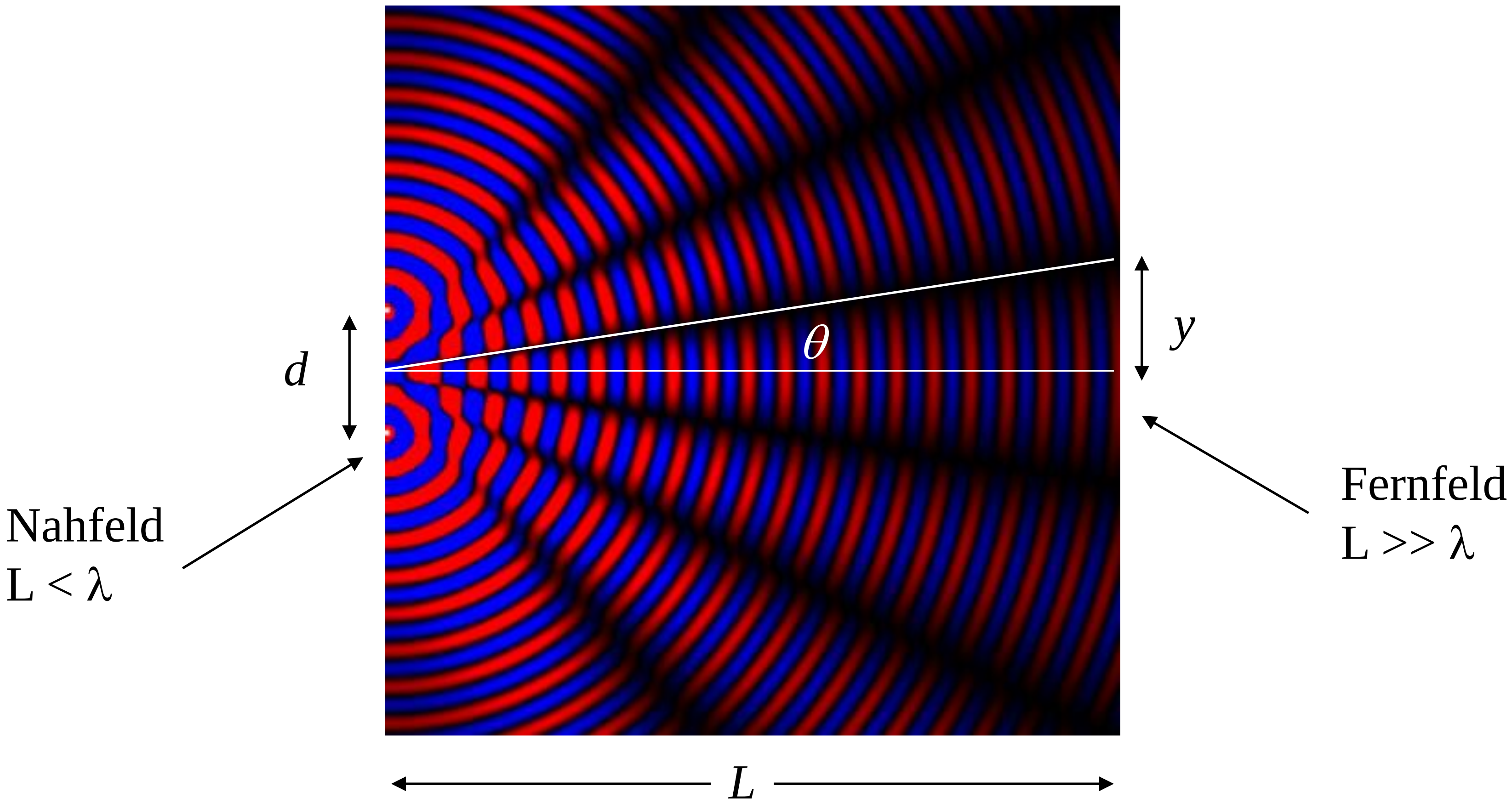
Intensität interferierender Oberflächenwellen



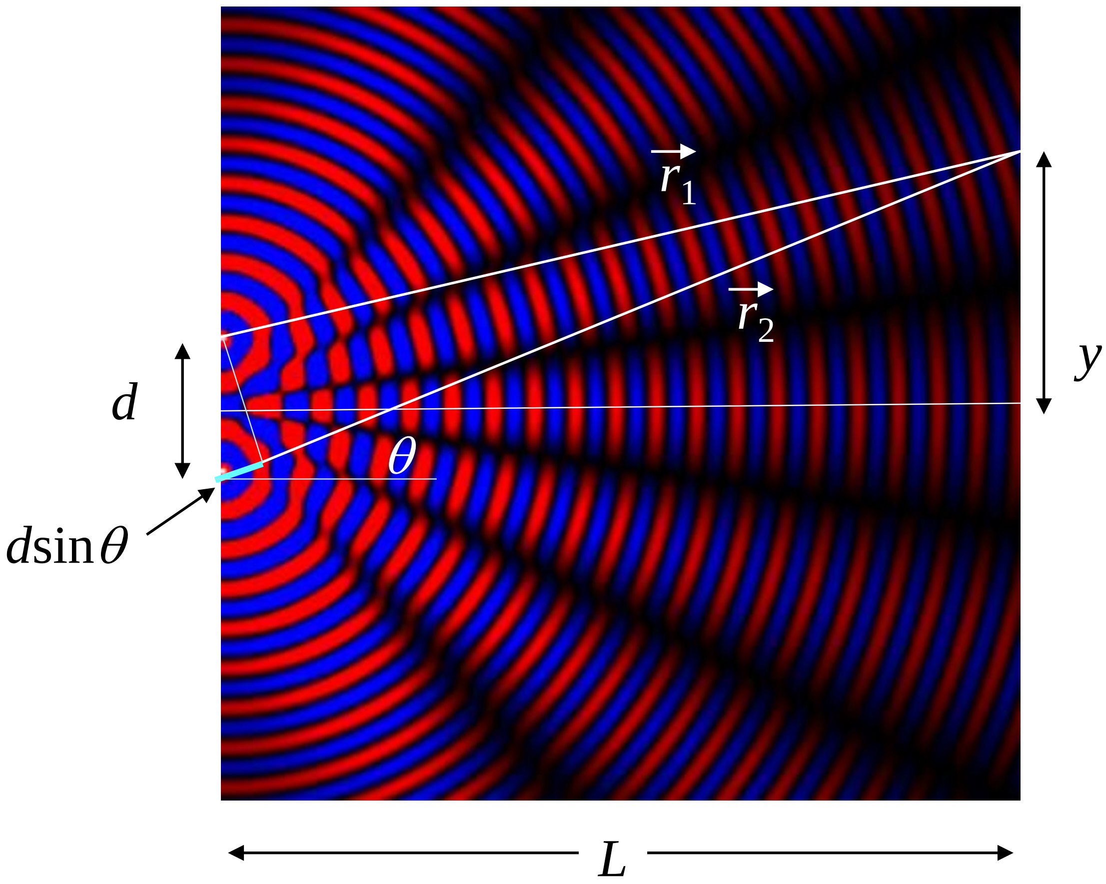
$$|A| = 1.07 \text{ [cm]}$$



Nahfeld / Fernfeld



Fernfeld

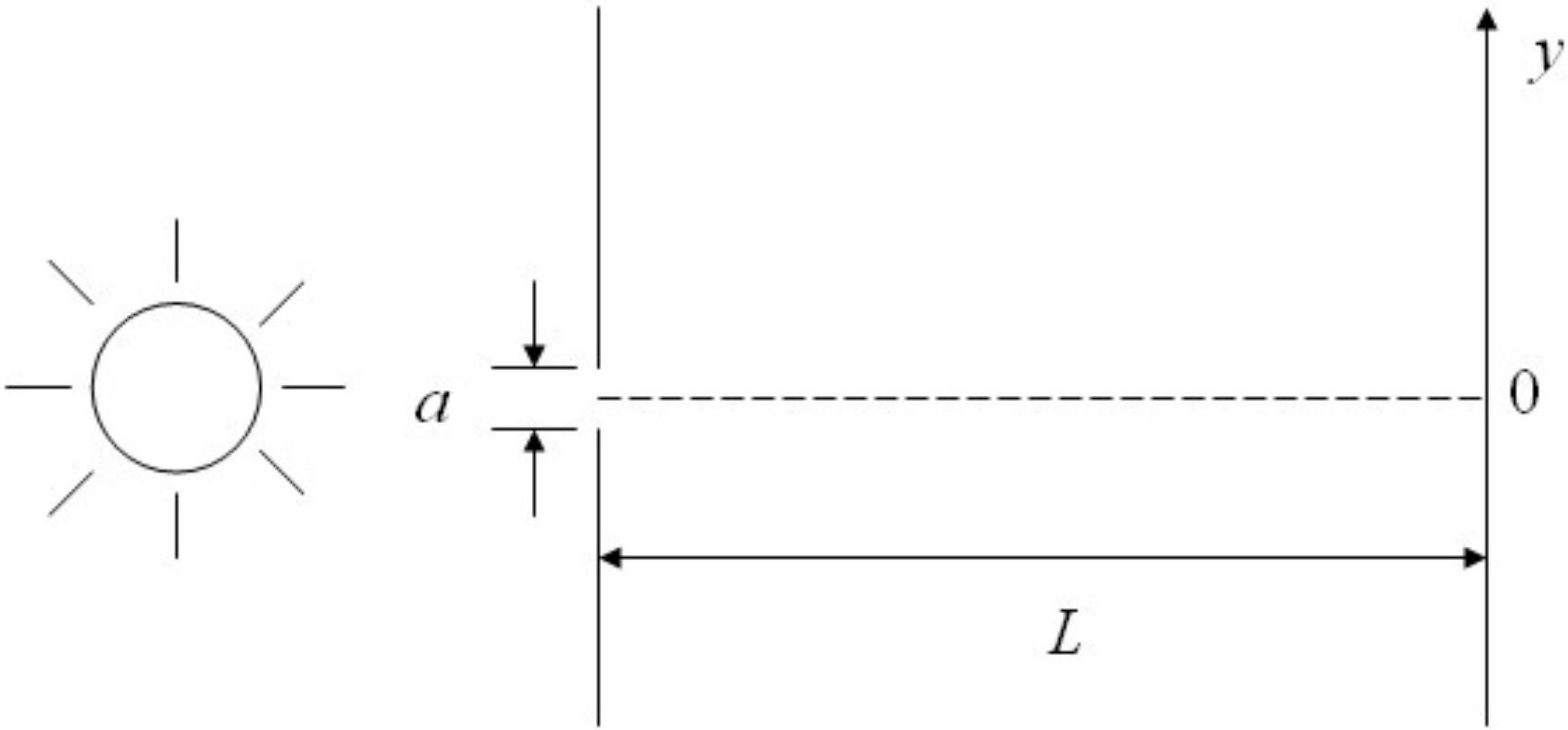


$$\theta \approx \frac{y}{L}$$

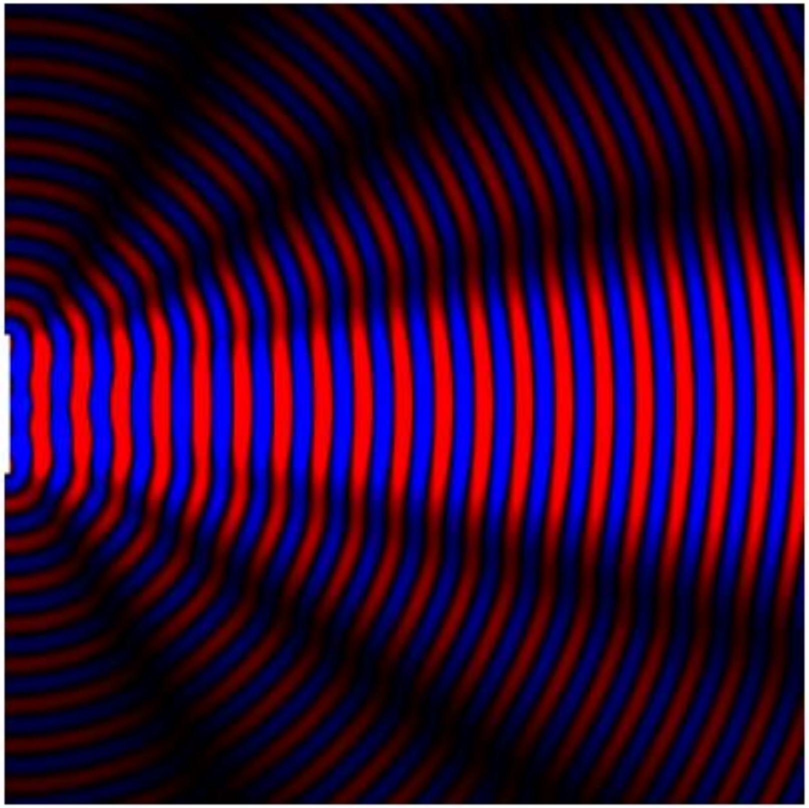
Konstruktive Interferenz: $|\vec{r}_2| - |\vec{r}_1| = n\lambda \approx d\sin\theta \approx \frac{yd}{L}$

Destruktive Interferenz: $|\vec{r}_2| - |\vec{r}_1| = \left(n + \frac{1}{2}\right)\lambda \approx d\sin\theta \approx \frac{yd}{L}$

Einfachspalt



Einfachspalt



$N =$ 40

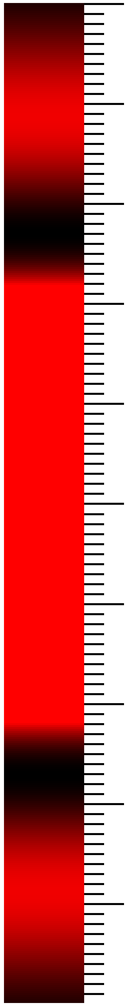
$\lambda =$ 0.3 [cm]

$a =$ 1 [cm]

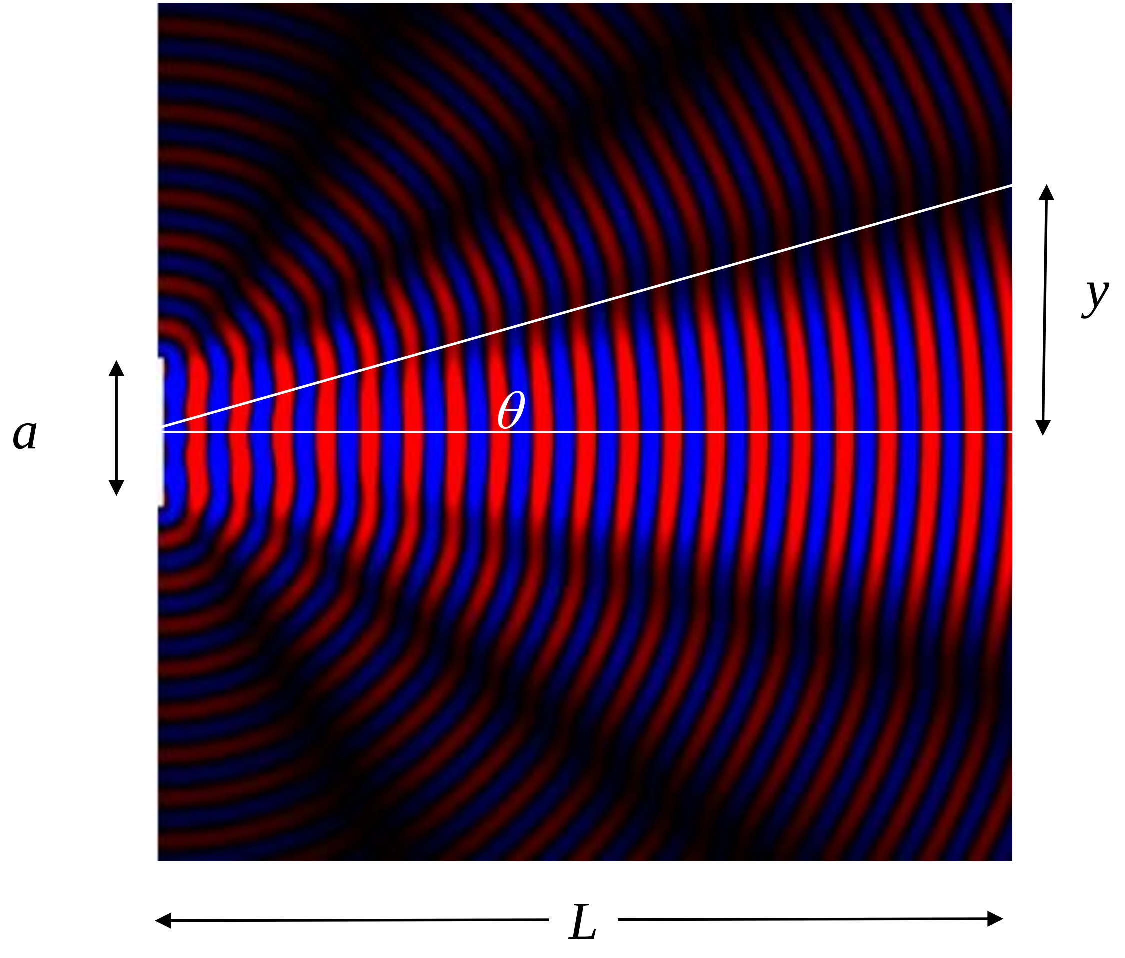
$T =$ 0.5 [s]

plot bei $t =$ 0 [s].

t - T/10 | t + T/10



Einfachspalt Beugung

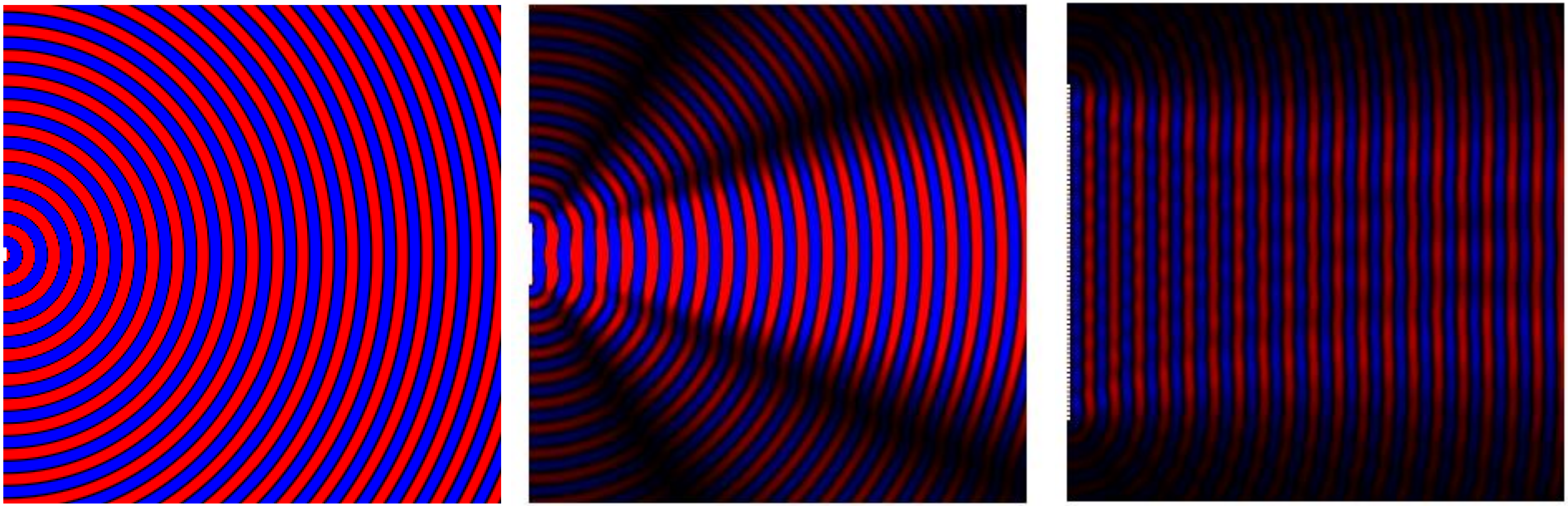


$$\theta \approx \frac{\lambda}{a}$$

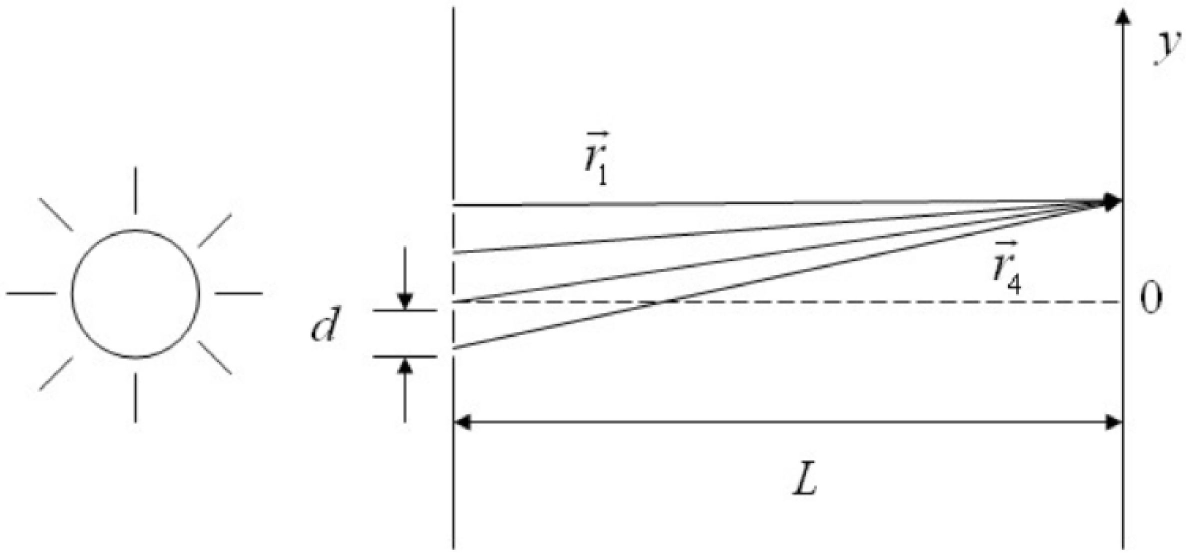
Destruktive Interferenz: $\left|\vec{r}_2\right| - \left|\vec{r}_1\right| = \frac{\lambda}{2} \approx \frac{a}{2}\sin\theta \approx \frac{ya}{2L}$

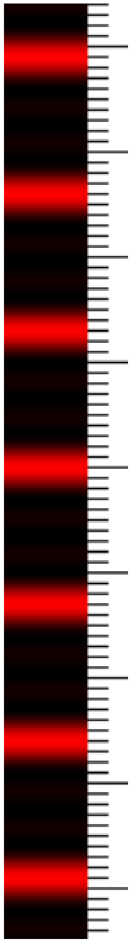
(Fernfeld)

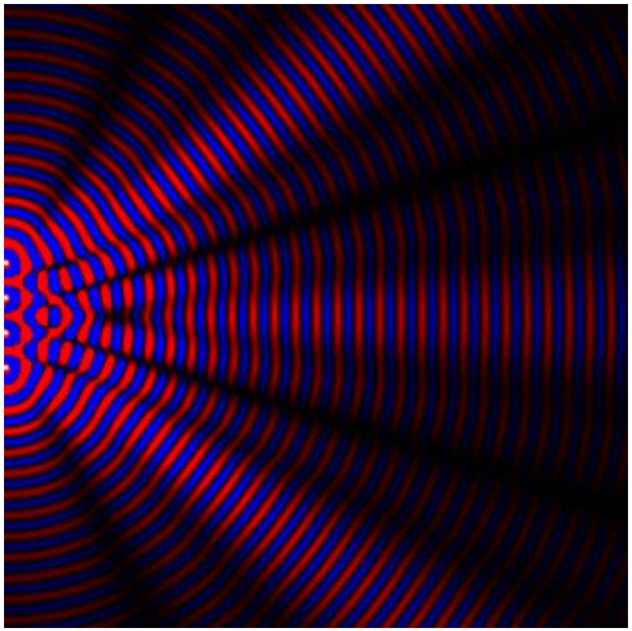
Beugung

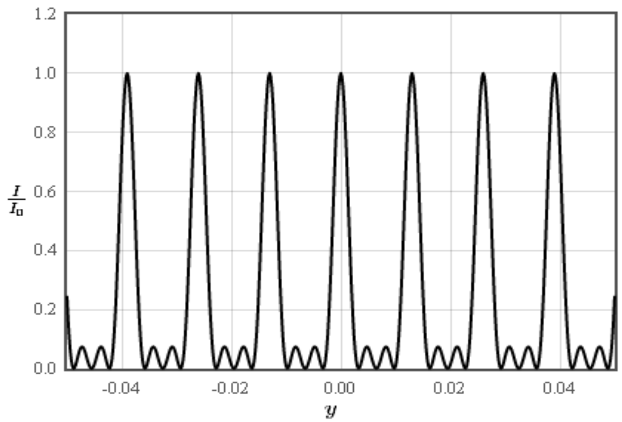


Interferenz am *N*-fach Spalt









Konstruktive Interferenz: $|\vec{r}_2| - |\vec{r}_1| = n\lambda \approx d\sin\theta \approx \frac{yd}{L}$

(Fernfeld)