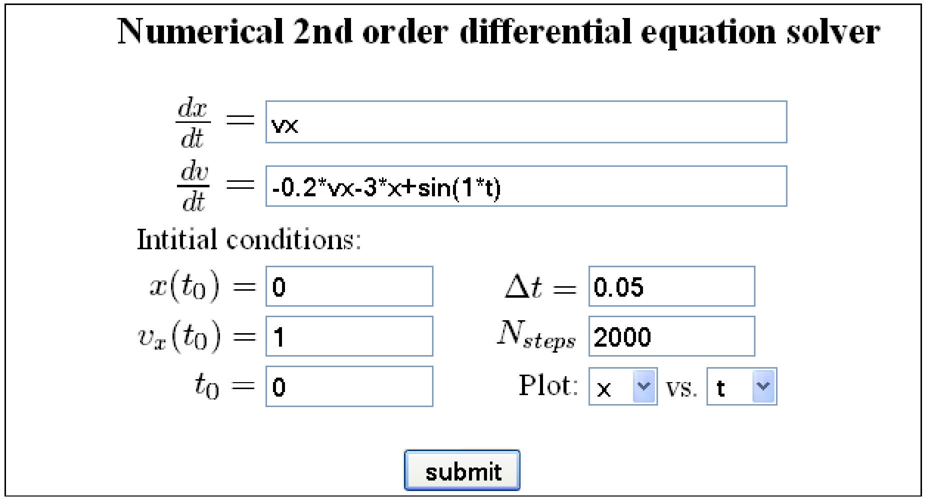
17. Schwingungen

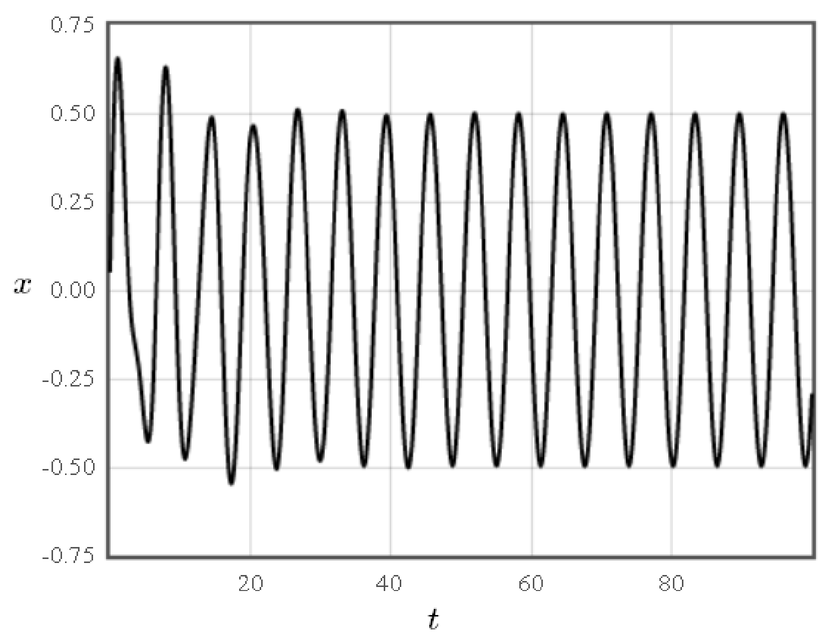
03 Dez. 2018

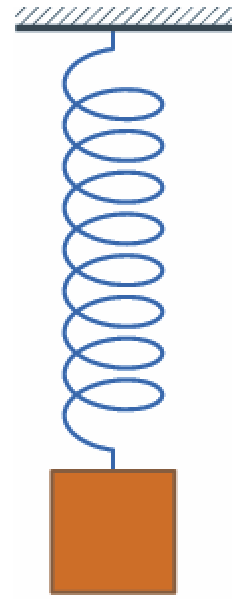
Resonanz

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx + F_0\cos(\omega t)$$

Resonanz


Numerical 2nd order differential equation solver
$\frac{dx}{dt} =$ vx
$\frac{dv}{dt} =$ -0.2*vx-3*x+sin(1*t)
Initial conditions:
$x(t_0) =$ 0
$\Delta t =$ 0.05
$v_x(t_0) =$ 1
N_{steps} 2000
$t_0 =$ 0
Plot: x vs. t
submit


0.75
0.50
0.25
x
0.00
-0.25
-0.50
-0.75
20
40
60
80
t

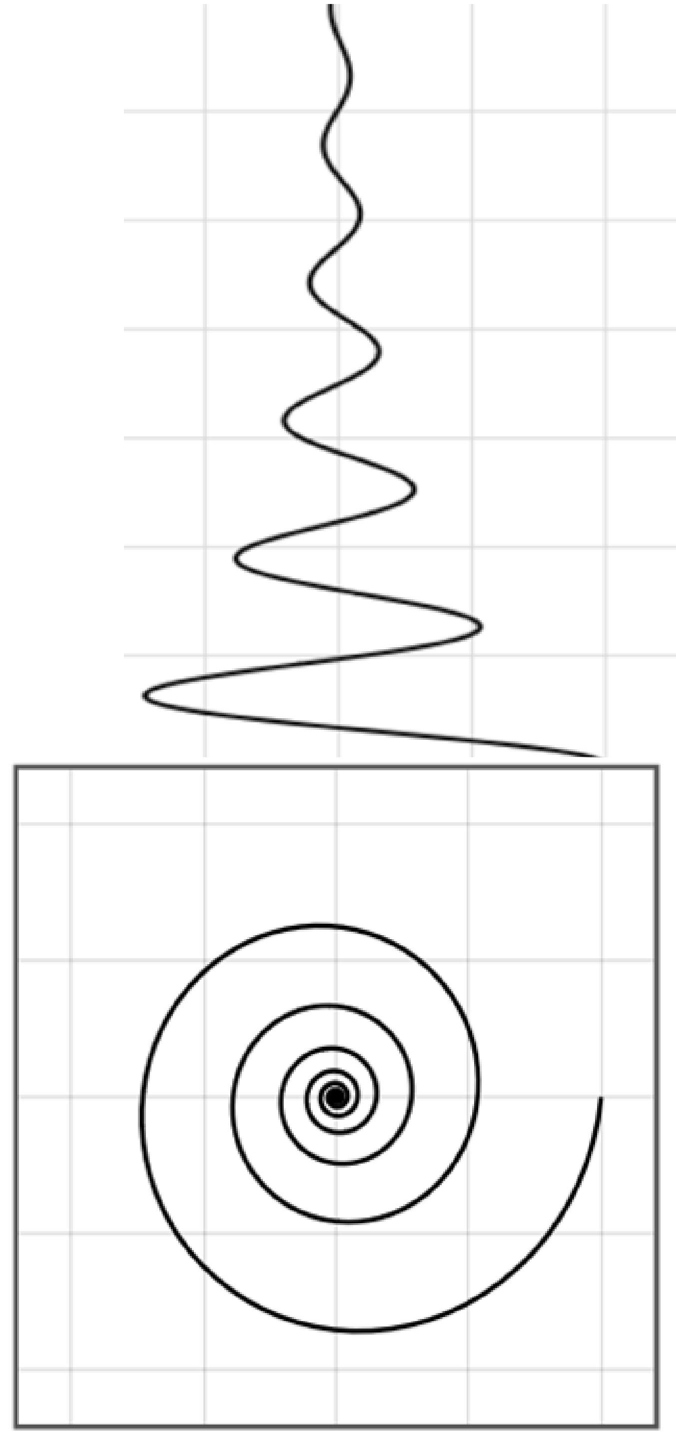


Resonanz

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=F_0\cos(\omega t)$$

$$z=x+iy$$

$$m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz=F_0e^{i\omega t}$$



$$m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz=F_0e^{i\omega t}$$

stationäre Lösung?

$$z(t)=Ae^{i\omega t}$$

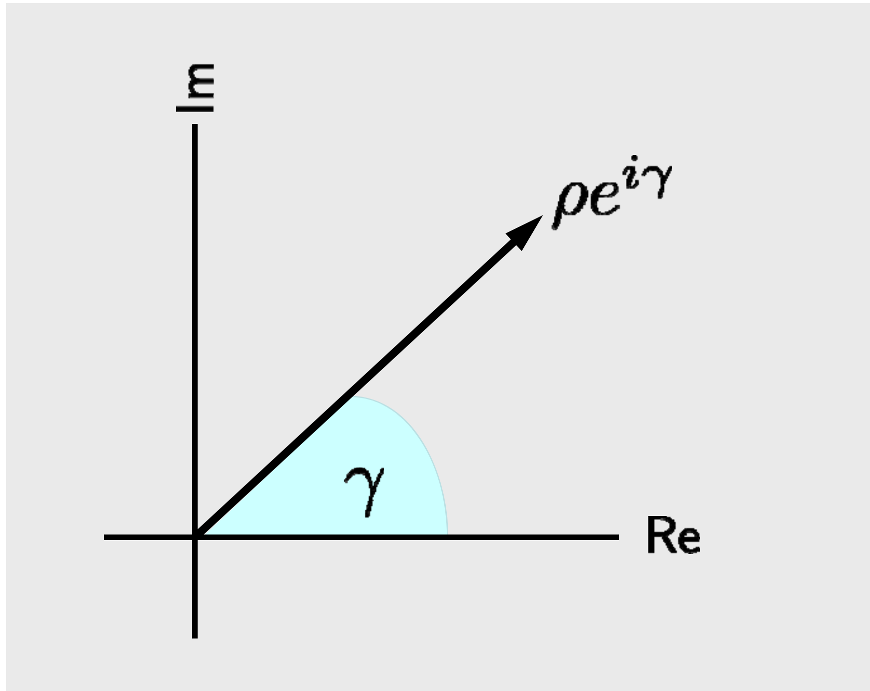
$$m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz=F_0e^{i\omega t}$$

$$z(t)=Ae^{i\omega t}$$

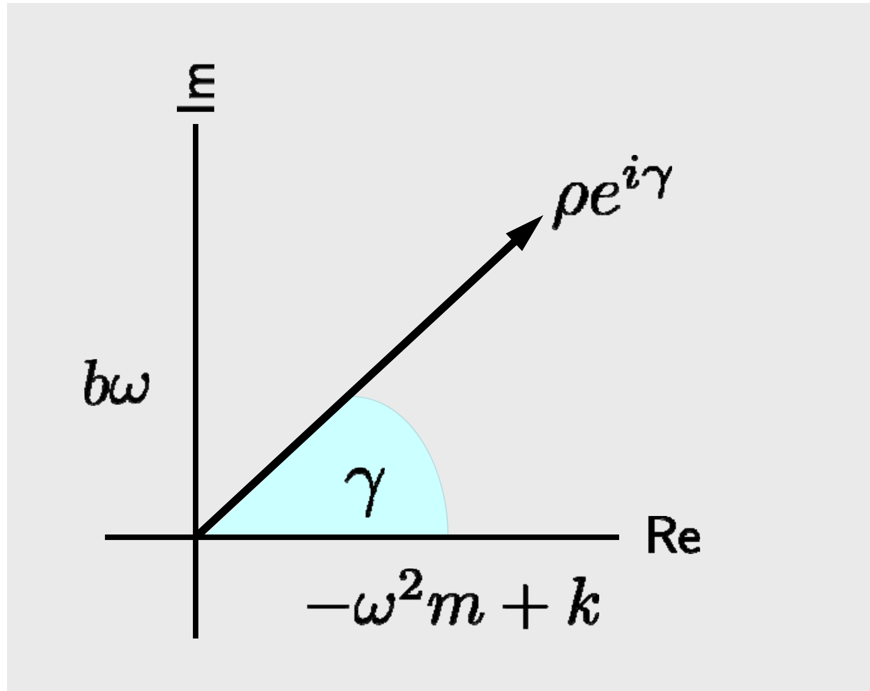
$$-A\omega^2 m\, e^{i\omega t}+iA\omega b\, e^{i\omega t}+kA\, e^{i\omega t}=F_0e^{i\omega t}$$

$$F_0=A\left(-\omega^2 m+i\omega b+k\right)$$

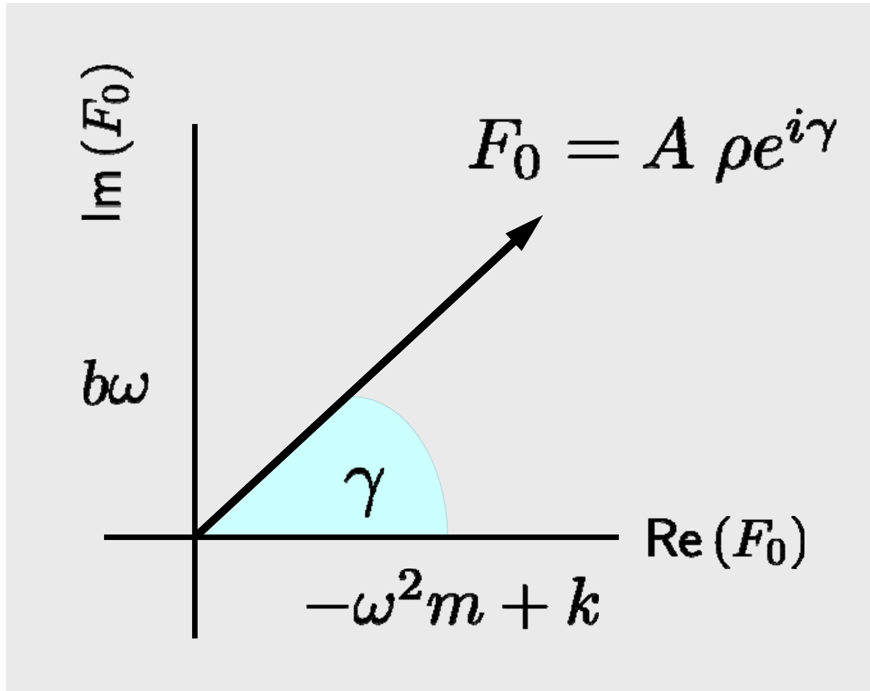
$$=A\,\rho e^{i\gamma}=A\,\rho\left(\cos\gamma+i\sin\gamma\right)$$



$$\rho e^{i\gamma} = -\omega^2 m + i\omega b + k$$



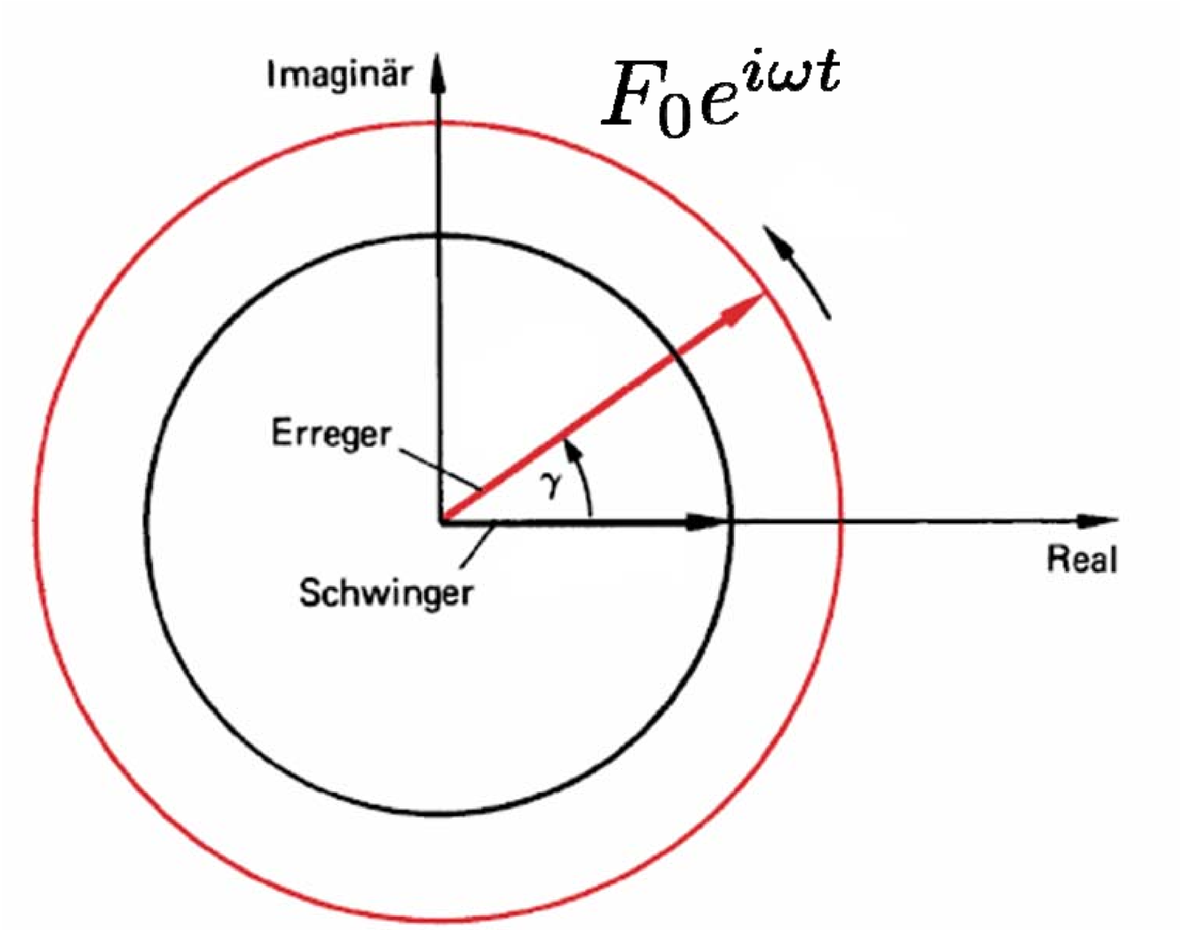
$$\tan\gamma = \frac{b\omega}{m(\frac{k}{m} - \omega^2)} = \frac{b\omega}{m(\omega_0^2 - \omega^2)}$$



$$F_0^2 = A^2\rho^2 = A^2\left(b^2\omega^2 + (k - m\omega^2)^2\right)$$

$$|A| = \frac{F_0}{\sqrt{b^2\omega^2 + (k - m\omega^2)^2}}$$

$$m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz=F_0e^{i\omega t}$$



$$z(t)=\frac{F_0}{\sqrt{b^2\omega^2+(k-m\omega^2)^2}}e^{-i\gamma}e^{i\omega t}$$

nach Hering

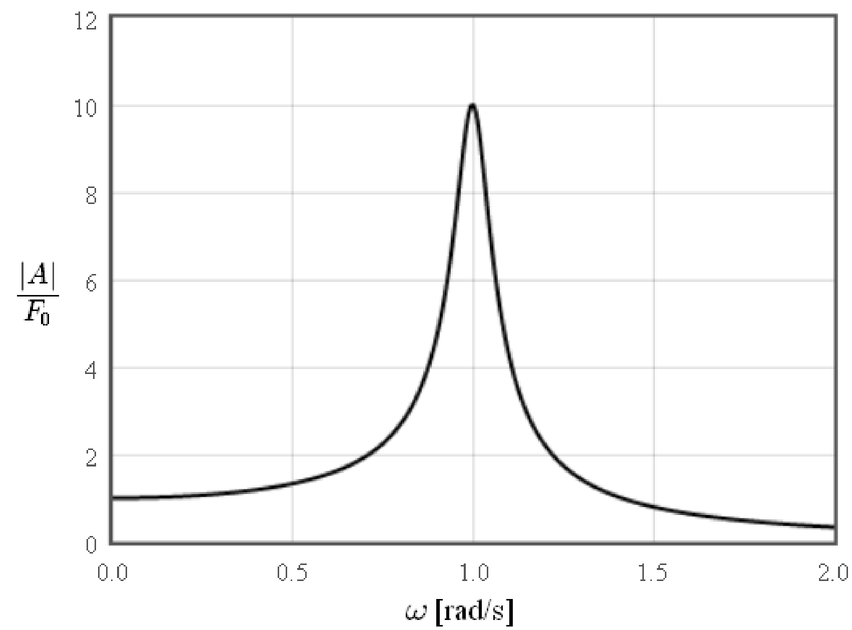
$m =$ 1 [kg] $b =$ 0.1 [N s/m] $k =$ 1 [N/m] Plot

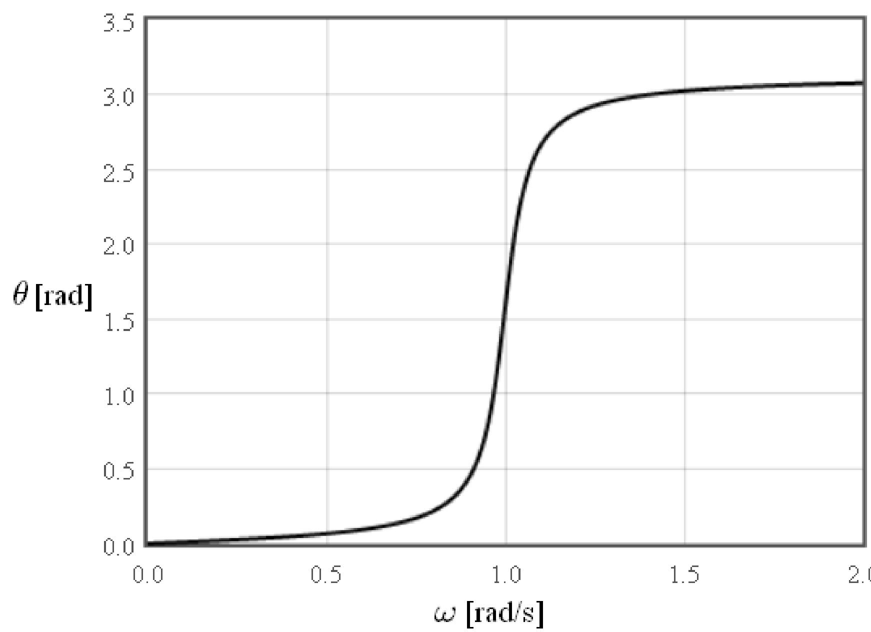
$Q = \frac{\sqrt{mk}}{b} =$ 10

$$\frac{|A|}{F_0} = \frac{1}{\sqrt{\left(k - m\omega^2\right)^2 + \omega^2 b^2}}$$

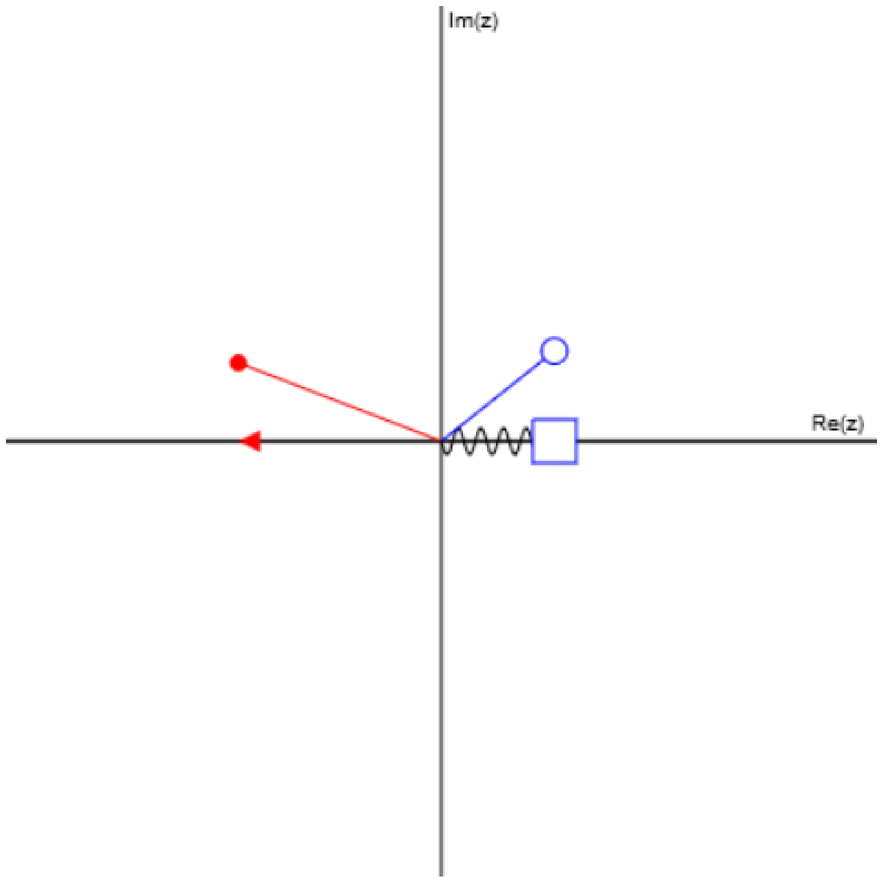
$$A = \frac{F_0}{\rho} e^{-i\gamma}$$

$$\tan\gamma = \frac{\omega b}{k - m\omega^2}$$





Resonanz



m =4 [kg]

b =1 [N s/m]

k =6 [N/m]

F_0 =1 [N]

ω =1.3 [rad/s]

$$\omega_0 = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = 1.22 \text{ [rad/s]} = 0.194 \text{ [Hz]}$$

$$\theta = \operatorname{atan}\left(\frac{\omega b}{k - m\omega^2}\right) = 2.10 \text{ [rad]} = 120 \text{ [deg]}$$

$$A = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + \omega^2 b^2}} = 0.664 \text{ [m]}$$

$$Q = \frac{\sqrt{mk}}{b} = 4.90$$

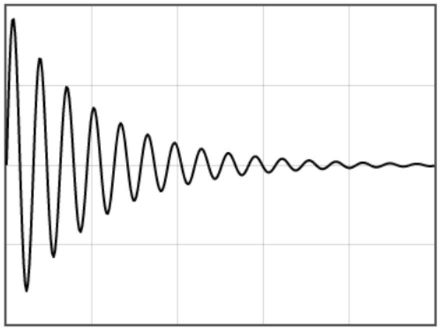
$F_0 e^{i\omega t}$ anzeigen: ☑ $Ae^{i(\omega t - \theta)}$ anzeigen: ☑

z anzeigen: ☐ x_2 anzeigen: ☐

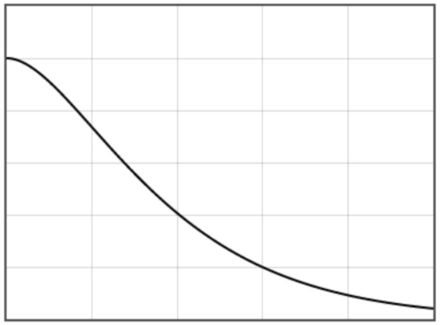
Gütefaktor

$$Q = \frac{\pi\tau}{T}$$

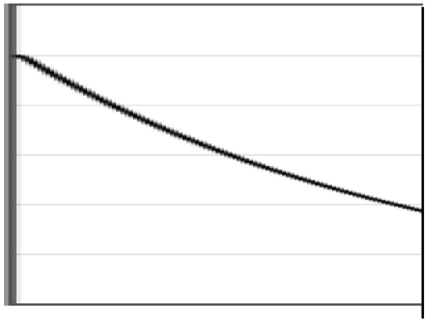
Schwingfall $Q > \frac{1}{2}$



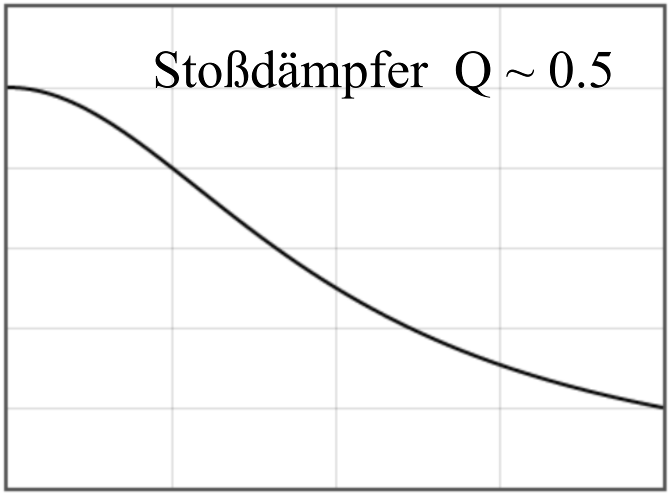
aperiodischer Grenzfall $Q = \frac{1}{2}$

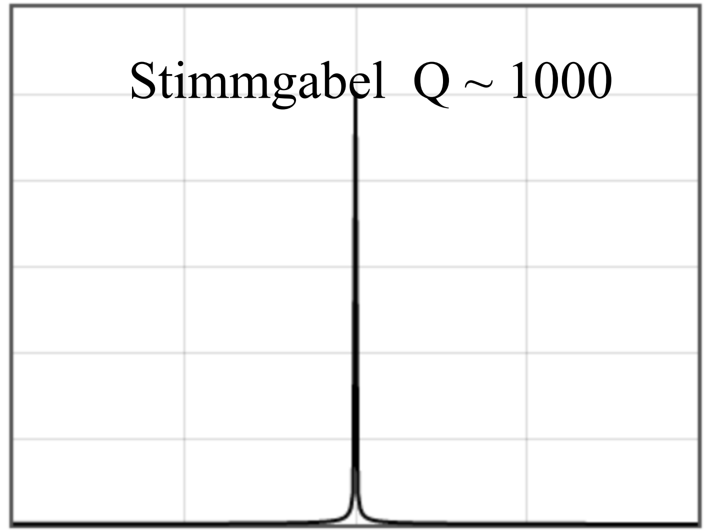


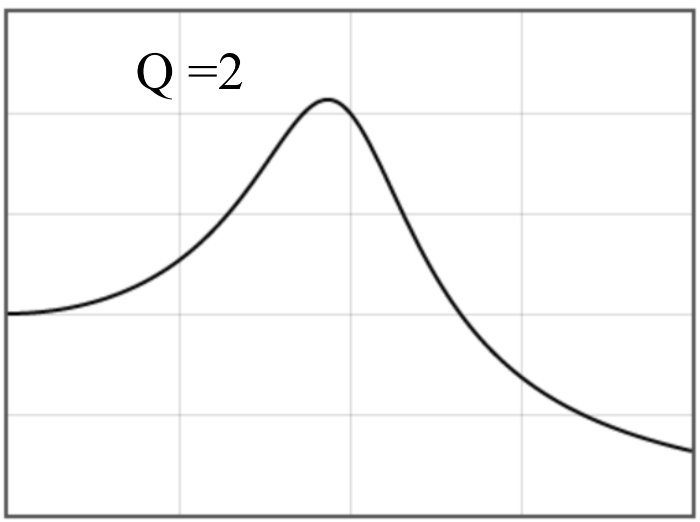
Kriechfall $Q < \frac{1}{2}$

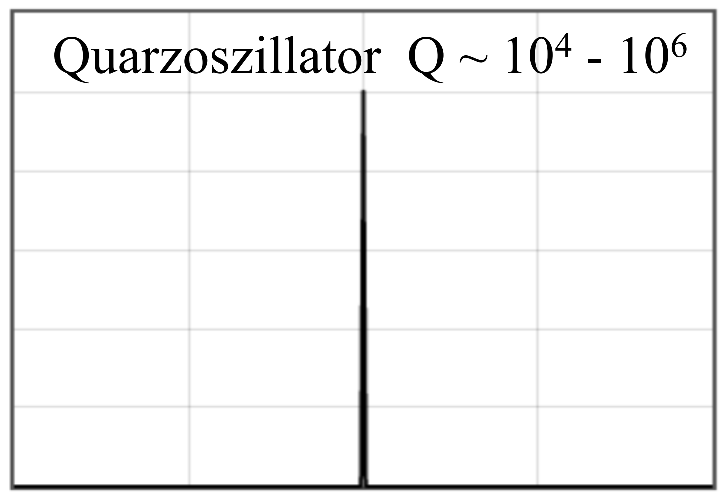


Gütefaktor


Stoßdämpfer Q ~ 0.5


Stimmgabel Q ~ 1000


Q =2


Quarzoszillator Q ~ 10^4 - 10^6

Q Faktor

Ein Kügelchen der Masse 56 Gramm wird auf eine Gitarrensaite gelegt. Wird die Saite gezupft, kann die Bewegung des Kügelchens so beschrieben werden:

$$x(t) = 0.007 \exp(-0.6t)(\sin(2637t + 4) + \cos(2637t + 5)) \text{ [m]}.$$

Hier ist t die Sekunden angegebene Zeit.

Wie lautet der Q Faktor für diese Schwingung?

$Q =$ ______ Lösung

Q Faktor

Ein Kügelchen der Masse 56 Gramm wird auf eine Gitarrensaite gelegt. Wird die Saite gezupft, kann die Bewegung des Kügelchens so beschrieben werden:

$$x(t) = 0.007\exp(-0.6t)(\sin(2637t + 4) + \cos(2637t + 5))\ [\mathrm{m}].$$

Hier ist t die Sekunden angegebene Zeit.

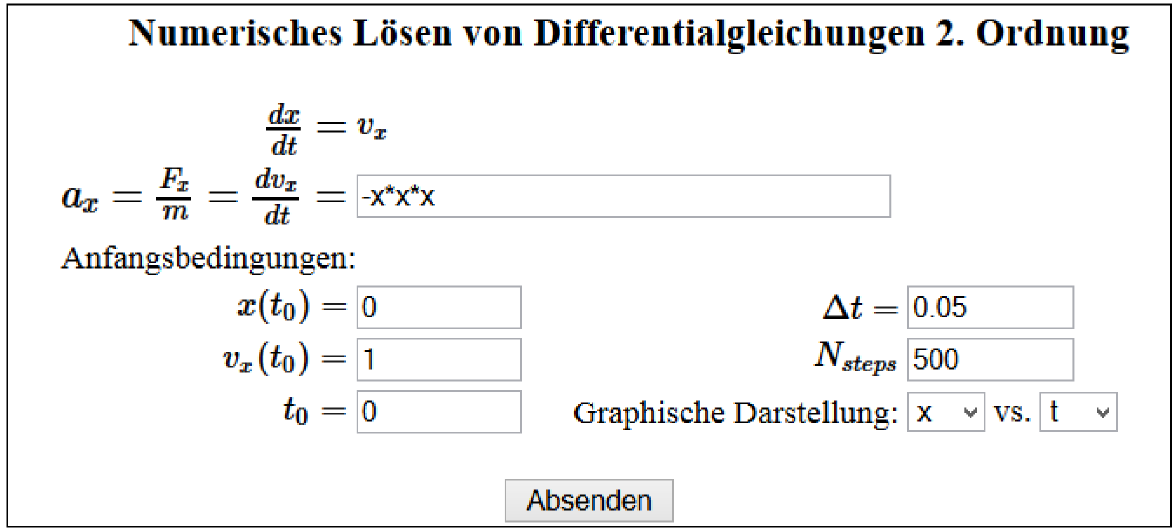
Wie lautet der Q Faktor für diese Schwingung?

$Q =$ ______ Lösung

Der Q −Faktor beschreibt, wieviele Perioden einer Schwingung in der Zeitspanne stattfinden, in welcher die Schwingungsamplitude um den Faktor $1/e$ gesunken ist. In unserem Falle ist die Periodendauer der Schwingung $T = \frac{2\pi}{2637} = 0.002383$ [s]. Es vergehen $\tau = 1.667$ s, bis die Schwingungsamplitude um den Faktor $1/e$ gesunken ist. Daher ist der Q −Faktor

$$Q = \frac{\pi\tau}{T} = \frac{\pi 1.667}{0.002383} = 2198.$$

Nichtlineare gewöhnliche Differentialgleichung



Numerisches Lösen von Differentialgleichungen 2. Ordnung

$$\frac{dx}{dt} = v_x$$

$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} = \text{-x*x*x}$$

Anfangsbedingungen:

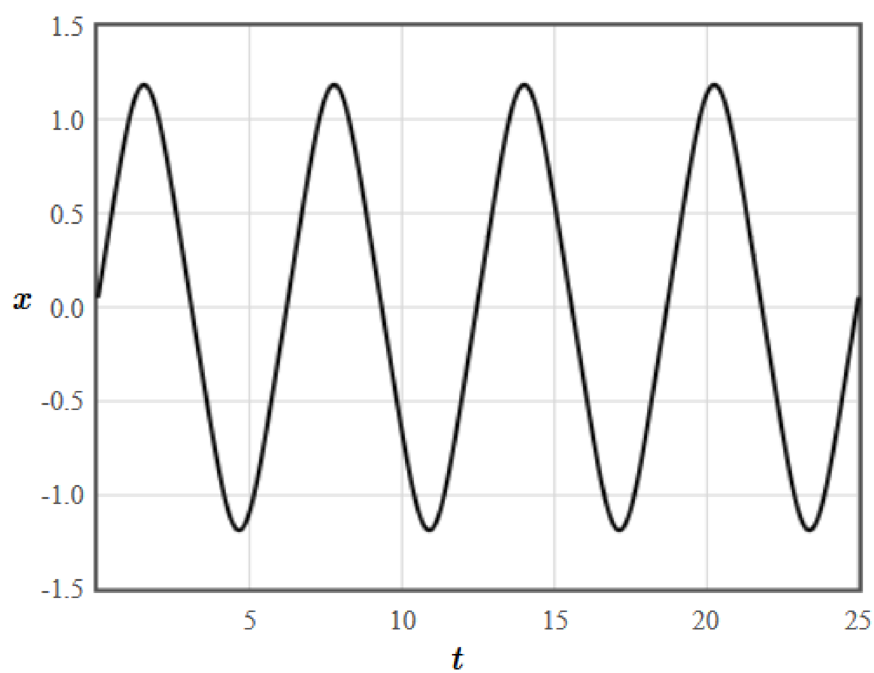
$x(t_0) =$ 0 $\quad \Delta t =$ 0.05

$v_x(t_0) =$ 1 $\quad N_{steps}$ 500

$t_0 =$ 0 $\quad$ Graphische Darstellung: x vs. t

Absenden

$$m\frac{d^2x}{dt^2} + kx^3 = 0$$



Nichtlineare gewöhnliche Differentialgleichung

Numerisches Lösen von Differentialgleichungen 2. Ordnung

$$\frac{dx}{dt} = v_x$$

$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} = \text{-0.3*vx*vx*vx-x}$$

Anfangsbedingungen:

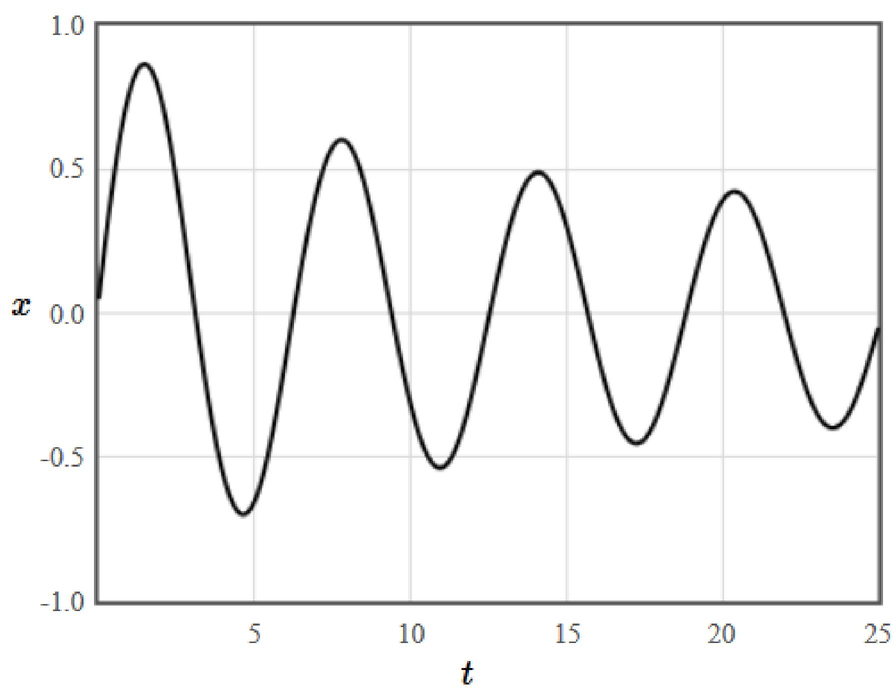
$x(t_0) = 0$ $\Delta t = 0.05$

$v_x(t_0) = 1$ N_{steps} 500

$t_0 = 0$ Graphische Darstellung: x vs. t

Absenden

$$m\frac{d^2x}{dt^2} + b\left(\frac{dx}{dt}\right)^3 + kx = 0$$



Nichtlineare gewöhnliche Differentialgleichung

Numerisches Lösen von Differentialgleichungen 2. Ordnung

$$\frac{dx}{dt} = v_x$$

$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} =$$ -0.3*vx*vx*vx-x*x*x

Anfangsbedingungen:

$x(t_0) =$ 0

$v_x(t_0) =$ 1

$t_0 =$ 0

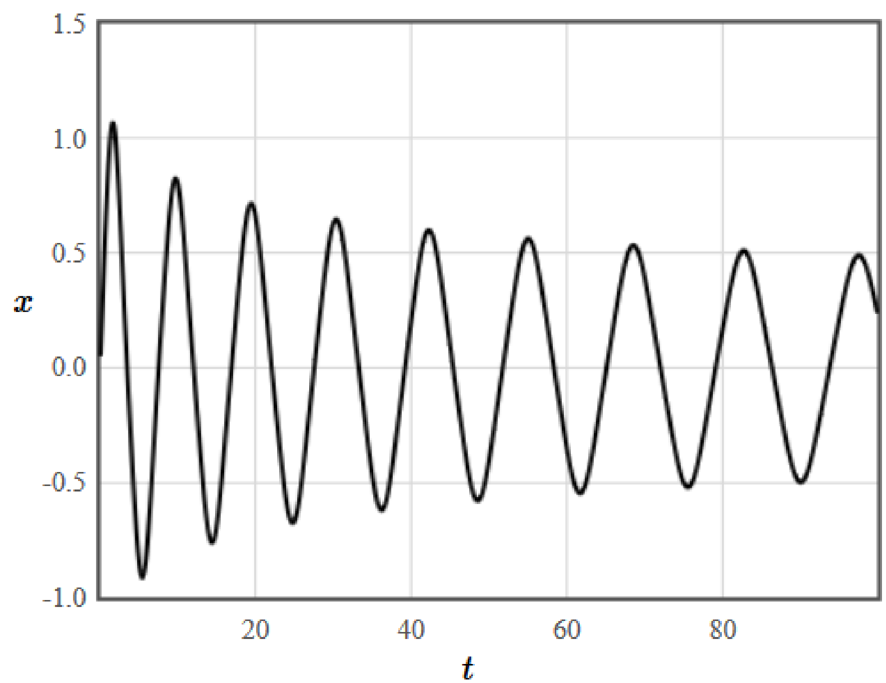
$\Delta t =$ 0.05

N_{steps} 2000

Graphische Darstellung: x vs. t

Absenden

$$m\frac{d^2x}{dt^2} + b\left(\frac{dx}{dt}\right)^3 + kx^3 = 0$$



Nichtlineare Schwinger

Numerisches Lösen von Differentialgleichungen 2. Ordnung

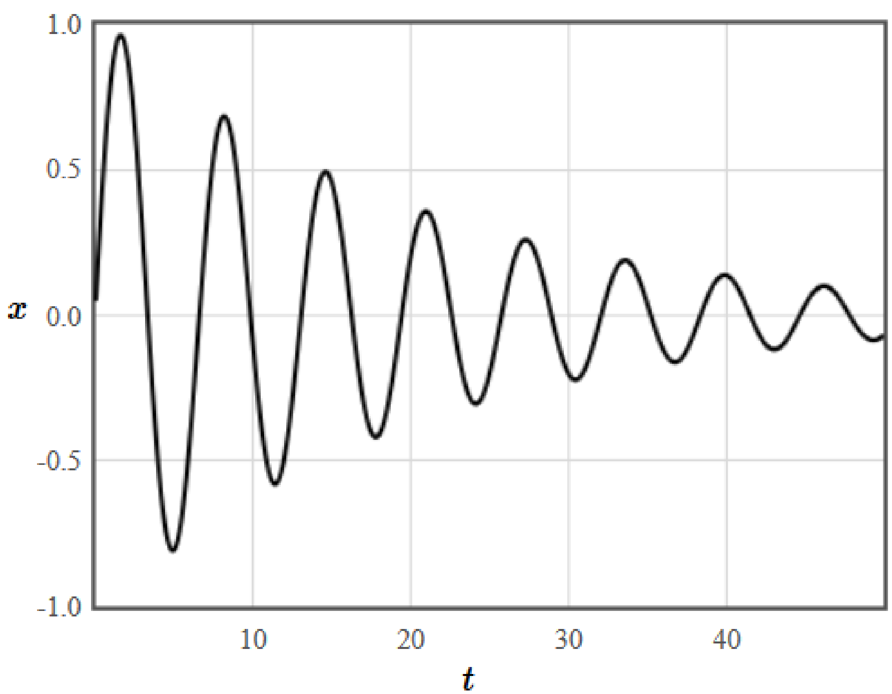
$$\frac{dx}{dt} = v_x$$

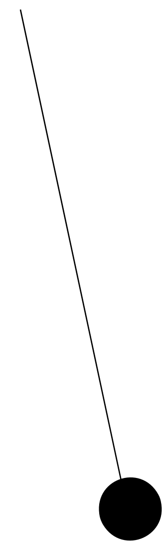
$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} =$$ -0.1*vx-sin(x)

Anfangsbedingungen:

$x(t_0) =$	0	$\Delta t =$	0.05
$v_x(t_0) =$	1	N_{steps}	1000
$t_0 =$	0	Graphische Darstellung:	x vs. t

Absenden





Pendel

Parametrisch erregte Schwingungen

Numerisches Lösen von Differentialgleichungen 2. Ordnung

$$\frac{dx}{dt} = v_x$$

$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} =$$ -0.1*vx-(1-cos(t))*sin(x)

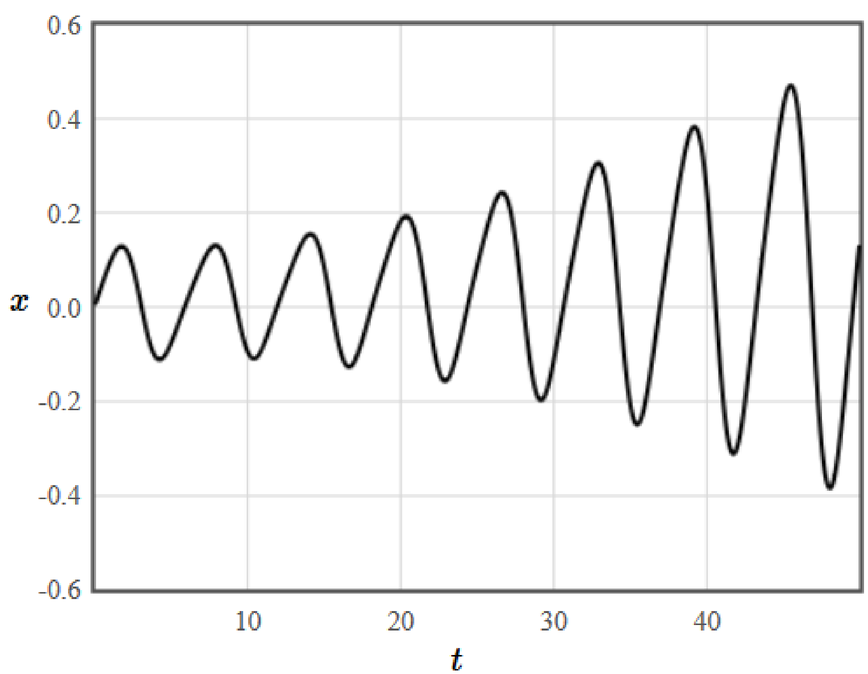
Anfangsbedingungen:

$x(t_0) =$ 0 $\Delta t =$ 0.05

$v_x(t_0) =$ 0.1 N_{steps} 1000

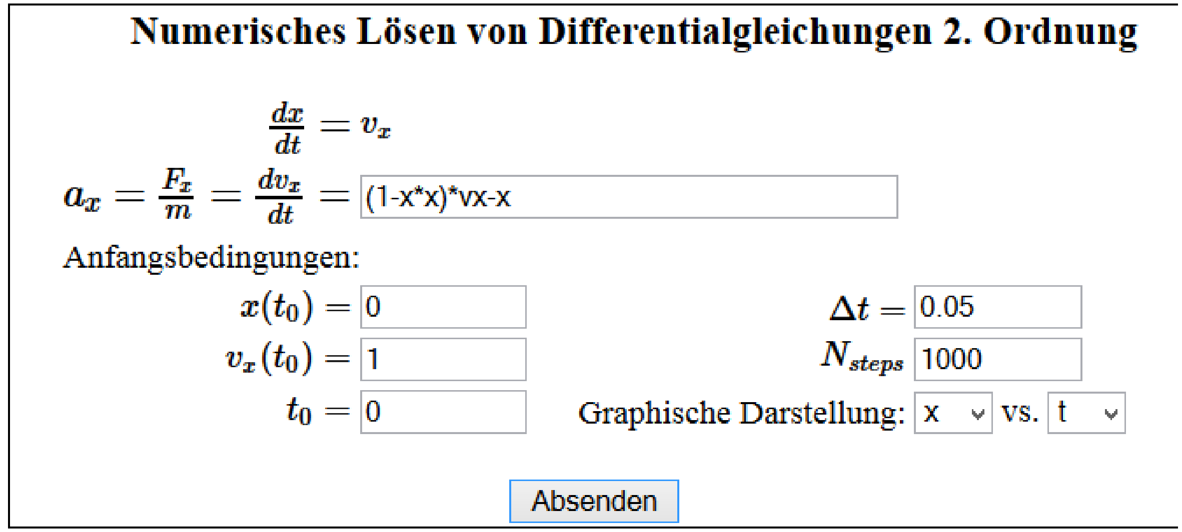
$t_0 =$ 0 Graphische Darstellung: x vs. t

Absenden



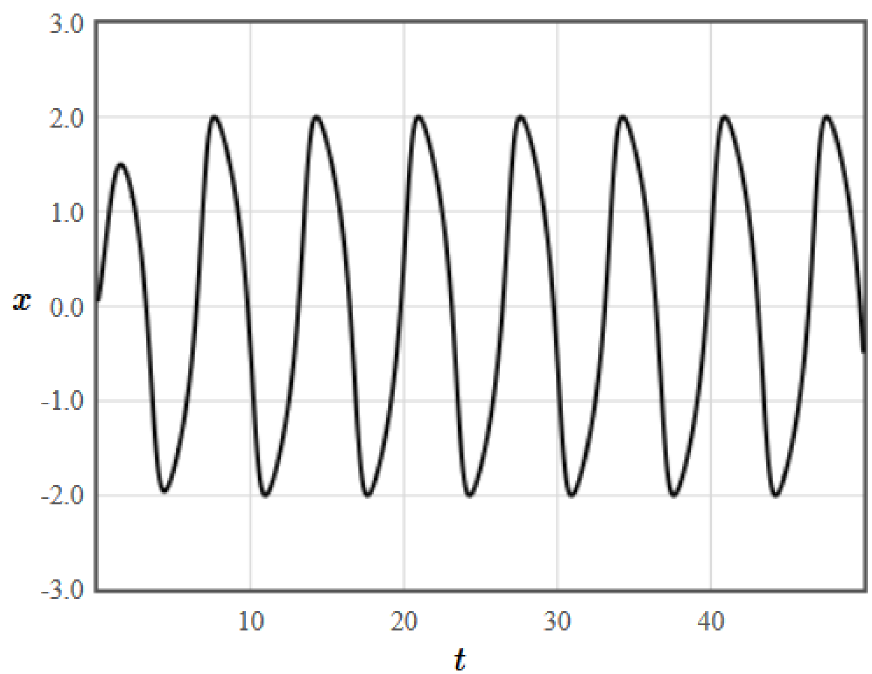
Kind auf einer Schaukel

Nichtlineare Schwinger



Van-der-Pol-System

$$\frac{d^2x}{dt^2} - \mu\left(1 - x^2\right)\frac{dx}{dt} + x = 0$$



negativer Reibungsfaktor für kleine *x*

Lineare gewöhnliche Differentialgleichung

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$

Lineare Differentialgleichung zweiter Ordnung

Nichtlineare Differentialgleichung zweiter Ordnung

$$m\left(\frac{d^2x}{dt^2}\right)^2+b\frac{dx}{dt}+kx=0$$

$$m\frac{d^2x}{dt^2}+b\left(\frac{dx}{dt}\right)^2+kx=0$$

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx^3=0$$

Nichtlineare gewöhnliche Differentialgleichung

Numerisches Lösen von Differentialgleichungen 2. Ordnung

$$\frac{dx}{dt} = v_x$$

$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} = \text{-x*x*x}$$

Anfangsbedingungen:

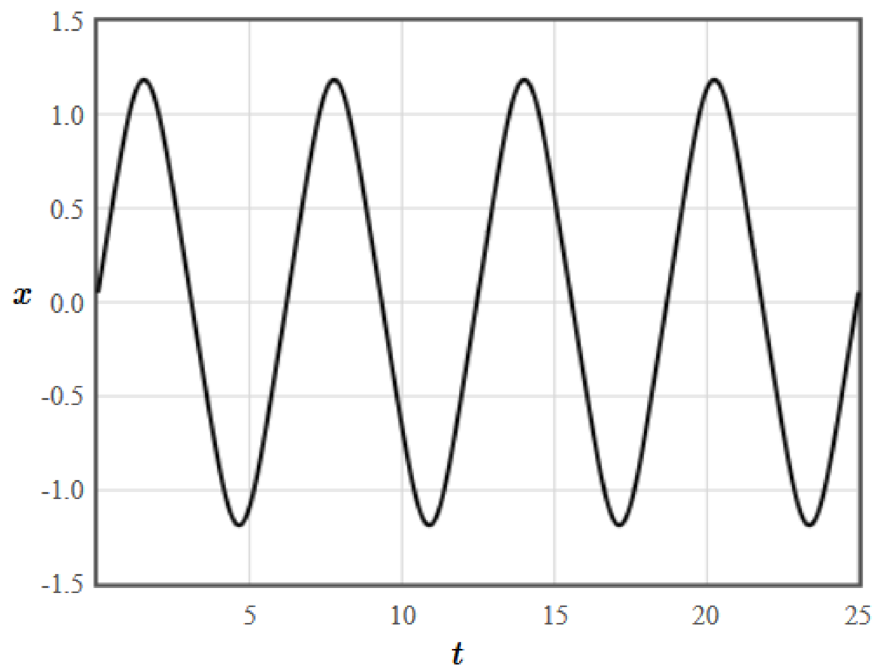
$x(t_0) = 0$ $\Delta t = 0.05$

$v_x(t_0) = 1$ N_{steps} 500

$t_0 = 0$ Graphische Darstellung: x vs. t

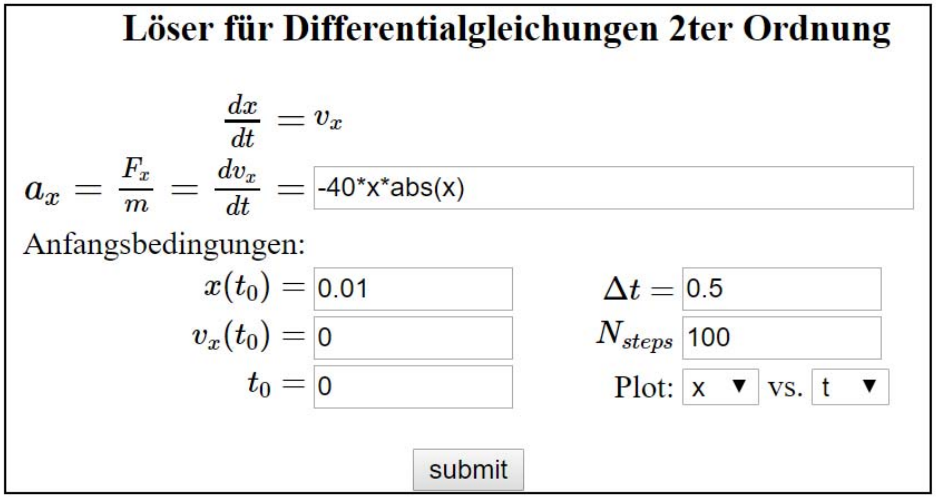
Absenden

$$m\frac{d^2x}{dt^2} + kx^3 = 0$$

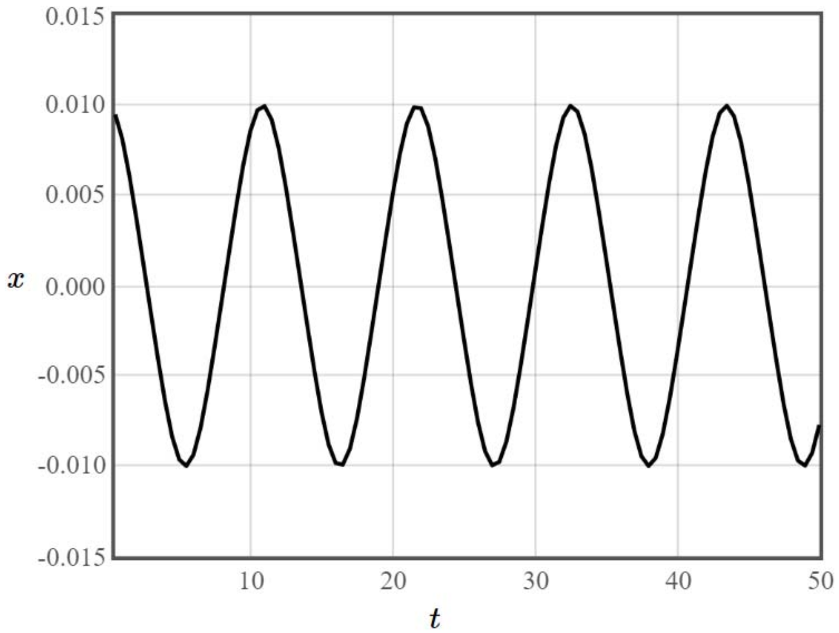


Nichtlineare gewöhnliche Differentialgleichung

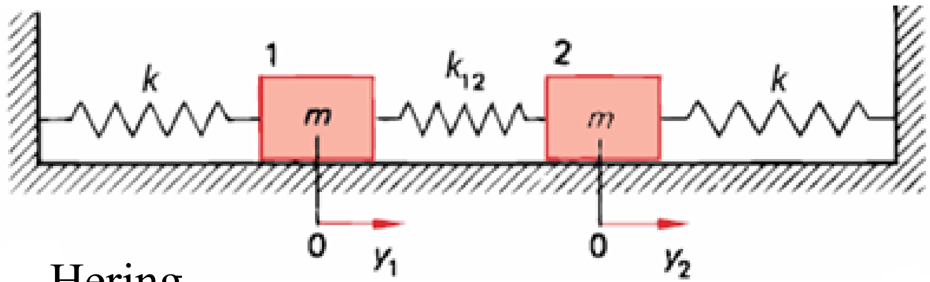
APP Masse – nichtlineare Feder



Harte Feder



gekoppeltes Schwingungssystem



Hering

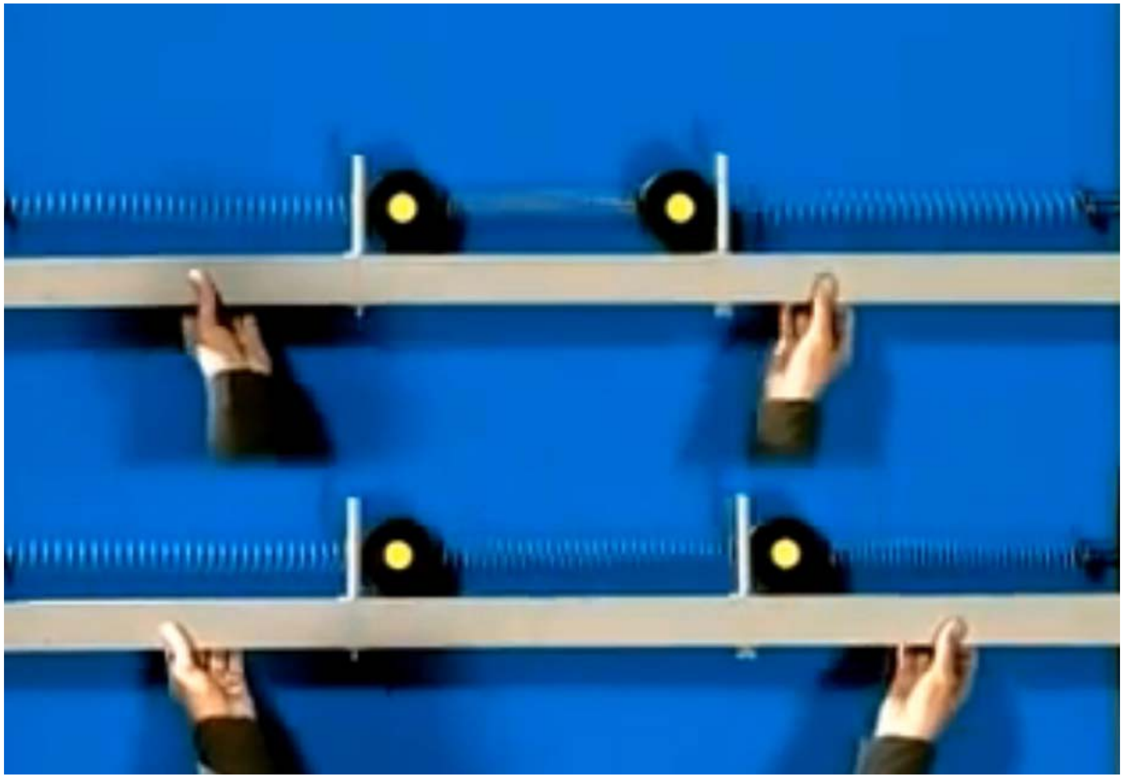
$$M\frac{d^2y_1}{dt^2} = -ky_1 + k_{12}\left(y_2 - y_1\right)$$

$$M\frac{d^2y_2}{dt^2} = -ky_2 + k_{12}\left(y_1 - y_2\right)$$

Eigenmoden: harmonische Bewegung

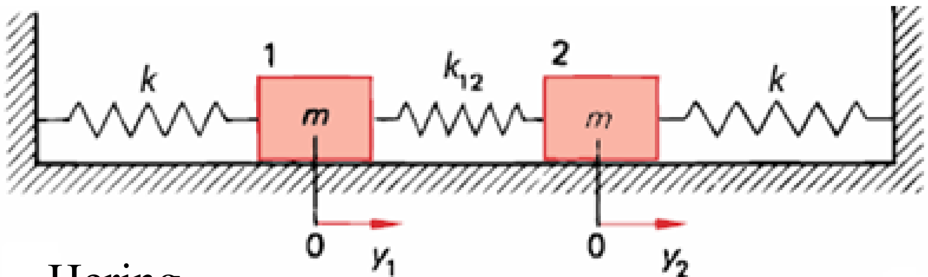
Alle Teile schwingen mit der gleichen Frequenz

zwei Eigenmoden



https://www.youtube.com/watch?v=8RV3gXm6j2I

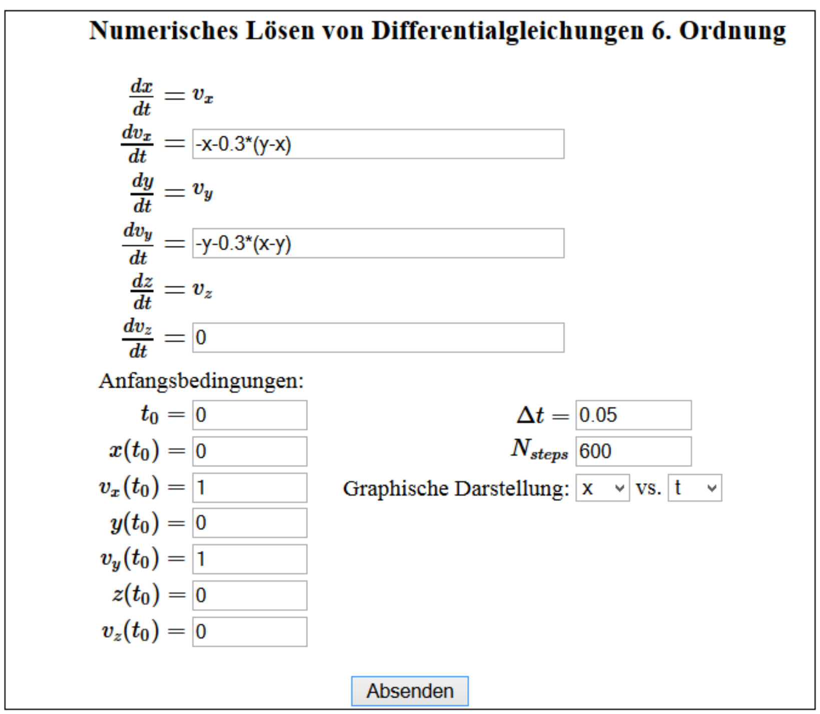
gekoppeltes Schwingungssystem

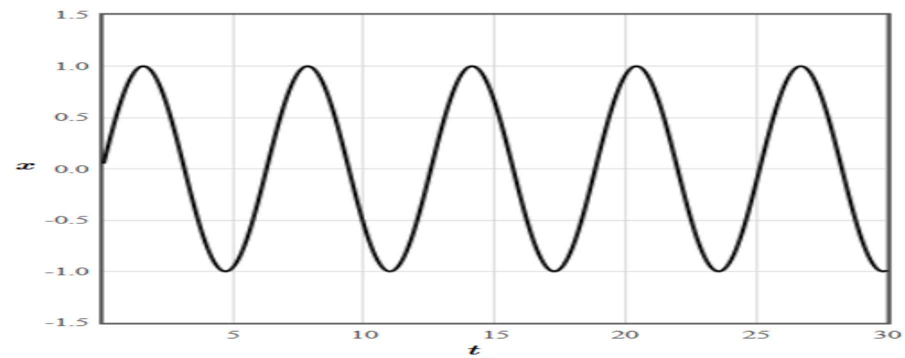


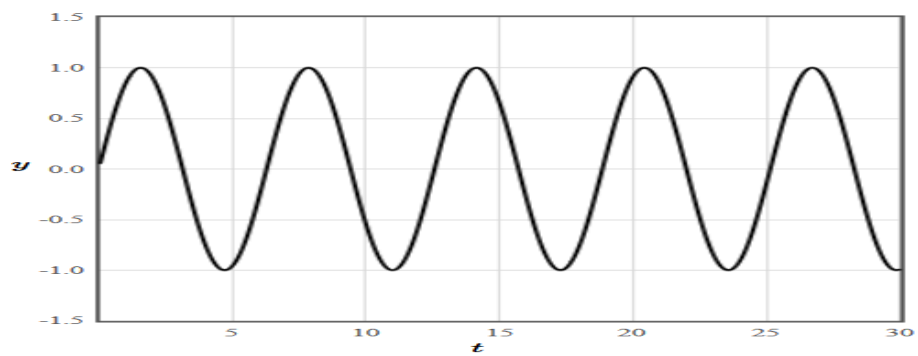
Hering

$$M\frac{d^2y_1}{dt^2} = -ky_1 + k_{12}\left(y_2 - y_1\right)$$

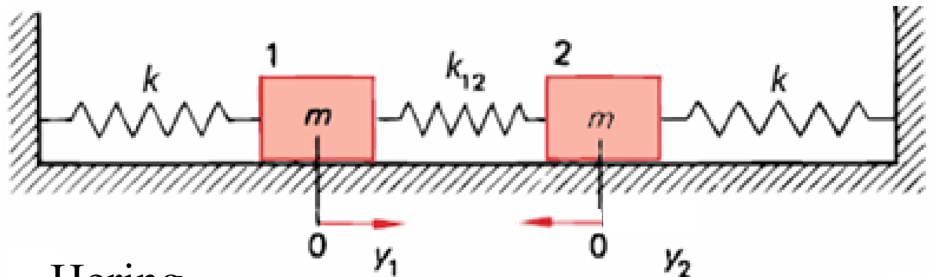
$$M\frac{d^2y_2}{dt^2} = -ky_2 + k_{12}\left(y_1 - y_2\right)$$







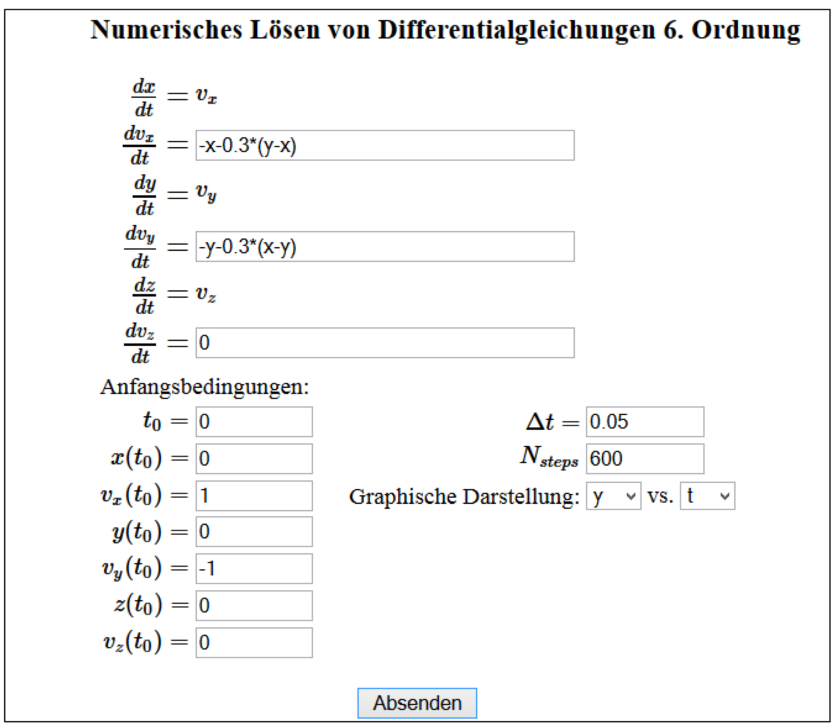
gekoppeltes Schwingungssystem



Hering

$$M\frac{d^2y_1}{dt^2} = -ky_1 + k_{12}\left(y_2 - y_1\right)$$

$$M\frac{d^2y_2}{dt^2} = -ky_2 + k_{12}\left(y_1 - y_2\right)$$



Numerisches Lösen von Differentialgleichungen 6. Ordnung

$\frac{dx}{dt} = v_x$

$\frac{dv_x}{dt} =$ -x-0.3*(y-x)

$\frac{dy}{dt} = v_y$

$\frac{dv_y}{dt} =$ -y-0.3*(x-y)

$\frac{dz}{dt} = v_z$

$\frac{dv_z}{dt} =$ 0

Anfangsbedingungen:

$t_0 =$ 0

$x(t_0) =$ 0

$v_x(t_0) =$ 1

$y(t_0) =$ 0

$v_y(t_0) =$ -1

$z(t_0) =$ 0

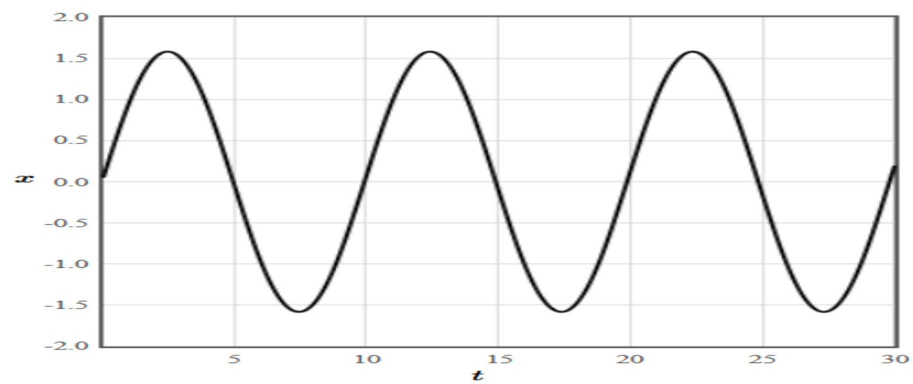
$v_z(t_0) =$ 0

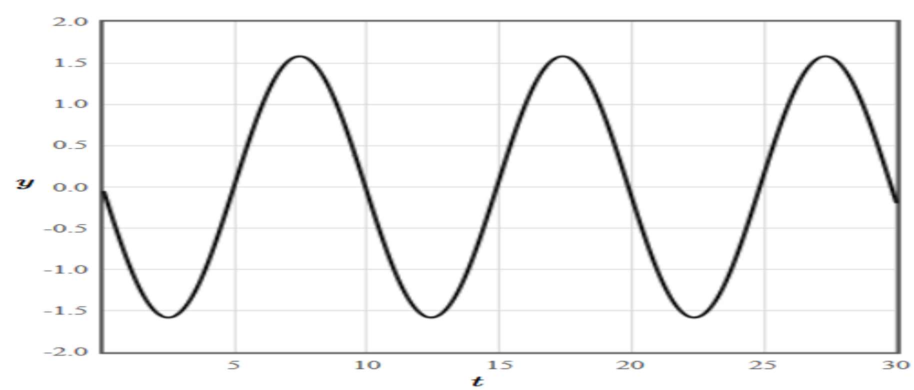
$\Delta t =$ 0.05

N_{steps} 600

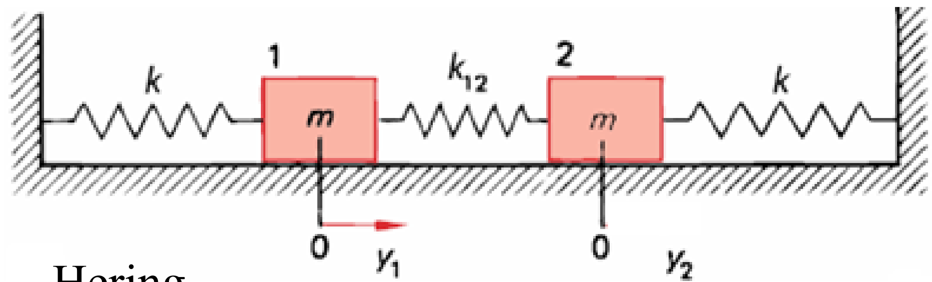
Graphische Darstellung: y vs. t

Absenden





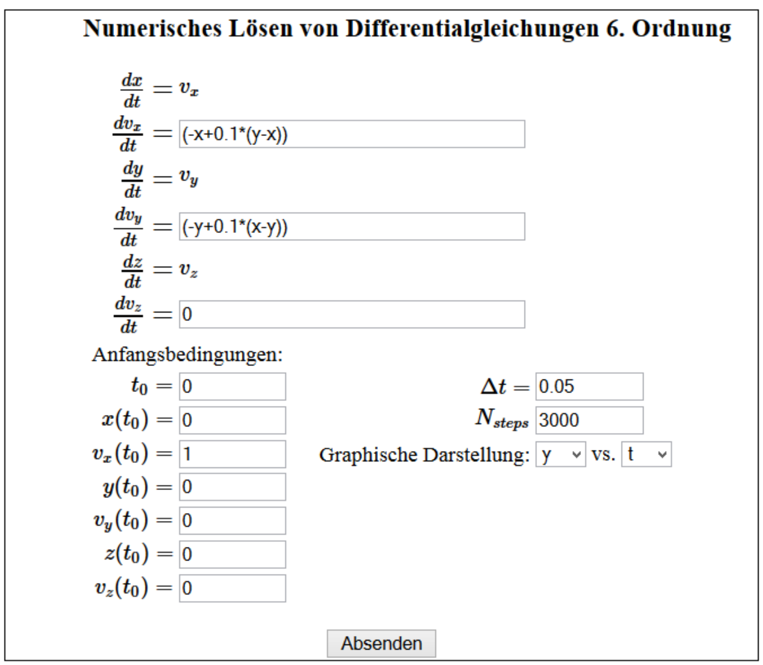
gekoppeltes Schwingungssystem



Hering

$$M\frac{d^2y_1}{dt^2} = -ky_1 + k_{12}\left(y_2 - y_1\right)$$

$$M\frac{d^2y_2}{dt^2} = -ky_2 + k_{12}\left(y_1 - y_2\right)$$



Numerisches Lösen von Differentialgleichungen 6. Ordnung

$\frac{dx}{dt} = v_x$

$\frac{dv_x}{dt} =$ (-x+0.1*(y-x))

$\frac{dy}{dt} = v_y$

$\frac{dv_y}{dt} =$ (-y+0.1*(x-y))

$\frac{dz}{dt} = v_z$

$\frac{dv_z}{dt} =$ 0

Anfangsbedingungen:

$t_0 =$ 0

$x(t_0) =$ 0

$v_x(t_0) =$ 1

$y(t_0) =$ 0

$v_y(t_0) =$ 0

$z(t_0) =$ 0

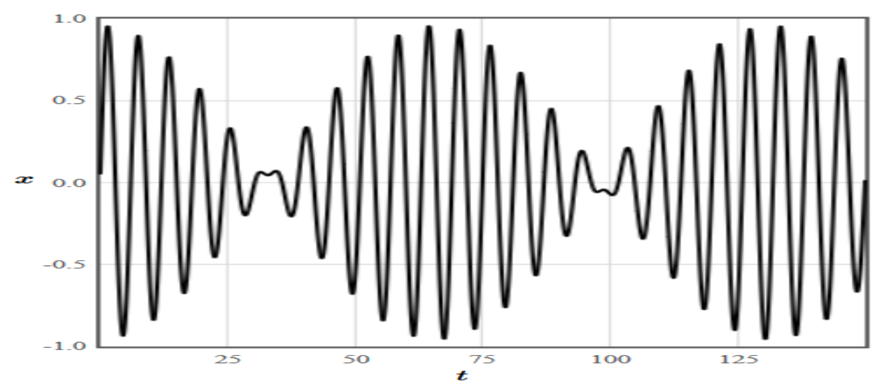
$v_z(t_0) =$ 0

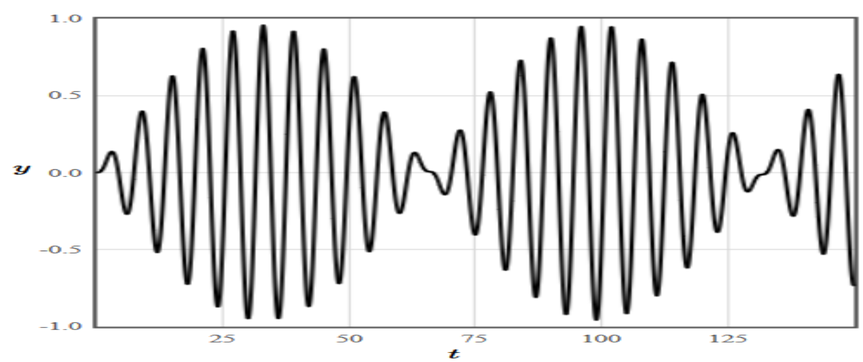
$\Delta t =$ 0.05

N_{steps} 3000

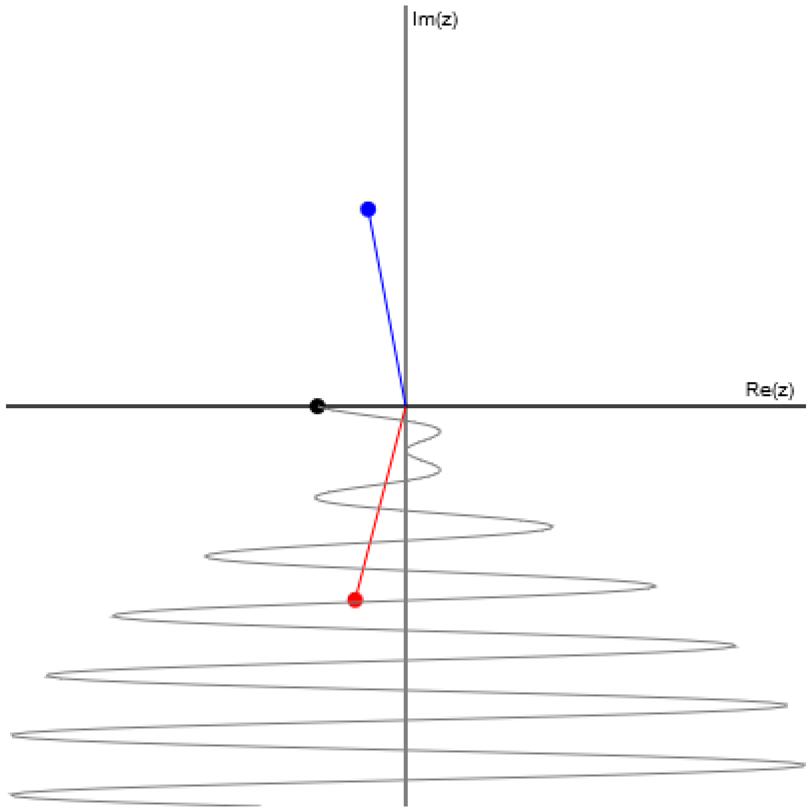
Graphische Darstellung: y vs. t

Absenden





Schwebung



$$x = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t).$$

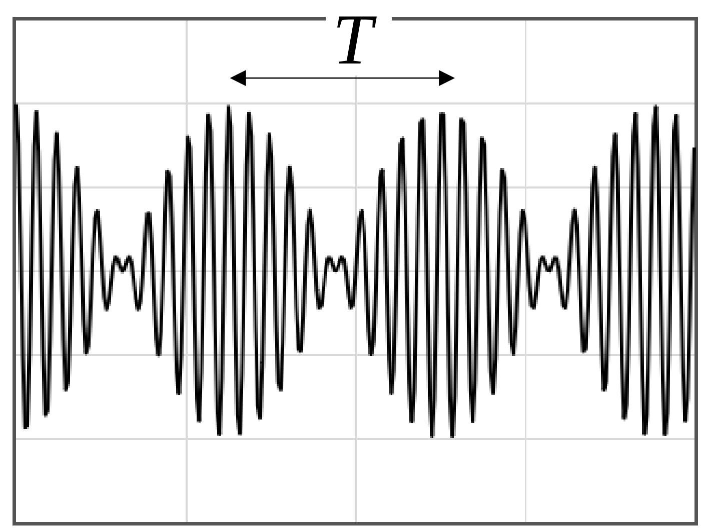
$A_1 = 1$ [m]

$A_2 = 1$ [m]

$\omega_1 = 1$ [rad/s]

$\omega_2 = 1.1$ [rad/s]

t = 0



$$\omega_1 T - \omega_2 T = 2\pi$$

$$T = \frac{2\pi}{\omega_1 - \omega_2}$$

http://lamp.tu-graz.ac.at/~hadley/physikm/apps/resonance/schwebung.de.php