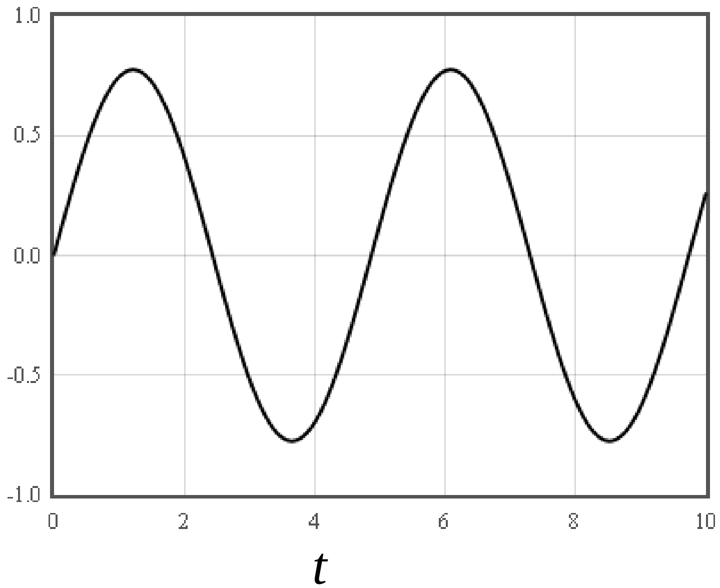
Schwingungen

Harmonische Schwingung

$$f = \frac{1}{T}$$



sinusförmig = harmonisch

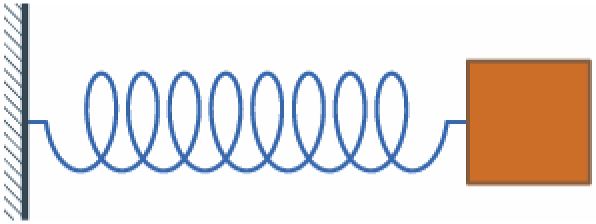
$$\sin(\omega t)$$

Radiant

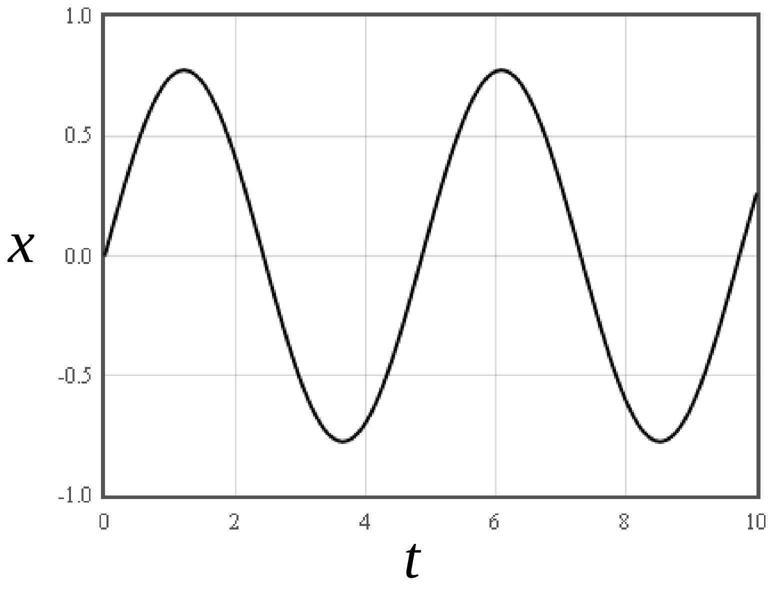
Kreisfrequenz $$\omega = 2\pi f = \frac{2\pi}{T}$$ rad/s

Freie Schwingung

$$m\frac{d^2x}{dt^2} = -kx$$



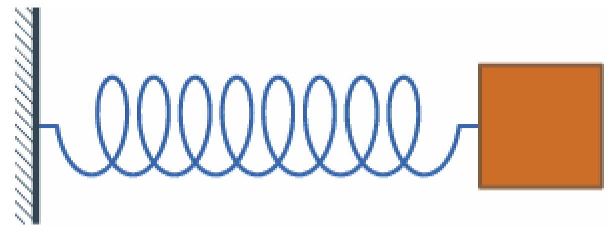
Lösung: $$x(t) = C_1 \sin\left(\omega_0 t\right) + C_2 \cos\left(\omega_0 t\right)$$



$$\omega_0 = ?$$

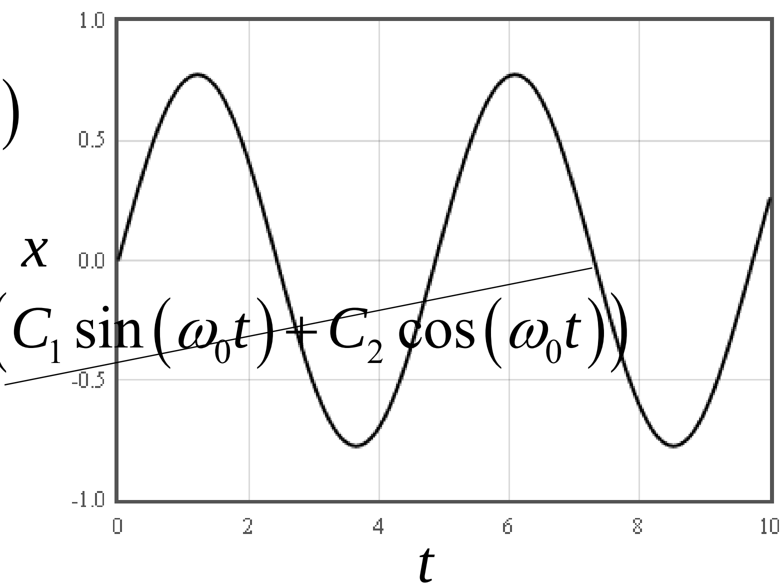
Freie Schwingung

$$m\frac{d^2x}{dt^2} = -kx$$



$$x(t) = C_1 \sin(\omega_0 t) + C_2 \cos(\omega_0 t)$$

$$\frac{d^2x}{dt^2} = -\omega_0^2 C_1 \sin(\omega_0 t) - \omega_0^2 C_2 \cos(\omega_0 t)$$



$$-m\omega_0^2 \left(C_1 \sin(\omega_0 t) + C_2 \cos(\omega_0 t)\right) = -k\left(C_1 \sin(\omega_0 t) + C_2 \cos(\omega_0 t)\right)$$

$$m\omega_0^2 = k$$

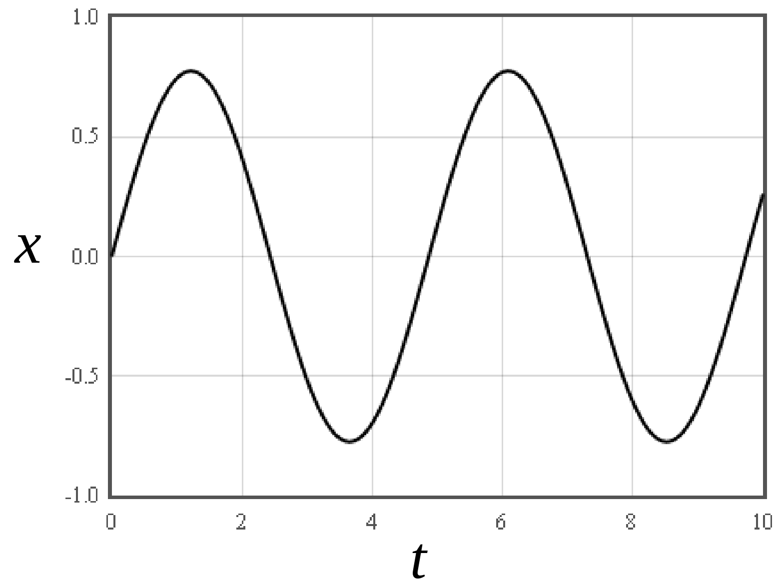
$$\omega_0 = \sqrt{\frac{k}{m}}$$

Anfangsbedingungen

$$x(t=0)=0 \qquad \frac{dx}{dt}(t=0)=1$$

$$x(t)=C_1\sin\left(\omega_0 t\right)+C_2\cos\left(\omega_0 t\right)$$

$$\frac{dx}{dt}=\omega_0 C_1\cos\left(\omega_0 t\right)-\omega_0 C_2\sin\left(\omega_0 t\right)$$



$$x(t=0)=0=C_2$$

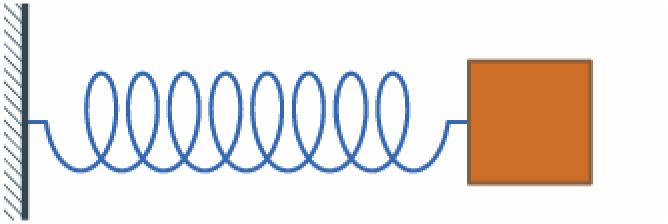
$$C_2=0$$

$$\frac{dx}{dt}(t=0)=1=\omega_0 C_1$$

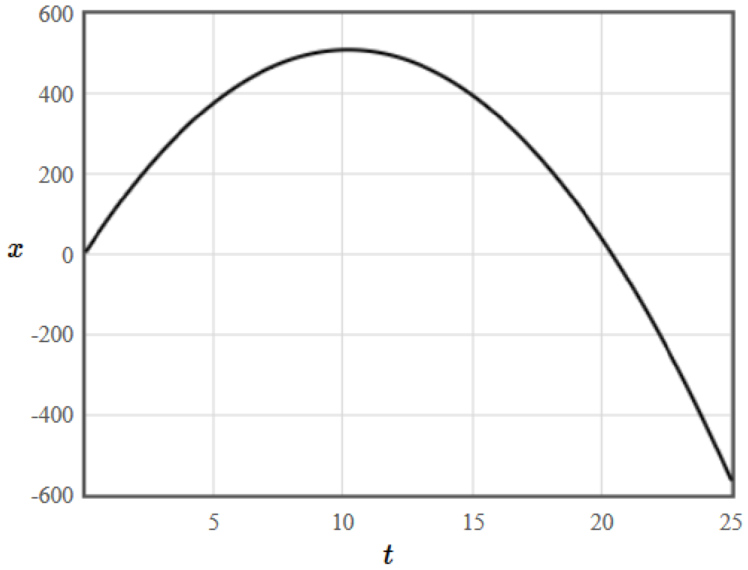
$$C_1=\frac{1}{\omega_0}$$

falsche Lösung

$$m\frac{d^2x}{dt^2} = -kx$$



$$x = v_0 t - \tfrac{1}{2} g t^2$$

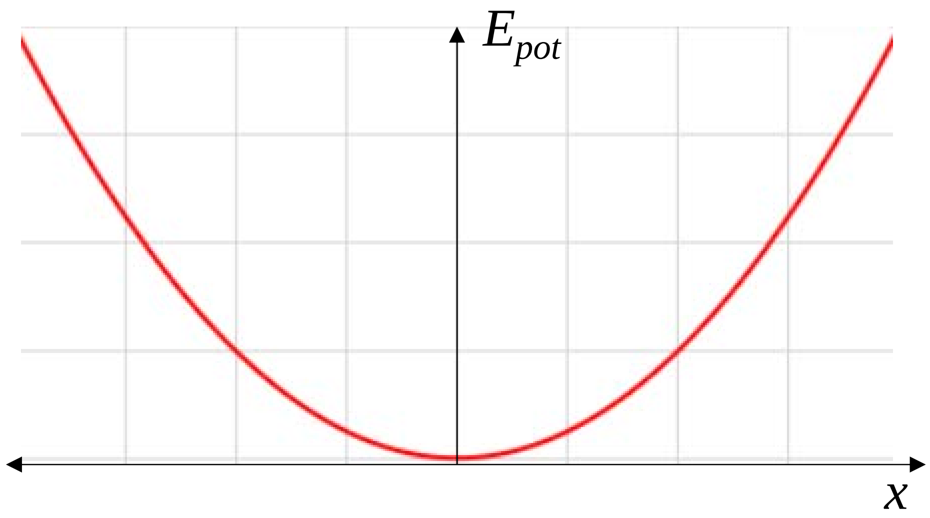


$$\frac{d^2x}{dt^2} = -g$$

$$-mg = -k(v_0 t - \tfrac{1}{2} g t^2)$$

andere Schwingungssysteme

E_{pot} hat ein Minimum bei dem Gleichgewichtspunkt

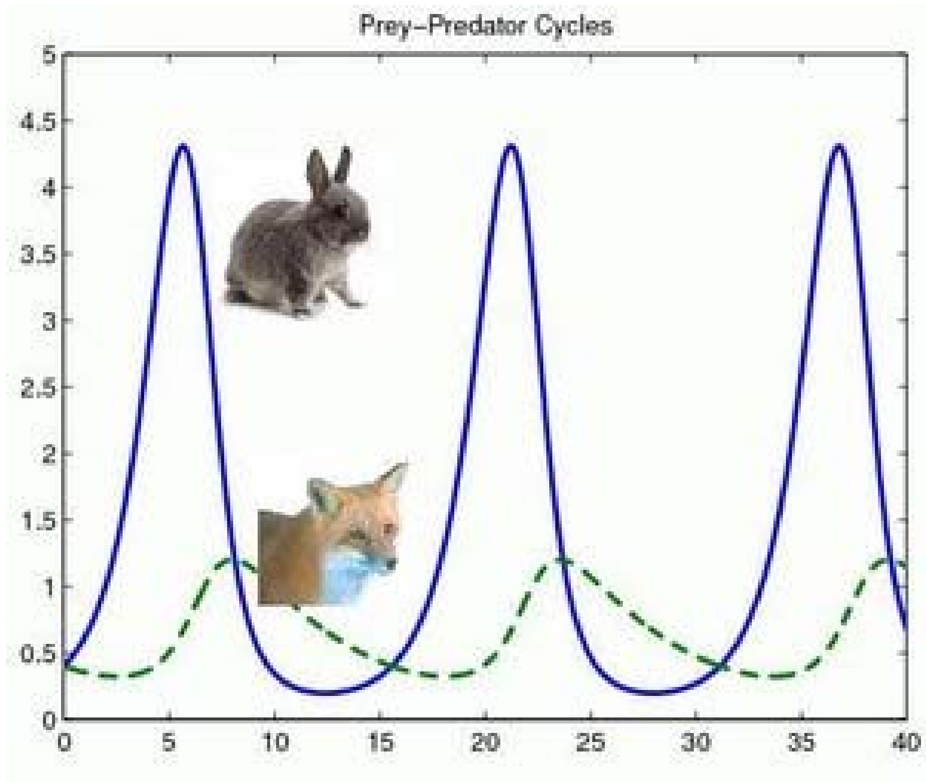


$$F_x = -\frac{dU}{dx}$$

Räuber-Beute-Gleichungen

$$\frac{dx}{dt} = (b - py)\,x$$

$$\frac{dy}{dt} = (ry - d)\,y$$



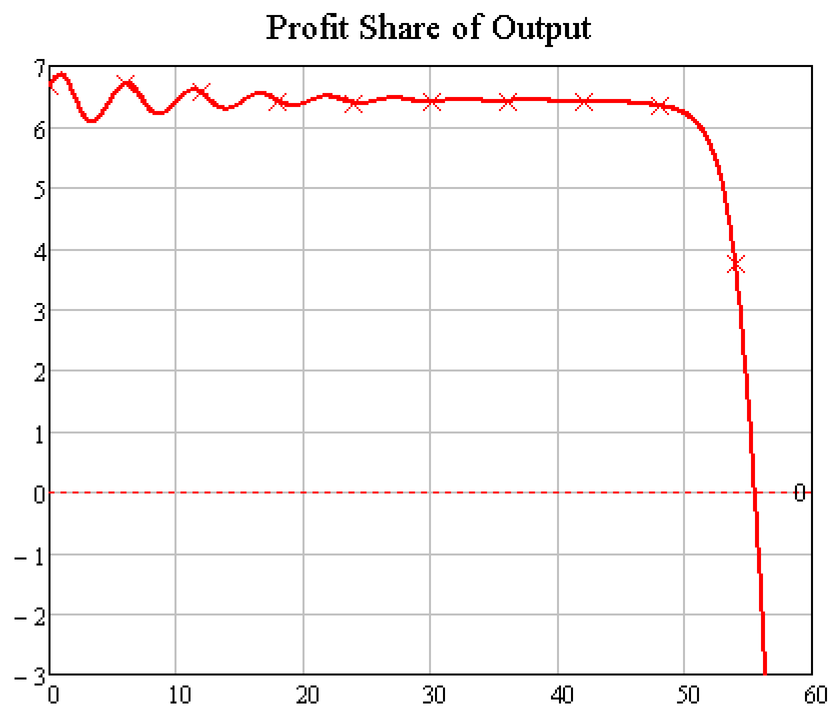
http://www.scholarpedia.org/article/Predator-prey_model

Konjunkturzyklen

$$\frac{d}{dt}\lambda = \lambda \cdot \left(\frac{Inv_{fn}(\pi_r)}{v} - (a + \beta + \gamma) \right)$$

$$\frac{d}{dt}\omega = \omega \cdot \left(\mathrm{Wage}_{\mathrm{fn}}(\lambda) - \alpha \right)$$

$$\frac{d}{dt}d = Inv_{fn}(\pi_r) - \pi_r - d \cdot \left(\frac{Inv_{fn}(\pi_r)}{v} - \gamma \right)$$

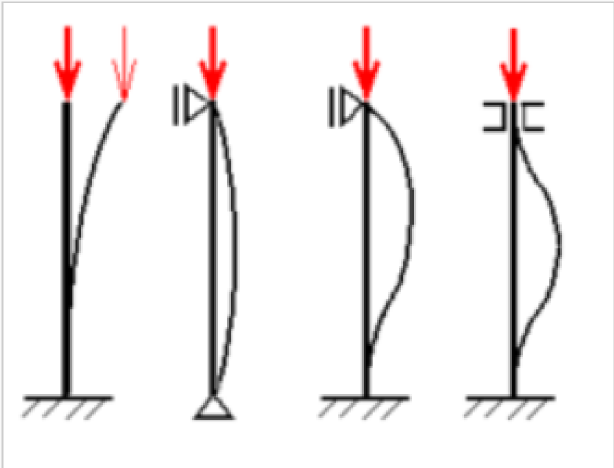


http://debunkingeconomics.com/2013/04/economics-and-the-powerful-faulty-analysis-economic-advice-and-the-imperatives-of-power/

Knicken

Eulersche Knickfälle (Biegeknicken) [Bearbeiten]

Nach Leonhard Euler, der das Knicken schlanker Stäbe als erster behandelt hat, sind vier Fälle für das Knicken des elastischen Stabes mit mittig wirkender Druckkraft und speziellen Randbedingungen benannt. Euler untersuchte das Gleichgewicht der Spannungen an bereits durch die eigentliche Belastung verformten Stäben, dieser Lösungsansatz war für seine Zeit neu und führte zu umfangreichen Erkenntnissen innerhalb der Stabilitätstheorie. In die Rechnung zum Nachweis der Knicksicherheit gehen sämtliche geometrischen, mechanischen und werkstoffseitigen Parameter des belasteten Bauteiles ein.



Die vier Eulerfälle mit folgenden Randbedingungen (v.l.n.r.):
(1) eingespannt/frei, (2) gelenkig/gelenkig,
(3) eingespannt/gelenkig,
(4) eingespannt/eingespannt

Die Knickkraft, zuweilen auch als *Eulerkraft* bezeichnet, kann für den elastischen Bereich durch eine einzige Formel dargestellt werden:

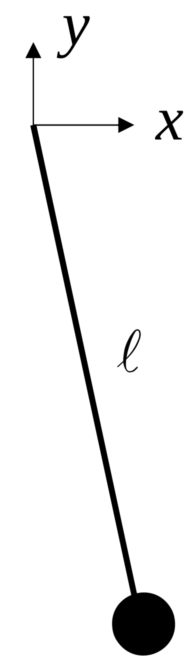
$$F_\mathrm{k} = \frac{\pi^2 EI}{s^2}$$

$$\frac{d^2 v}{dx^2} + \left(\frac{P}{EI}\right) v = 0$$

http://de.wikipedia.org/wiki/Knicken

Pendel

$$x^2 + y^2 = \ell^2$$



$$E_{pot} = -mgy = -mg\sqrt{\ell^2 - x^2}$$

$$F_x = -\frac{\partial E_{pot}}{\partial x} = \frac{1}{2}\frac{mg\left(-2x\right)}{\sqrt{\ell^2 - x^2}}$$

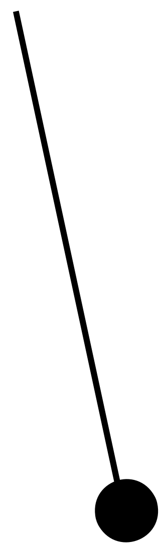
für $\quad x \ll y \approx \ell$

$$F_x \approx -\frac{mgx}{\ell}$$

Pendel

$$m\frac{d^2x}{dt^2} \approx -\frac{mg}{\ell}x$$

$$\omega_0 = \sqrt{\frac{g}{\ell}}$$



Schwingungssysteme

für kleine Amplitudenschwankungen und niedrigen Geschwindigkeiten die Differentialgleichung für eine schwingende Systeme ist

$$a\frac{d^2x}{dt^2}+b\frac{dx}{dt}+cx=d$$

Lineare gewöhnliche Differentialgleichung zweiter Ordnung

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$a_x = -\frac{b}{m}v_x - \frac{k}{m}x$$

Numerisches Lösen von Differentialgleichungen 2. Ordnung

$\frac{dx}{dt} = v_x$

$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} =$ -0.1*vx-sin(x)+sin(3*t)

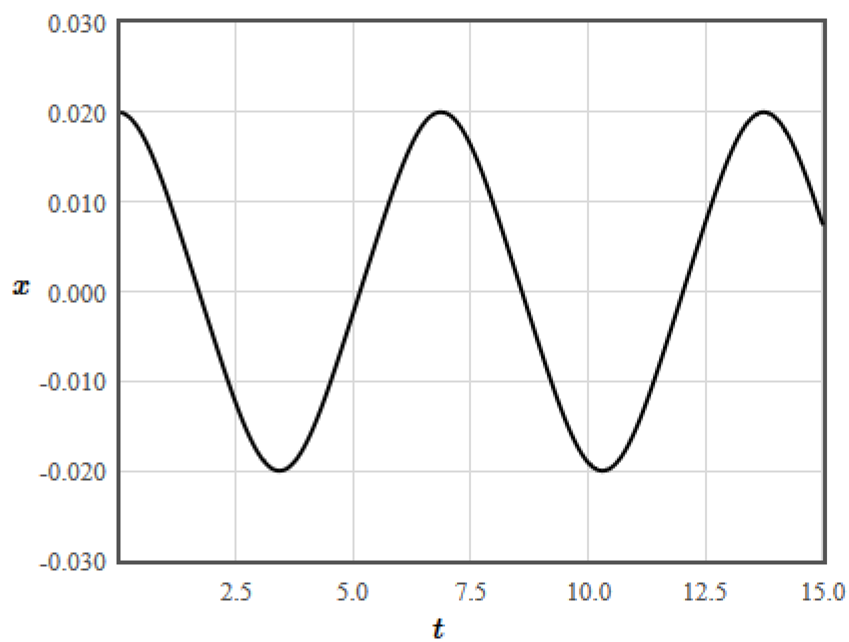
Anfangsbedingungen:

$x(t_0) =$ 0 $\Delta t =$ 0.05

$v_x(t_0) =$ 1 N_{steps} 500

$t_0 =$ 0 Graphische Darstellung: vx vs. x

Absenden



Differentialgleichungen zweiter Ordnung

Lösung Differentialgleichungen zweiter Ordnung

$$a\,\frac{d^2x}{dt^2} + b\,\frac{dx}{dt} + cx = d,$$

$a =$ 1

$b =$ 3

$c =$ 1

$d =$ 0

Anfangsbedingungen:

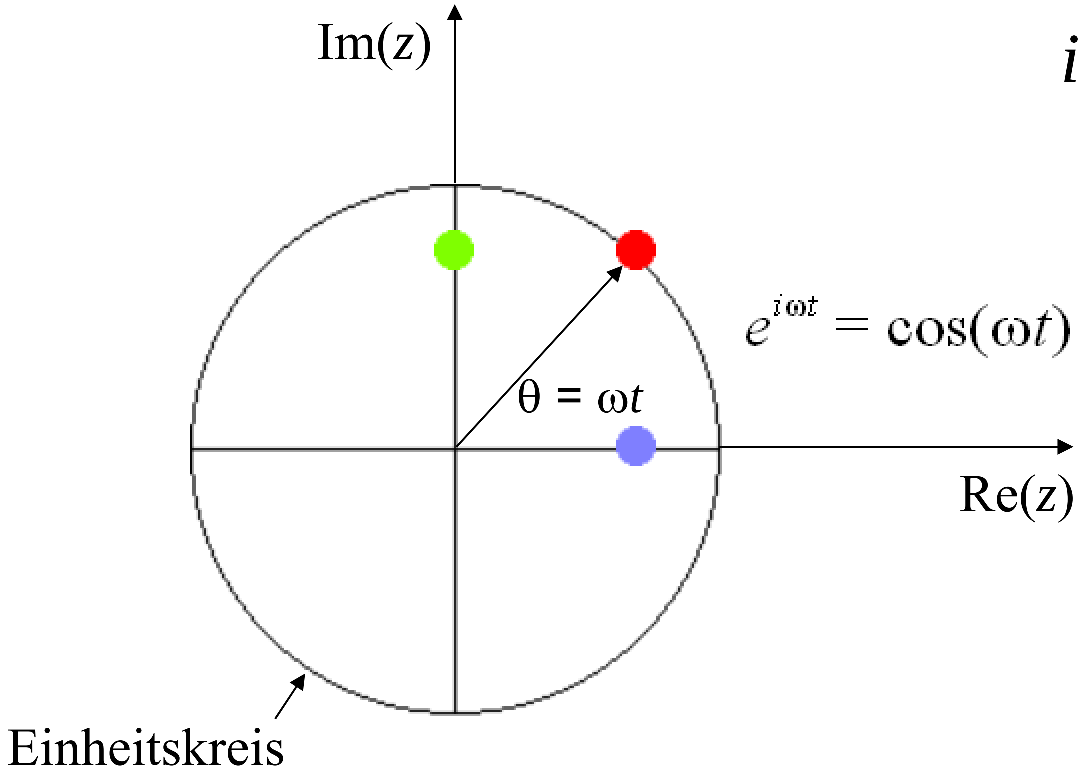
$x(t_0) =$ 1

$\frac{dx}{dt}(t_0) =$ 0

$t_0 =$ 0

Lösung

Euler'sche Formel $e^{i\theta} = \cos\theta + i\sin\theta$



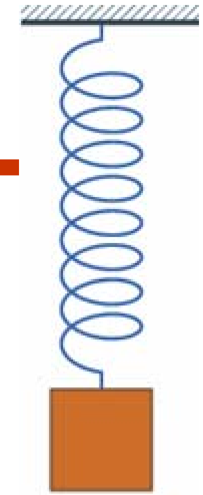
$$i^2 = -1 = \left(j^2\right)$$

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$

ω = Kreisfrequenz

$$\left|e^{i\theta}\right| = \sqrt{e^{-i\theta}e^{i\theta}} = \sqrt{e^0} = 1 = \sqrt{\left(\cos\theta - i\sin\theta\right)\left(\cos\theta + i\sin\theta\right)} = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

Freie Schwingung



$$m\frac{d^2x}{dt^2}+kx=-mg$$

Lösung Differentialgleichungen zweiter Ordnung

$$a\,\frac{d^2x}{dt^2}+b\,\frac{dx}{dt}+cx=d,$$

$a =$ 1

$b =$ 0

$c =$ 1

$d =$ -9.81

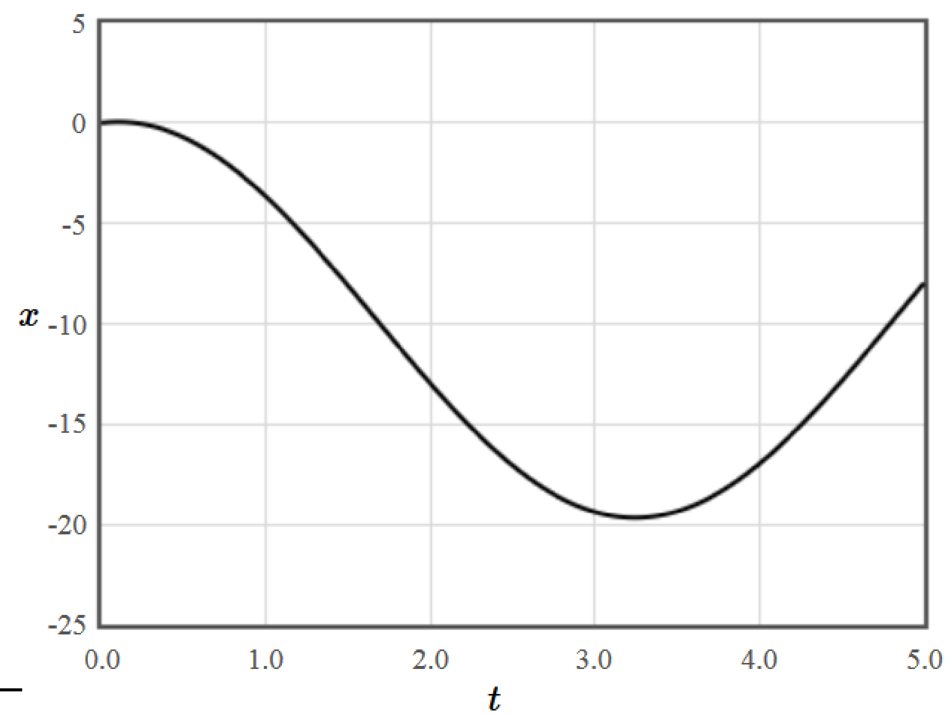
Anfangsbedingungen:

$x(t_0) =$ 0

$\frac{dx}{dt}(t_0) =$ 1

$t_0 =$ 0

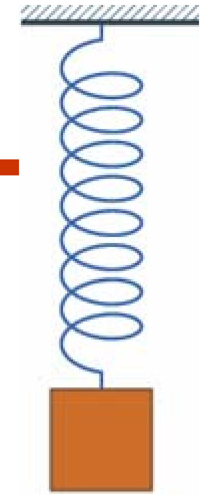
Lösung

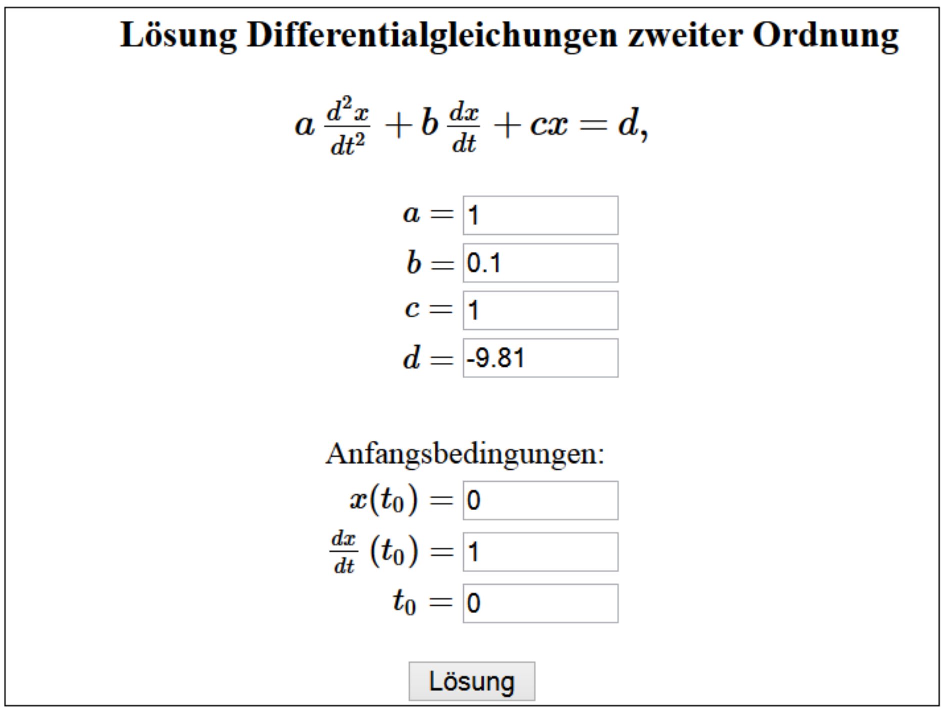


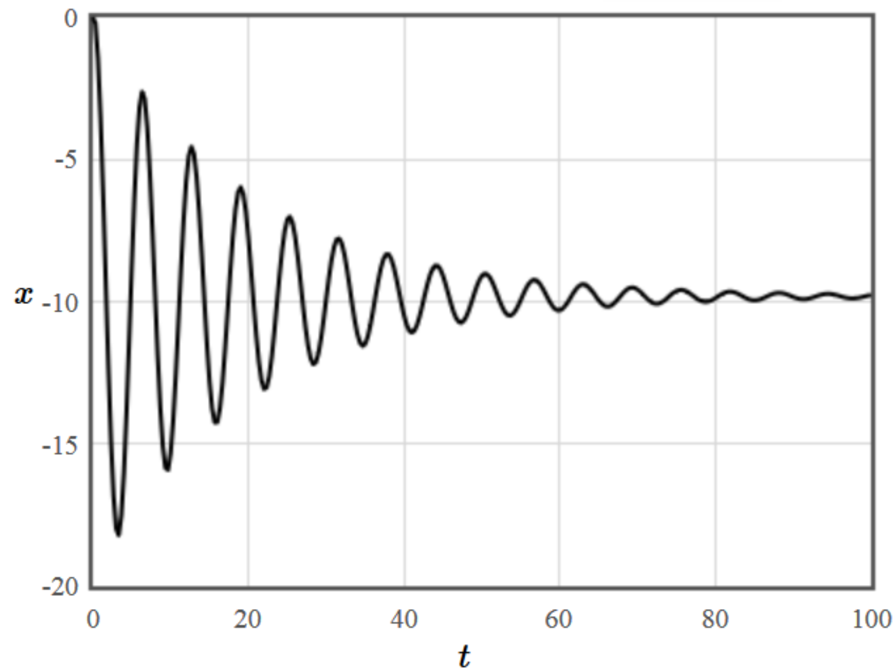
$$\omega_0=\sqrt{\frac{k}{m}}$$

$b^2 < 4km$ Schwingfall

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=-mg$$



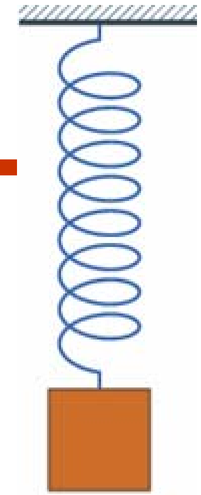




$$\tau=\frac{2m}{b}$$

$$\omega=\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}$$

$b^2 = 4km$ aperiodischer Grenzfall



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = -mg$$

Lösung Differentialgleichungen zweiter Ordnung

$$a\,\frac{d^2x}{dt^2} + b\,\frac{dx}{dt} + cx = d,$$

$a =$ 1

$b =$ 2

$c =$ 1

$d =$ -9.81

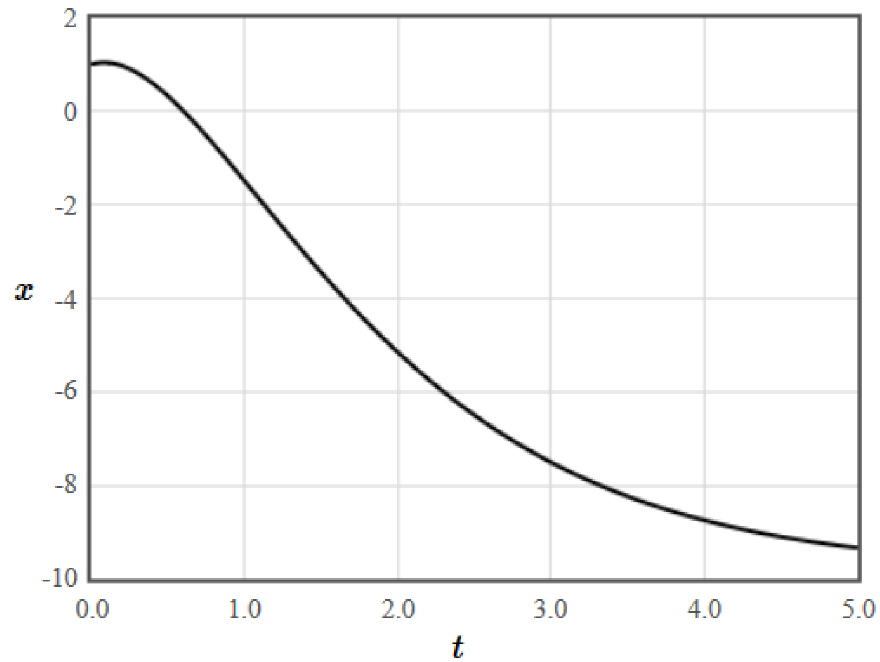
Anfangsbedingungen:

$x(t_0) =$ 1

$\frac{dx}{dt}(t_0) =$ 1

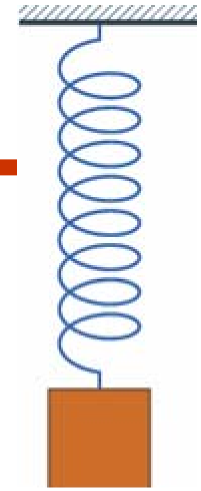
$t_0 =$ 0

Lösung

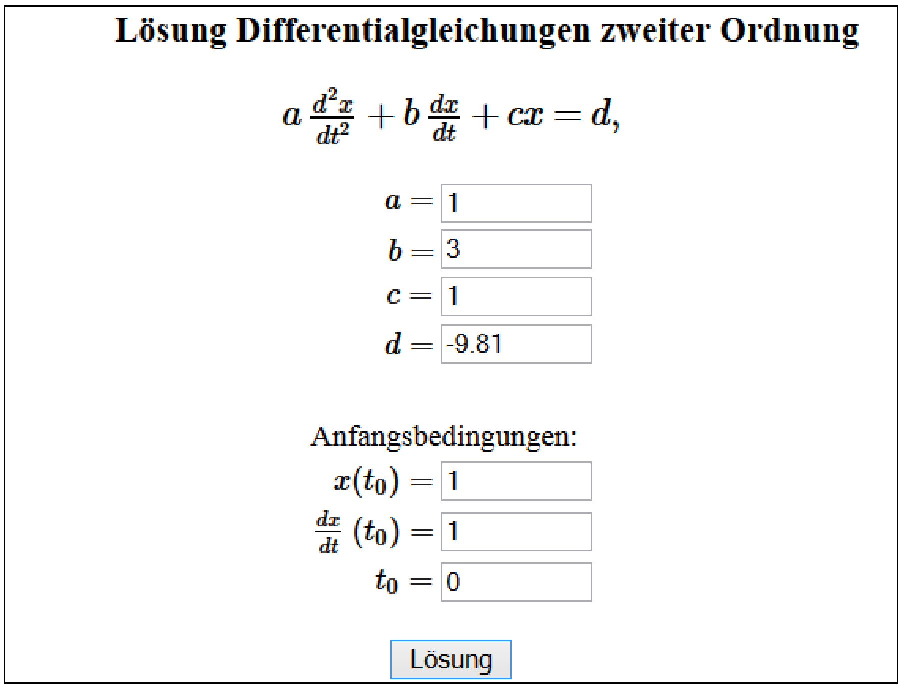


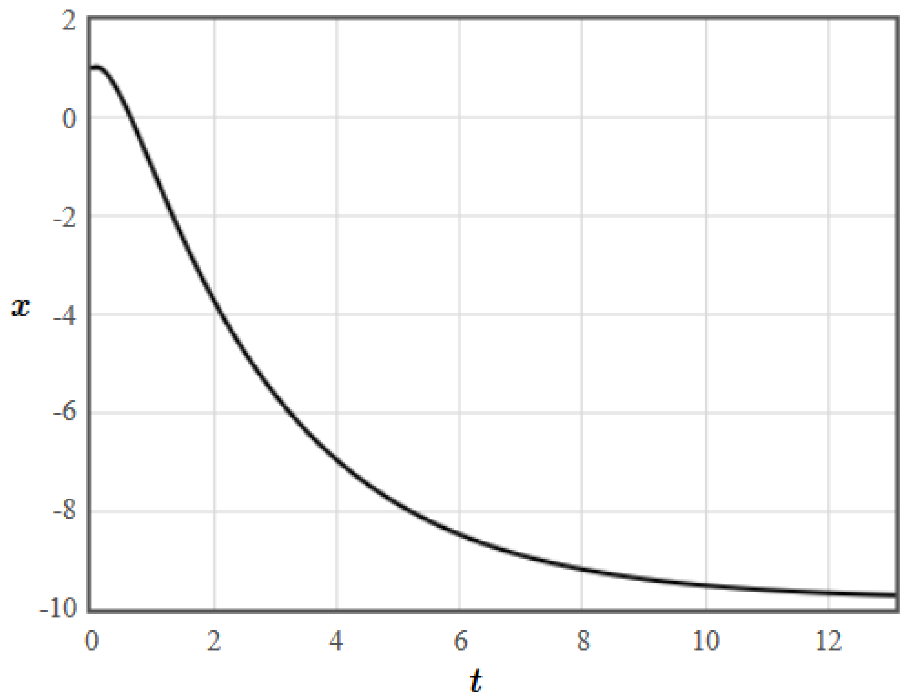
$$\tau = \frac{2m}{b}$$

$b^2 > 4km$ Kriechfall



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = -mg$$

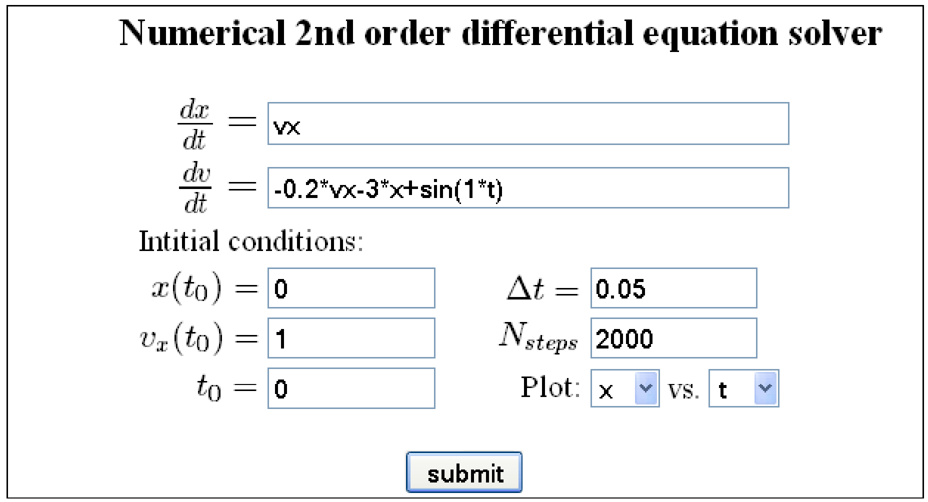




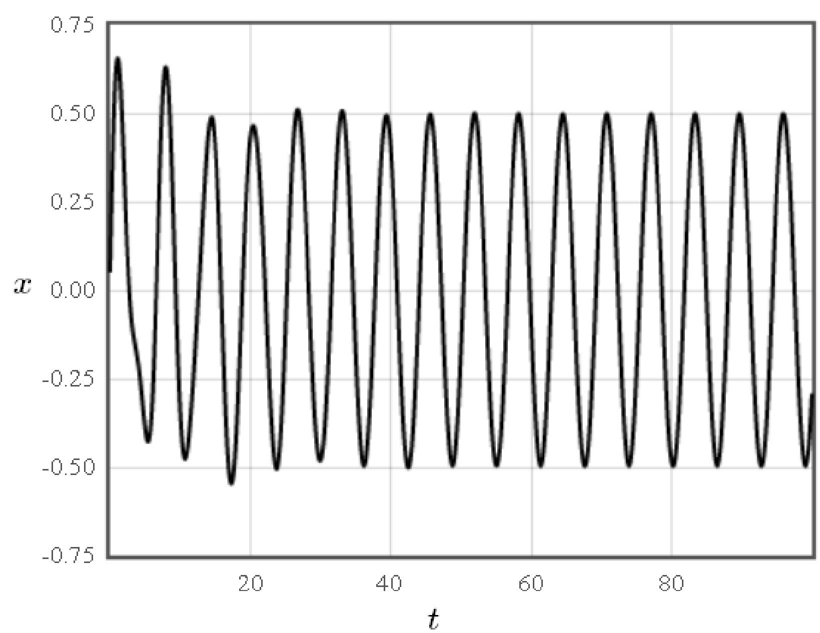
$$\tau_1 = \frac{-1}{\lambda_1} = \frac{2m}{b + \sqrt{b^2 - 4km}}$$

$$\tau_2 = \frac{-1}{\lambda_2} = \frac{2m}{b - \sqrt{b^2 - 4km}}$$

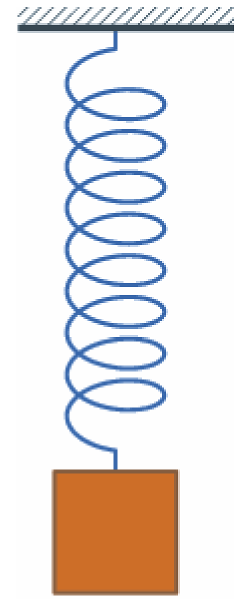
Resonaz



Numerical 2nd order differential equation solver
$\frac{dx}{dt}$ = vx
$\frac{dv}{dt}$ = -0.2*vx-3*x+sin(1*t)
Initial conditions:
$x(t_0)$ = 0
Δt = 0.05
$v_x(t_0)$ = 1
N_{steps} = 2000
t_0 = 0
Plot: x vs. t
submit

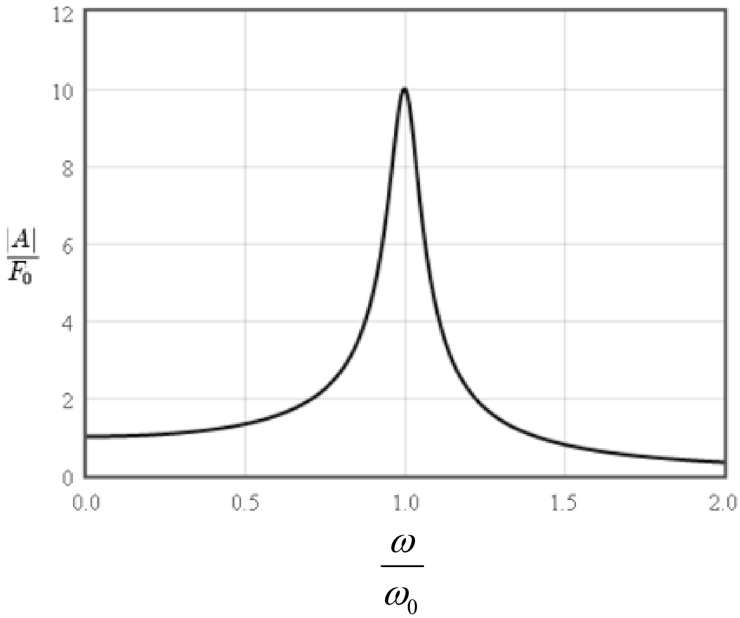


0.75
0.50
0.25
x
0.00
-0.25
-0.50
-0.75
20
40
60
80
t



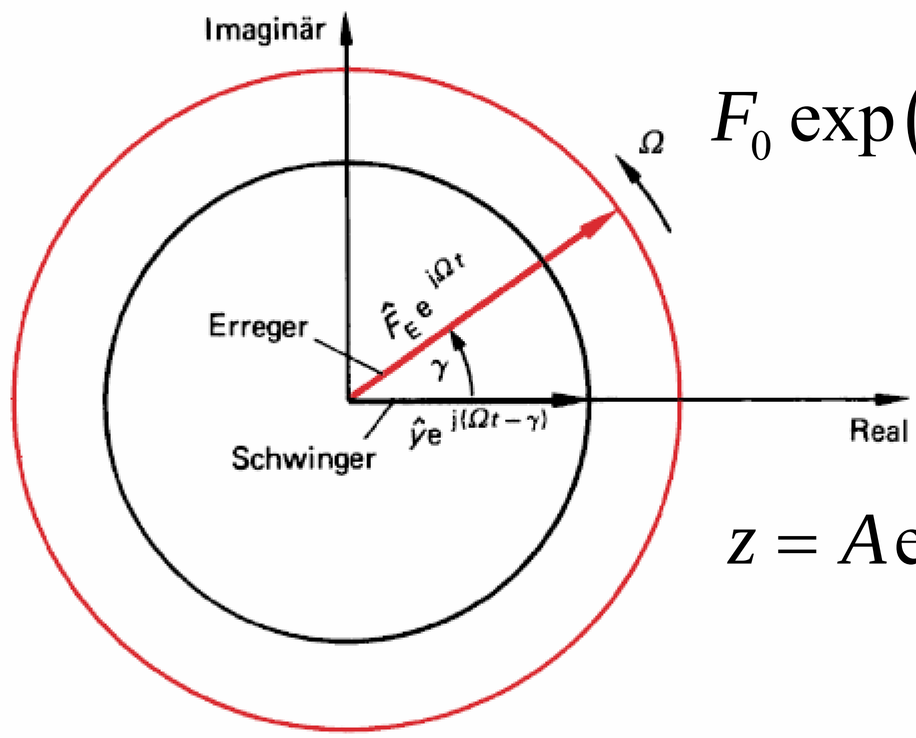
Resonaz

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx + F_0\cos(\omega t)$$



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos(\omega t)$$

$$m\frac{d^2z}{dt^2}+b\frac{dz}{dt}+kz=F_0e^{i\omega t}$$



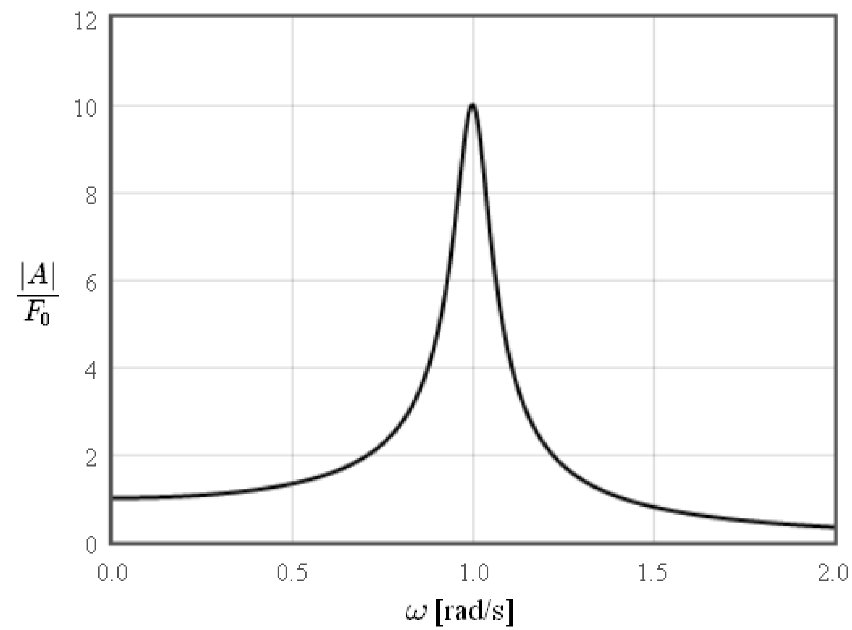
$$F_0\exp(i\omega t)$$

$$z=A\exp(i\omega t-\gamma)$$

$m =$ 1 [kg] $b =$ 0.1 [N s/m] $k =$ 1 [N/m] Plot

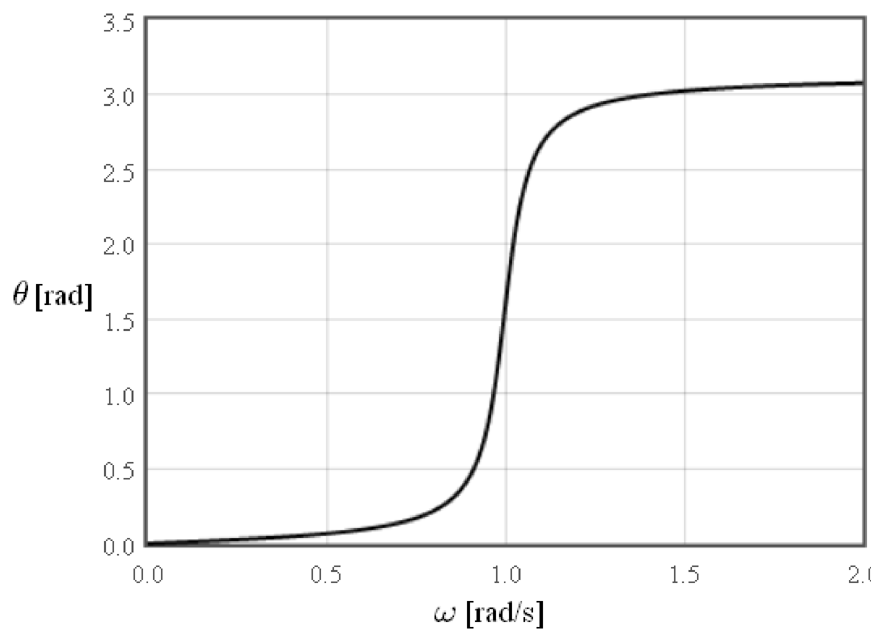
$Q = \frac{\sqrt{mk}}{b} =$ 10

$$\frac{|A|}{F_0} = \frac{1}{\sqrt{\left(k - m\omega^2\right)^2 + \omega^2 b^2}}$$

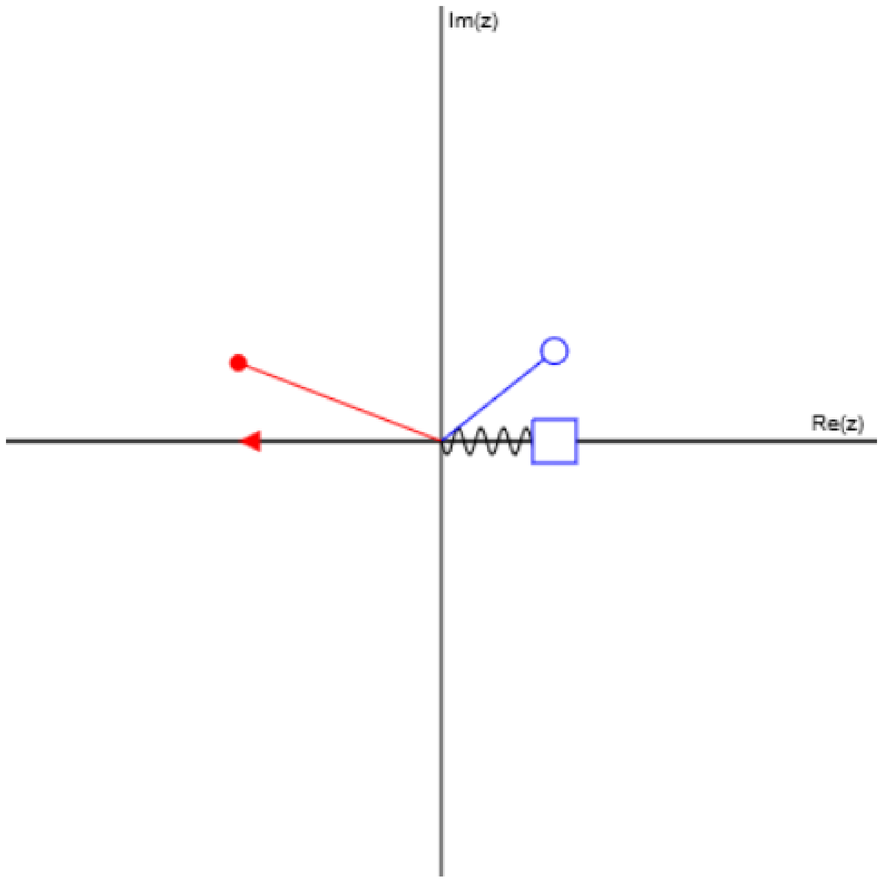


$$A = \frac{F_0}{\rho} e^{-i\theta}$$

$$\tan\theta = \frac{\omega b}{k - m\omega^2}$$



Resonaz



m =4 [kg]

b =1 [N s/m]

k =6 [N/m]

F_0 =1 [N]

ω =1.3 [rad/s]

$$\omega_0 = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = 1.22 \text{ [rad/s]} = 0.194 \text{ [Hz]}$$

$$\theta = \text{atan}\left(\frac{\omega b}{k - m\omega^2}\right) = 2.10 \text{ [rad]} = 120 \text{ [deg]}$$

$$A = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + \omega^2 b^2}} = 0.664 \text{ [m]}$$

$$Q = \frac{\sqrt{mk}}{b} = 4.90$$

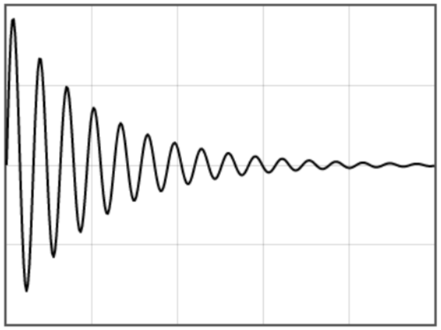
$F_0 e^{i\omega t}$ anzeigen: ☑ $Ae^{i(\omega t - \theta)}$ anzeigen: ☑

z anzeigen: ☐ x_2 anzeigen: ☐

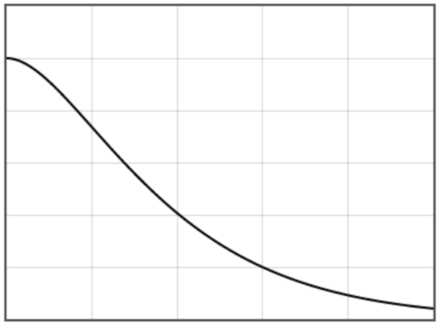
Gütefaktor

$$Q = \frac{\pi \tau}{T}$$

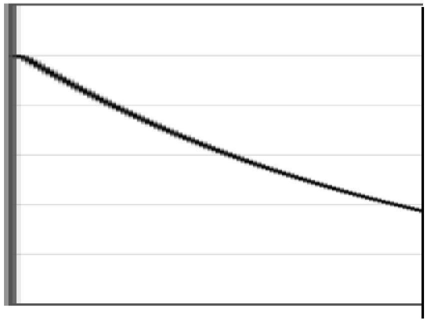
Schwingfall $Q > \frac{1}{2}$



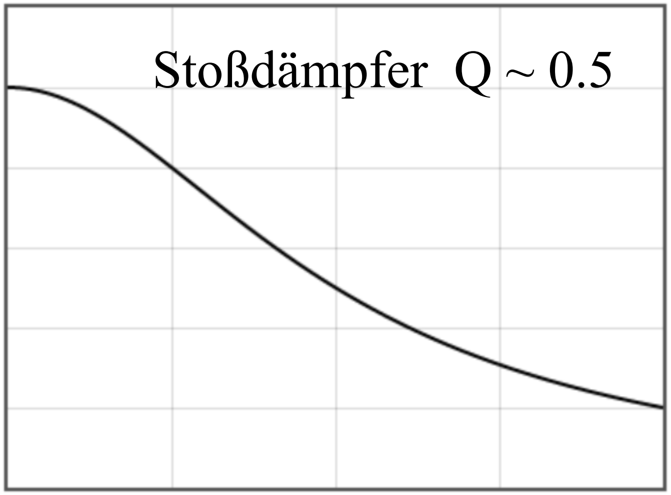
aperiodischer Grenzfall $Q = \frac{1}{2}$

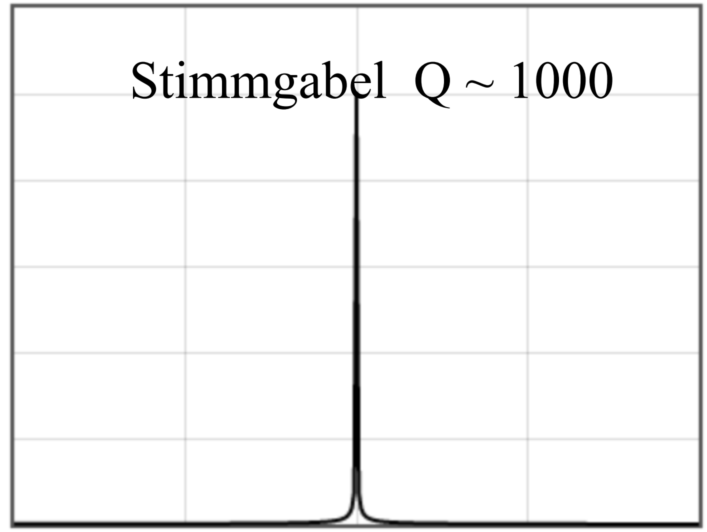


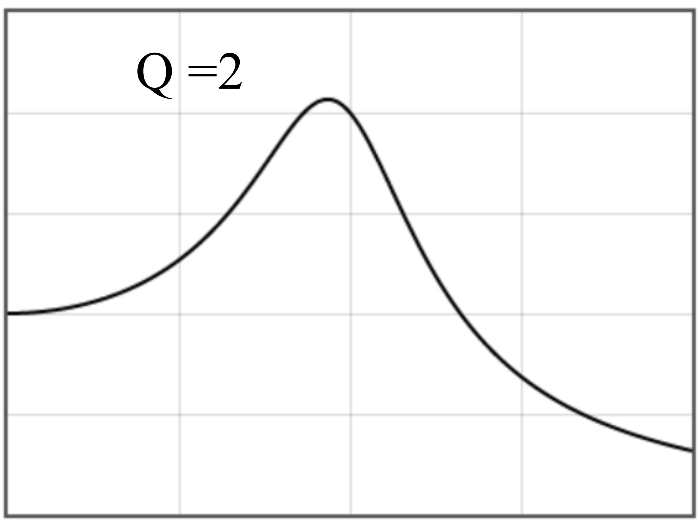
Kriechfall $Q < \frac{1}{2}$

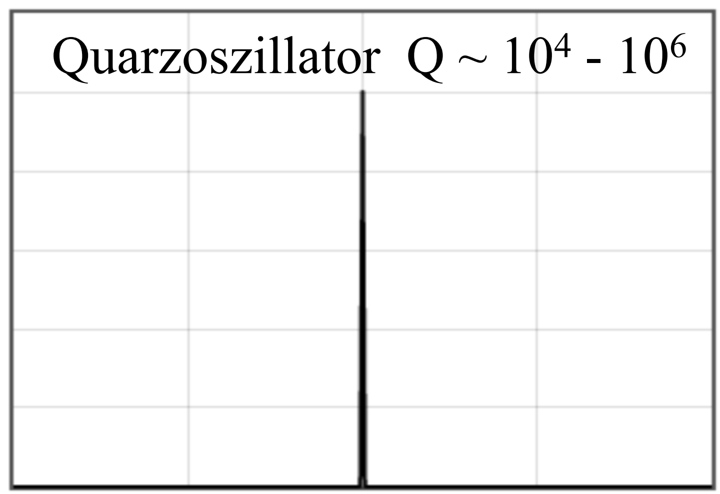


Gütefaktor


Stoßdämpfer Q ~ 0.5


Stimmgabel Q ~ 1000


Q =2


Quarzoszillator Q ~ 10^4 - 10^6

Lineare gewöhnliche Differentialgleichung

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$

Lineare Differentialgleichung zweiter Ordnung

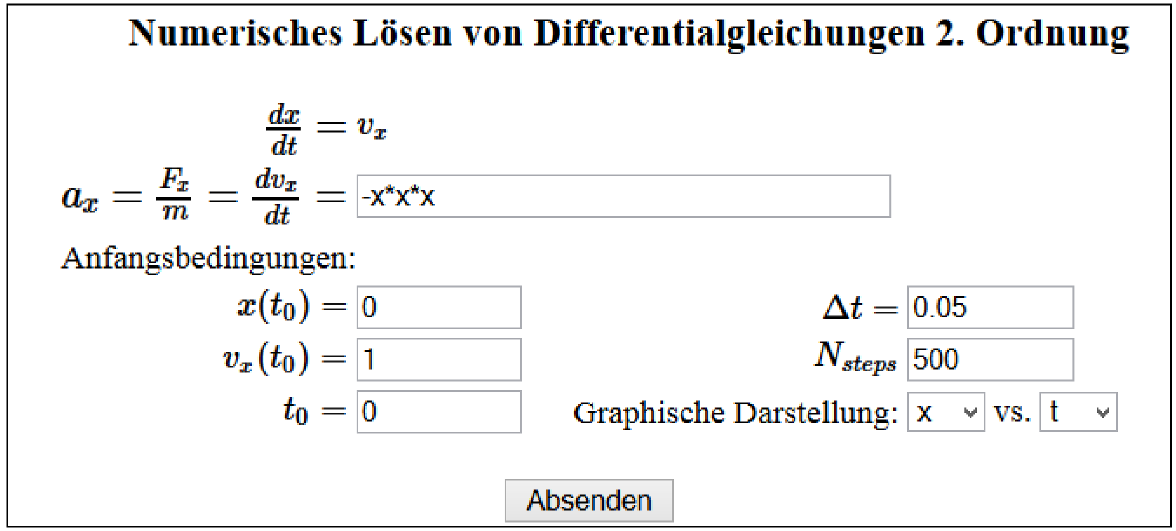
Nichtlineare Differentialgleichung zweiter Ordnung

$$m\left(\frac{d^2x}{dt^2}\right)^2+b\frac{dx}{dt}+kx=0$$

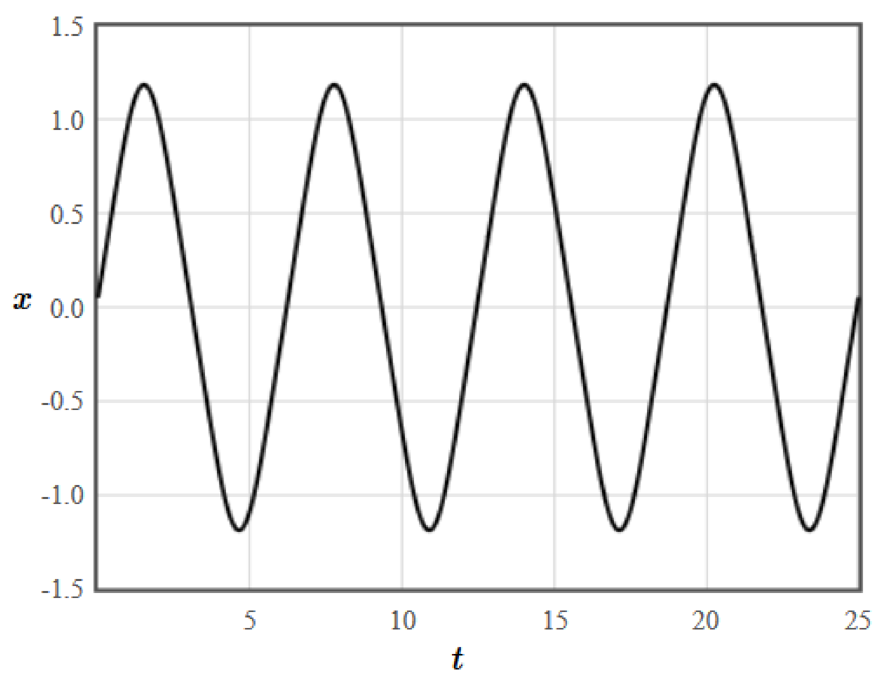
$$m\frac{d^2x}{dt^2}+b\left(\frac{dx}{dt}\right)^2+kx=0$$

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx^3=0$$

Nichtlineare gewöhnliche Differentialgleichung



$$m\frac{d^2x}{dt^2} + kx^3 = 0$$



Nichtlineare gewöhnliche Differentialgleichung

Numerisches Lösen von Differentialgleichungen 2. Ordnung

$$\frac{dx}{dt} = v_x$$

$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} = \text{-0.3*vx*vx*vx-x}$$

Anfangsbedingungen:

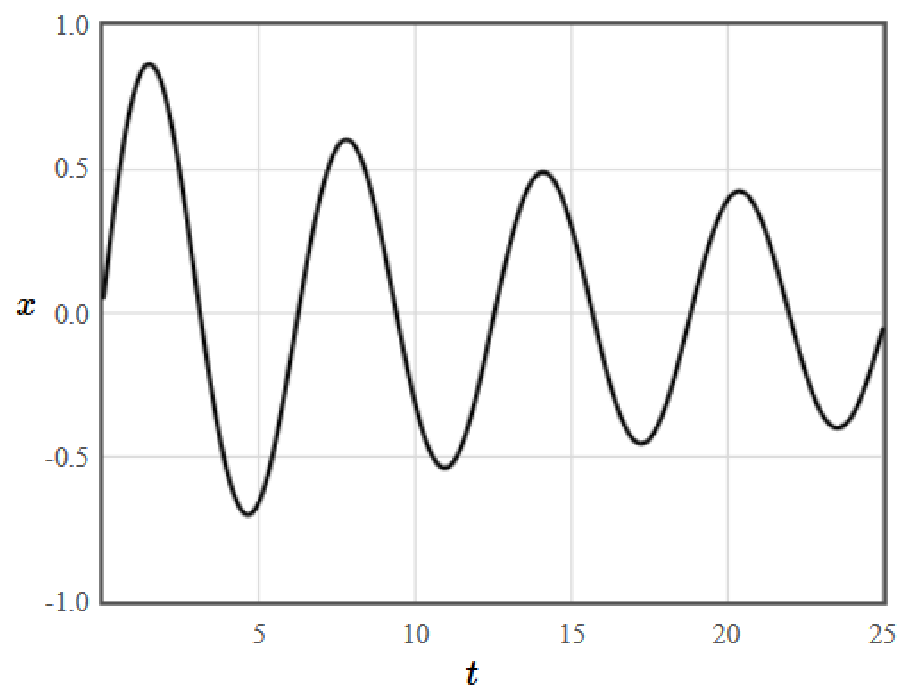
$x(t_0) = 0$ $\Delta t = 0.05$

$v_x(t_0) = 1$ N_{steps} 500

$t_0 = 0$ Graphische Darstellung: x vs. t

Absenden

$$m\frac{d^2x}{dt^2} + b\left(\frac{dx}{dt}\right)^3 + kx = 0$$



Nichtlineare gewöhnliche Differentialgleichung

Numerisches Lösen von Differentialgleichungen 2. Ordnung

$$\frac{dx}{dt} = v_x$$

$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} = \text{-0.3*vx*vx*vx-x*x*x}$$

Anfangsbedingungen:

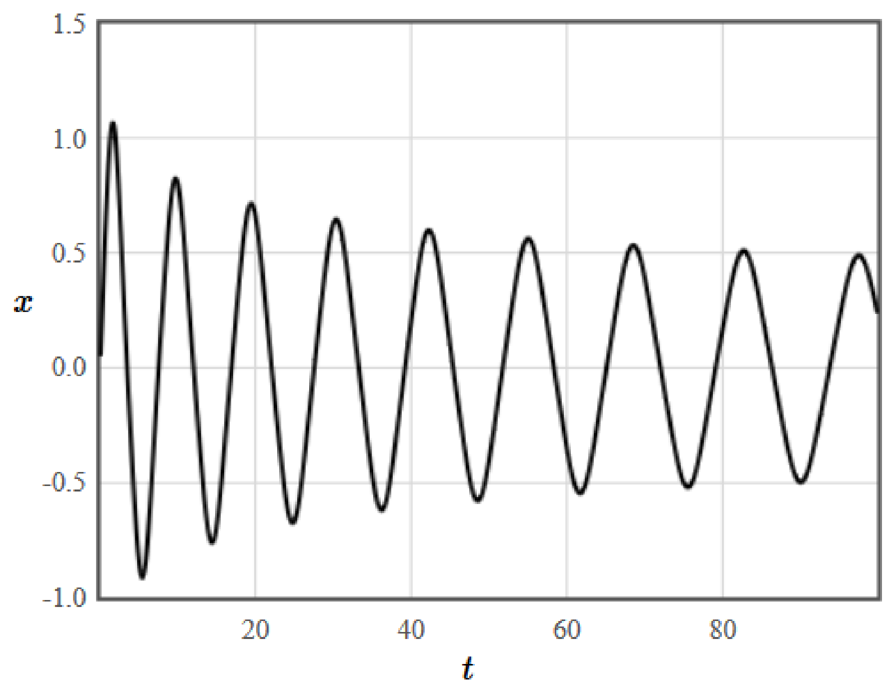
$x(t_0) =$ 0 $\Delta t =$ 0.05

$v_x(t_0) =$ 1 N_{steps} 2000

$t_0 =$ 0 Graphische Darstellung: x vs. t

Absenden

$$m\frac{d^2x}{dt^2} + b\left(\frac{dx}{dt}\right)^3 + kx^3 = 0$$



Nichtlineare Schwinger

Numerisches Lösen von Differentialgleichungen 2. Ordnung

$$\frac{dx}{dt} = v_x$$

$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} = $$ -0.1*vx-sin(x)

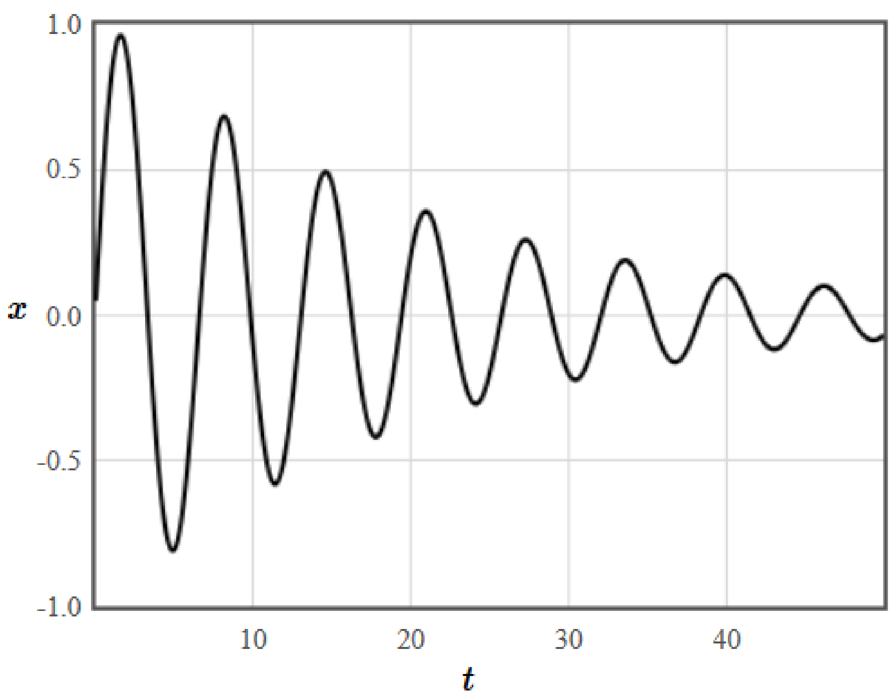
Anfangsbedingungen:

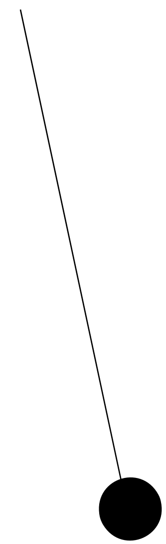
$x(t_0) =$ 0 $\Delta t =$ 0.05

$v_x(t_0) =$ 1 N_{steps} 1000

$t_0 =$ 0 Graphische Darstellung: x vs. t

Absenden





Pendel

Parametrisch erregte Schwingungen

Numerisches Lösen von Differentialgleichungen 2. Ordnung

$$\frac{dx}{dt} = v_x$$

$$a_x = \frac{F_x}{m} = \frac{dv_x}{dt} = \text{-0.1*vx-(1-cos(t))*sin(x)}$$

Anfangsbedingungen:

$x(t_0) =$ 0

$v_x(t_0) =$ 0.1

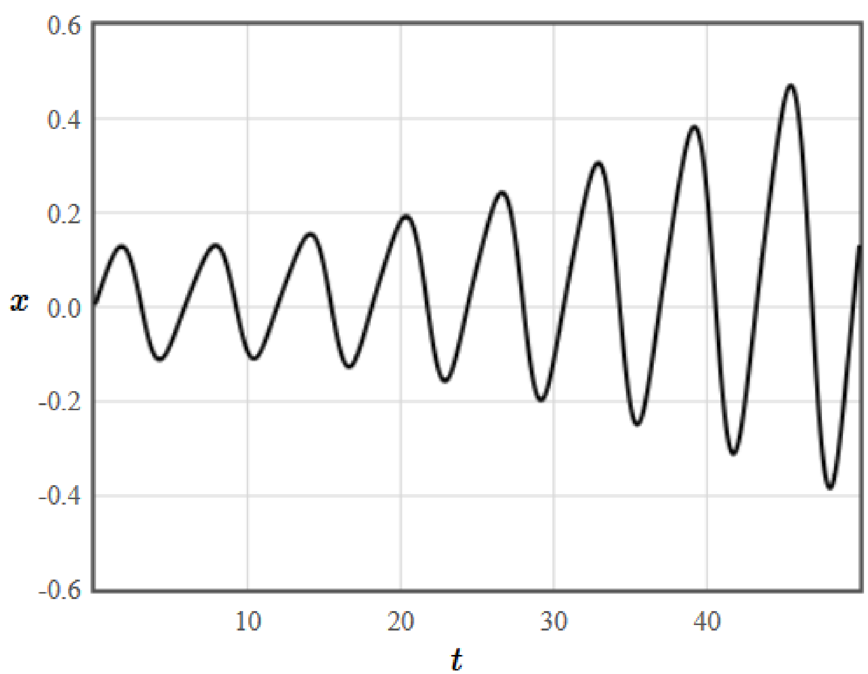
$t_0 =$ 0

$\Delta t =$ 0.05

N_{steps} 1000

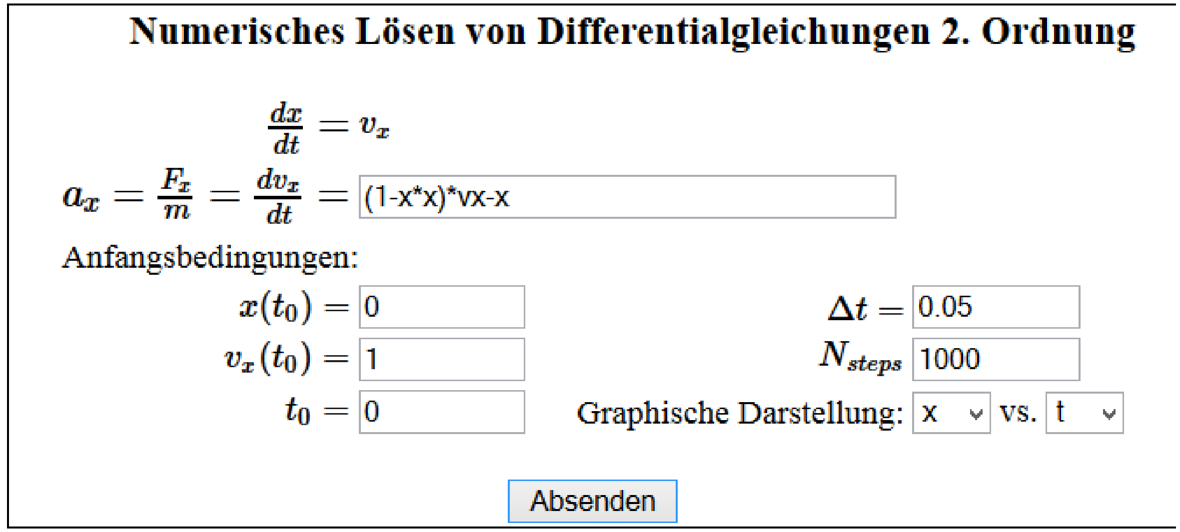
Graphische Darstellung: x vs. t

Absenden



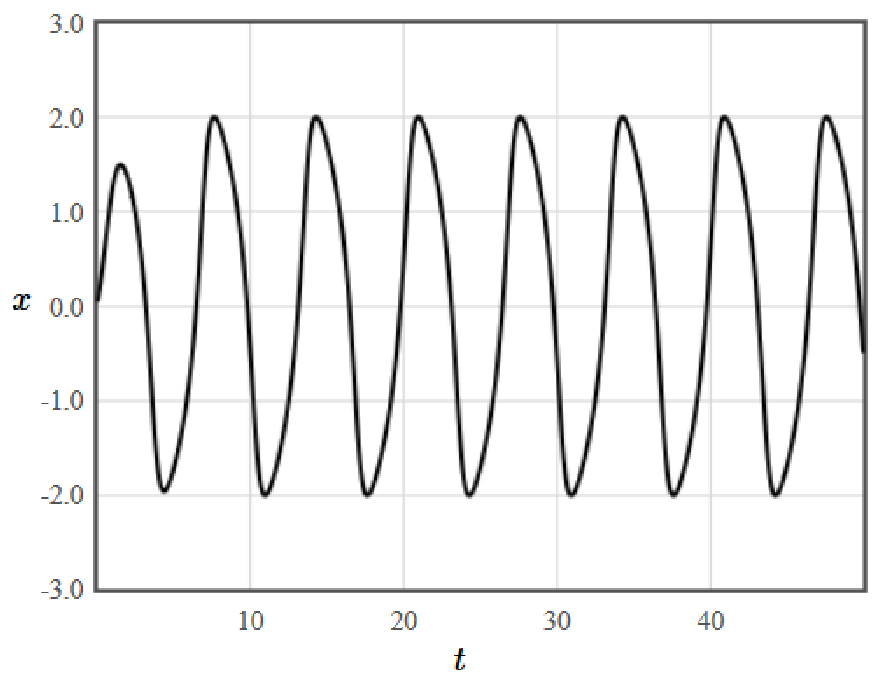
Kind auf einer Schaukel

Nichtlineare Schwinger



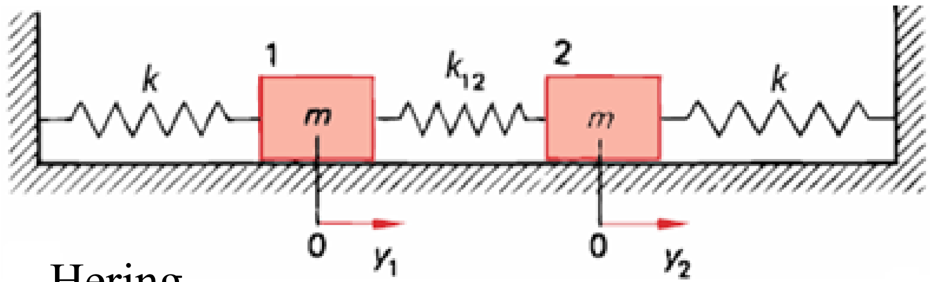
Van-der-Pol-System

$$\frac{d^2x}{dt^2} - \mu\left(1 - x^2\right)\frac{dx}{dt} + x = 0$$



negativen Reibungs für kleine *x*

gekoppeltes Schwingungssystem



Hering

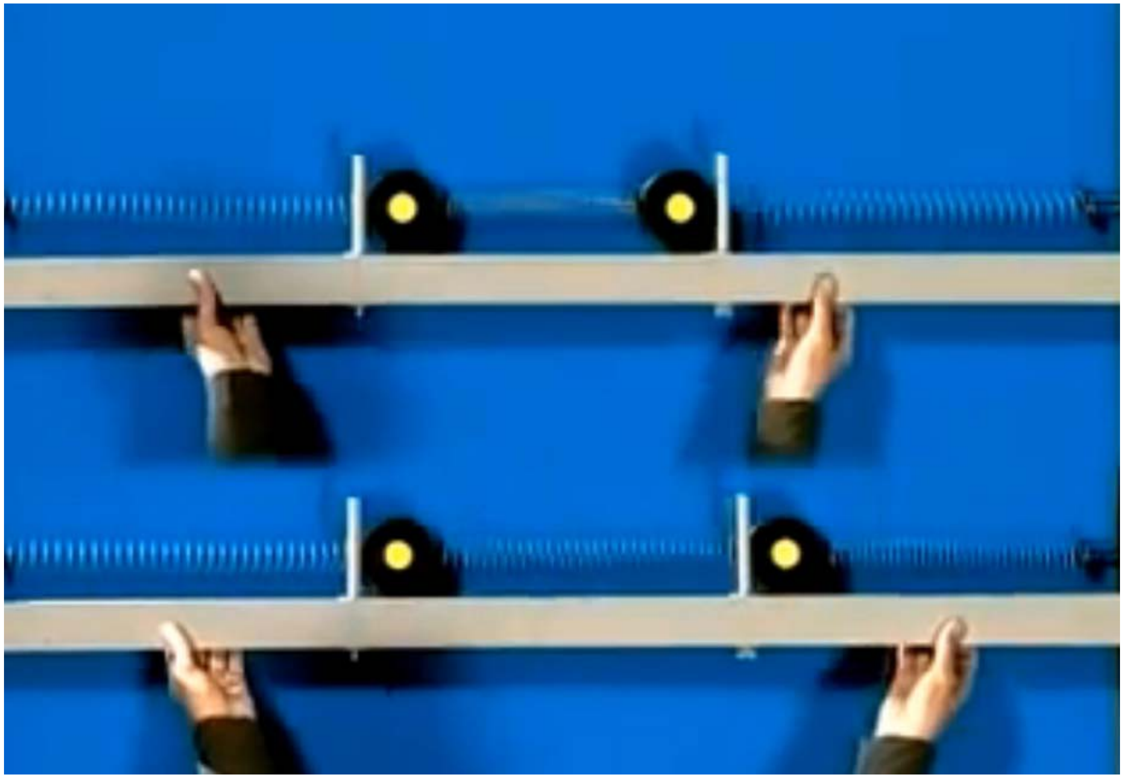
$$M\frac{d^2y_1}{dt^2} = -ky_1 + k_{12}\left(y_2 - y_1\right)$$

$$M\frac{d^2y_2}{dt^2} = -ky_2 + k_{12}\left(y_1 - y_2\right)$$

Eigenmoden: harmonische Bewegung

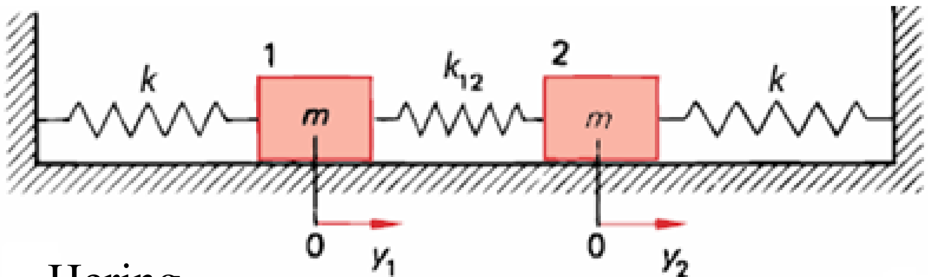
Alle Teile schwingen mit der gleichen Frequenz

zwei Eigenmoden



https://www.youtube.com/watch?v=8RV3gXm6j2I

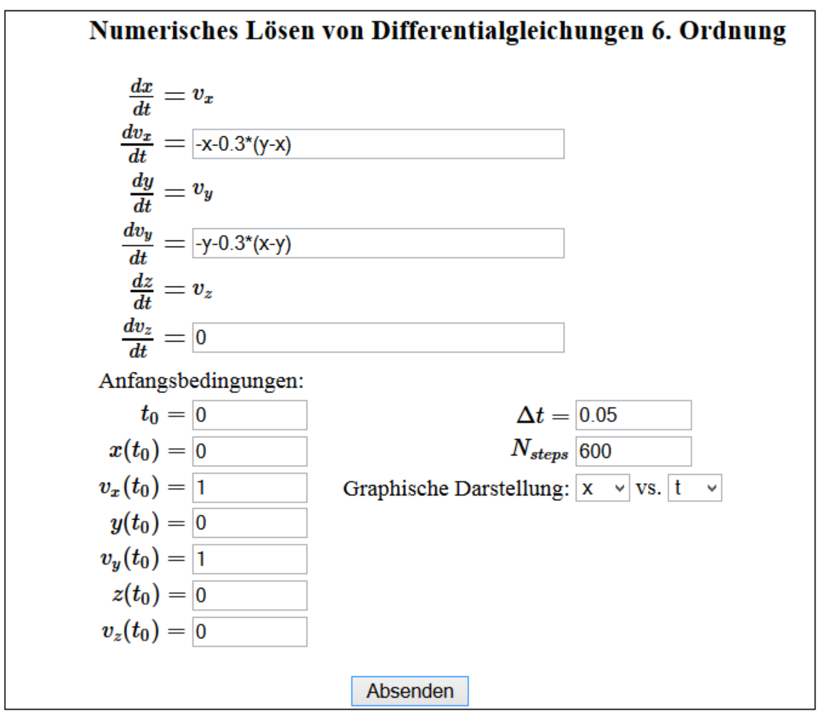
gekoppeltes Schwingungssystem



Hering

$$M\frac{d^2y_1}{dt^2} = -ky_1 + k_{12}\left(y_2 - y_1\right)$$

$$M\frac{d^2y_2}{dt^2} = -ky_2 + k_{12}\left(y_1 - y_2\right)$$



Numerisches Lösen von Differentialgleichungen 6. Ordnung

$\frac{dx}{dt} = v_x$

$\frac{dv_x}{dt} =$ -x-0.3*(y-x)

$\frac{dy}{dt} = v_y$

$\frac{dv_y}{dt} =$ -y-0.3*(x-y)

$\frac{dz}{dt} = v_z$

$\frac{dv_z}{dt} =$ 0

Anfangsbedingungen:

$t_0 =$ 0

$x(t_0) =$ 0

$v_x(t_0) =$ 1

$y(t_0) =$ 0

$v_y(t_0) =$ 1

$z(t_0) =$ 0

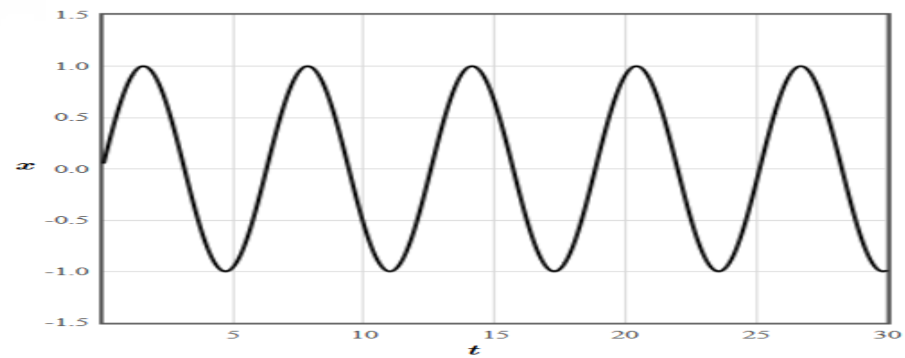
$v_z(t_0) =$ 0

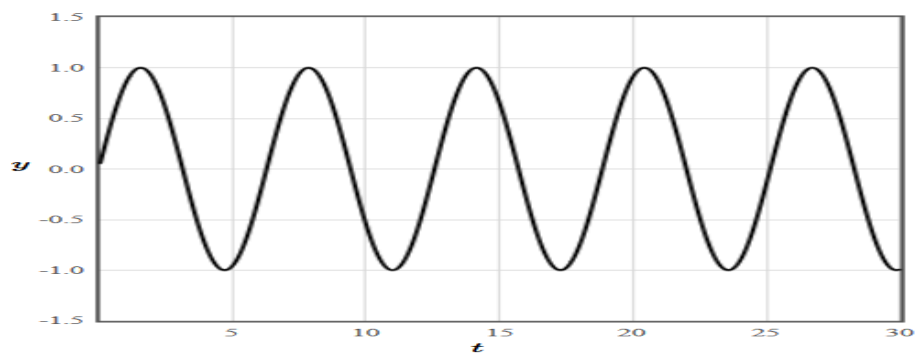
$\Delta t =$ 0.05

N_{steps} 600

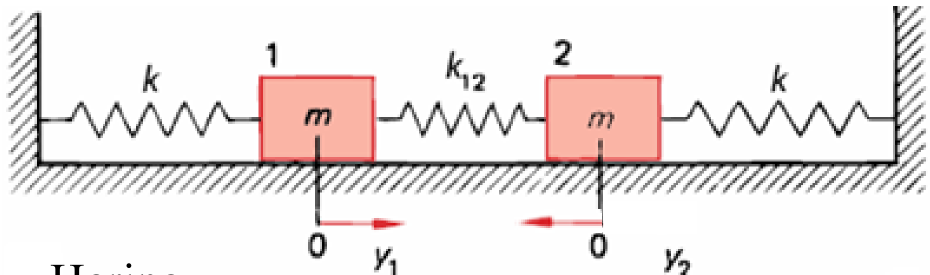
Graphische Darstellung: x vs. t

Absenden





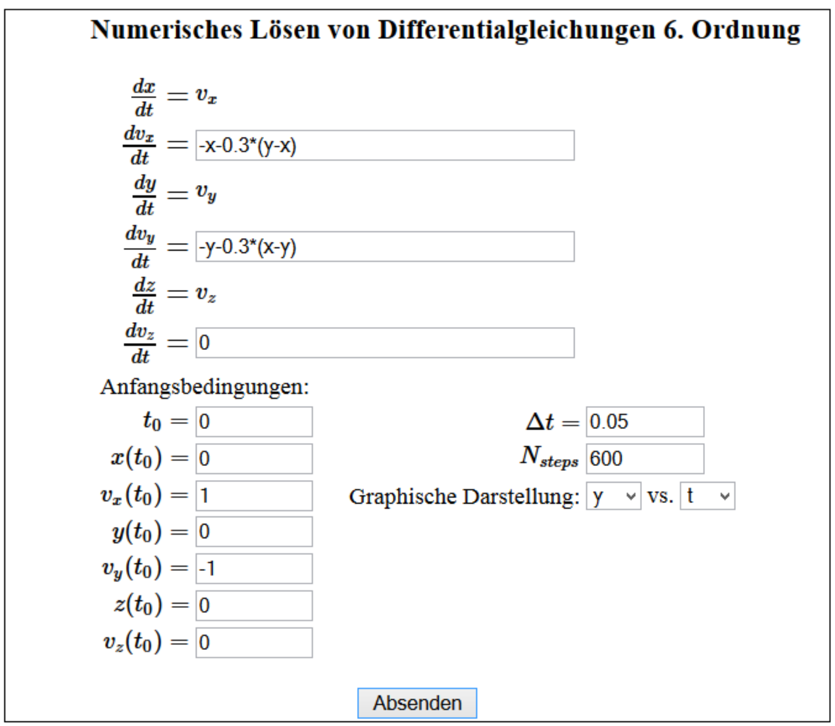
gekoppeltes Schwingungssystem



Hering

$$M\frac{d^2y_1}{dt^2} = -ky_1 + k_{12}\left(y_2 - y_1\right)$$

$$M\frac{d^2y_2}{dt^2} = -ky_2 + k_{12}\left(y_1 - y_2\right)$$



Numerisches Lösen von Differentialgleichungen 6. Ordnung

$\frac{dx}{dt} = v_x$

$\frac{dv_x}{dt} =$ -x-0.3*(y-x)

$\frac{dy}{dt} = v_y$

$\frac{dv_y}{dt} =$ -y-0.3*(x-y)

$\frac{dz}{dt} = v_z$

$\frac{dv_z}{dt} =$ 0

Anfangsbedingungen:

$t_0 =$ 0

$x(t_0) =$ 0

$v_x(t_0) =$ 1

$y(t_0) =$ 0

$v_y(t_0) =$ -1

$z(t_0) =$ 0

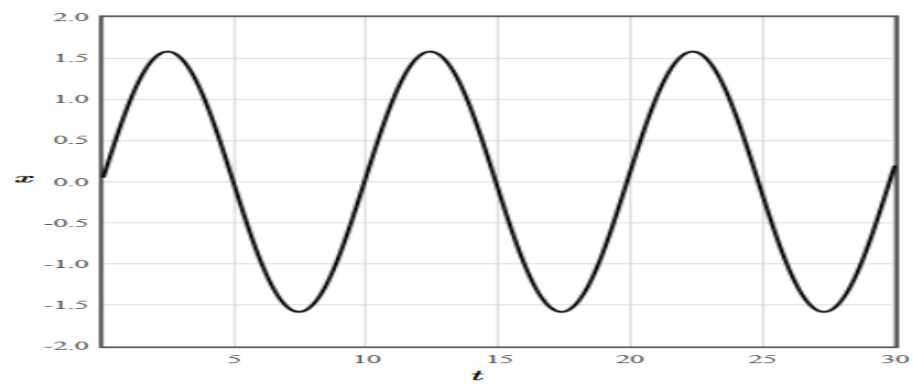
$v_z(t_0) =$ 0

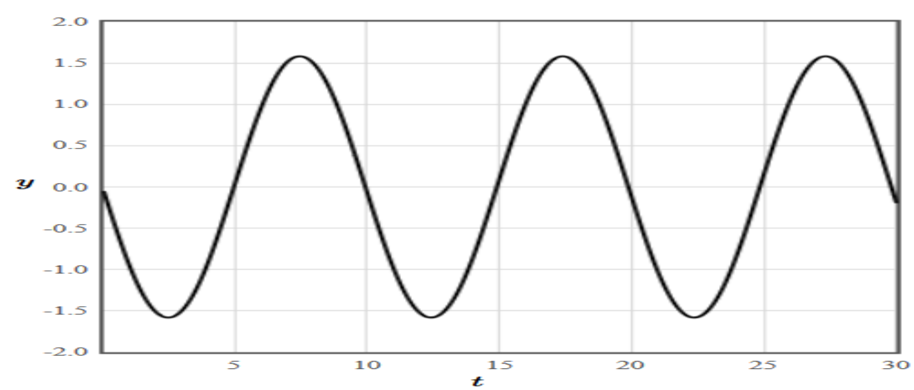
$\Delta t =$ 0.05

N_{steps} 600

Graphische Darstellung: y vs. t

Absenden

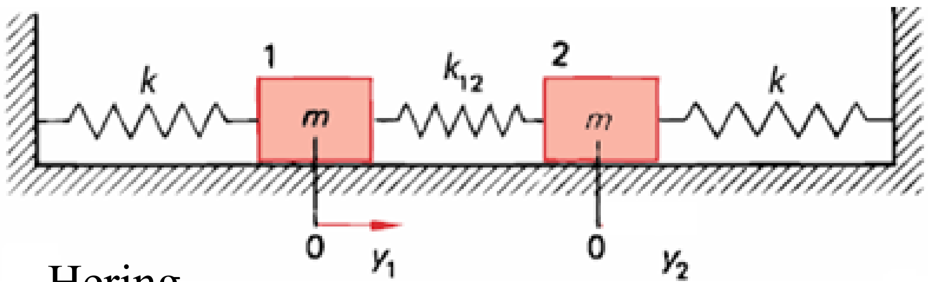




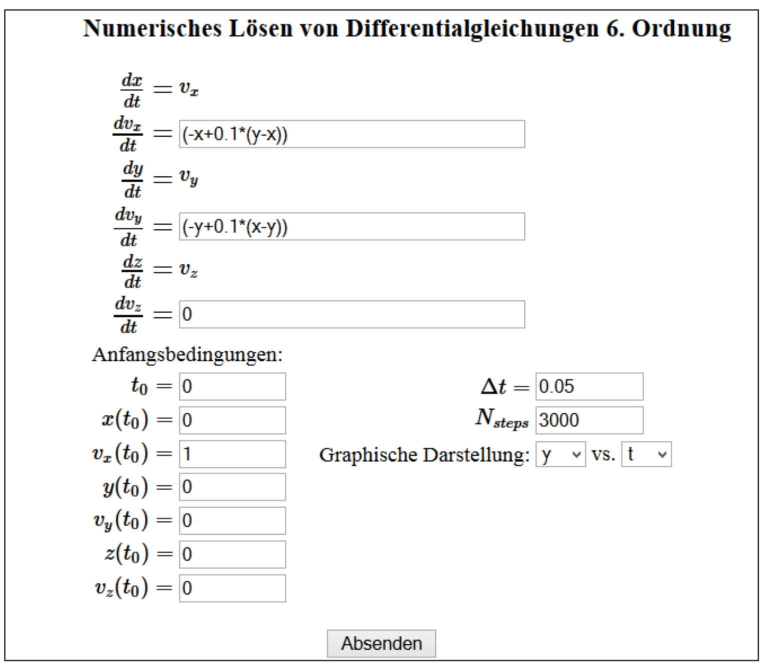
gekoppeltes Schwingungssystem

$$M\frac{d^2y_1}{dt^2} = -ky_1 + k_{12}\left(y_2 - y_1\right)$$

$$M\frac{d^2y_2}{dt^2} = -ky_2 + k_{12}\left(y_1 - y_2\right)$$



Hering



Numerisches Lösen von Differentialgleichungen 6. Ordnung

$\frac{dx}{dt} = v_x$

$\frac{dv_x}{dt} =$ (-x+0.1*(y-x))

$\frac{dy}{dt} = v_y$

$\frac{dv_y}{dt} =$ (-y+0.1*(x-y))

$\frac{dz}{dt} = v_z$

$\frac{dv_z}{dt} =$ 0

Anfangsbedingungen:

$t_0 =$ 0 | $\Delta t =$ 0.05

$x(t_0) =$ 0 | N_{steps} 3000

$v_x(t_0) =$ 1 | Graphische Darstellung: y vs. t

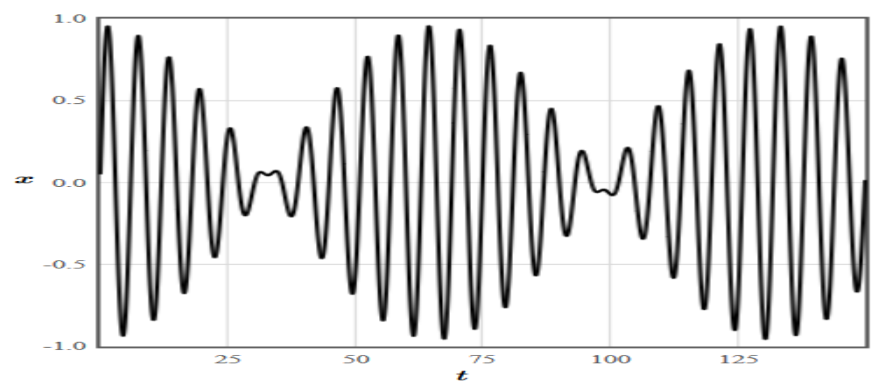
$y(t_0) =$ 0

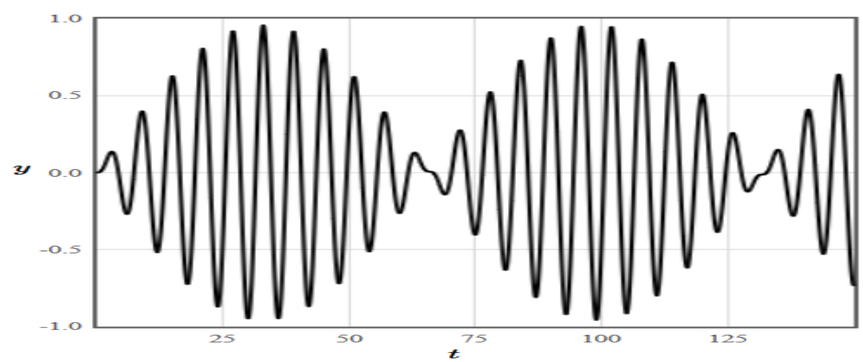
$v_y(t_0) =$ 0

$z(t_0) =$ 0

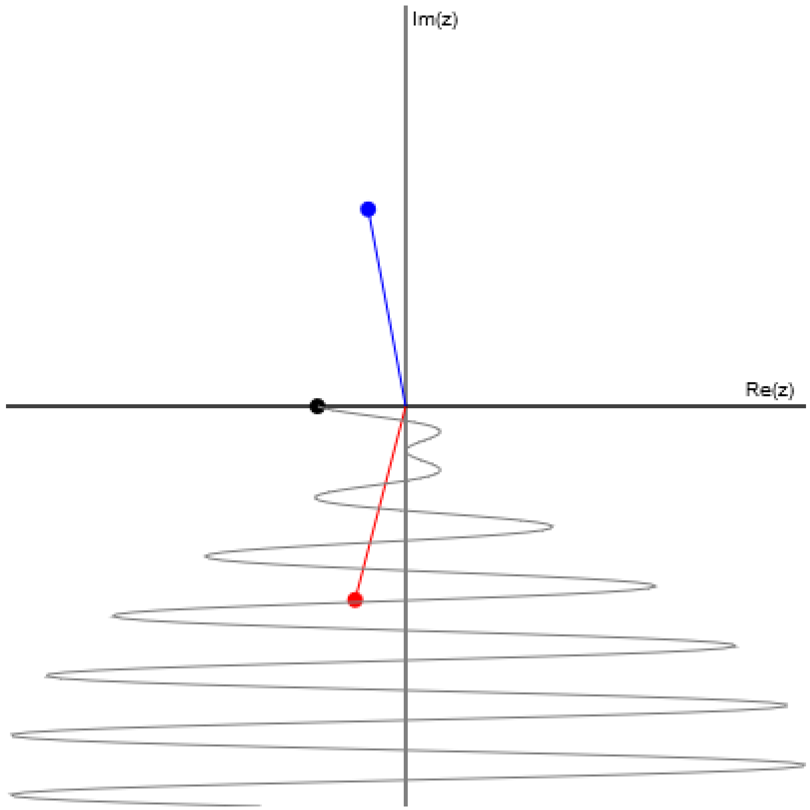
$v_z(t_0) =$ 0

Absenden





Schwebung



$$x = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t).$$

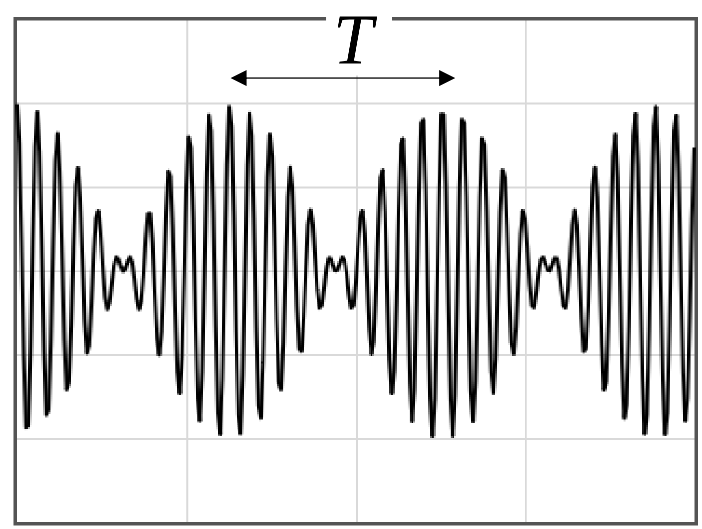
$A_1 = 1$ [m]

$A_2 = 1$ [m]

$\omega_1 = 1$ [rad/s]

$\omega_2 = 1.1$ [rad/s]

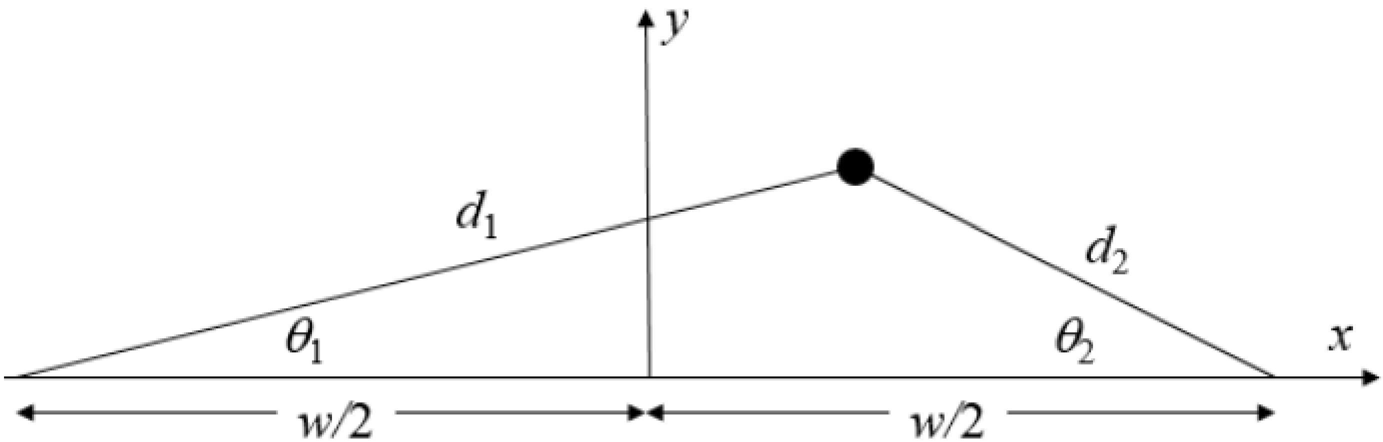
t = 0

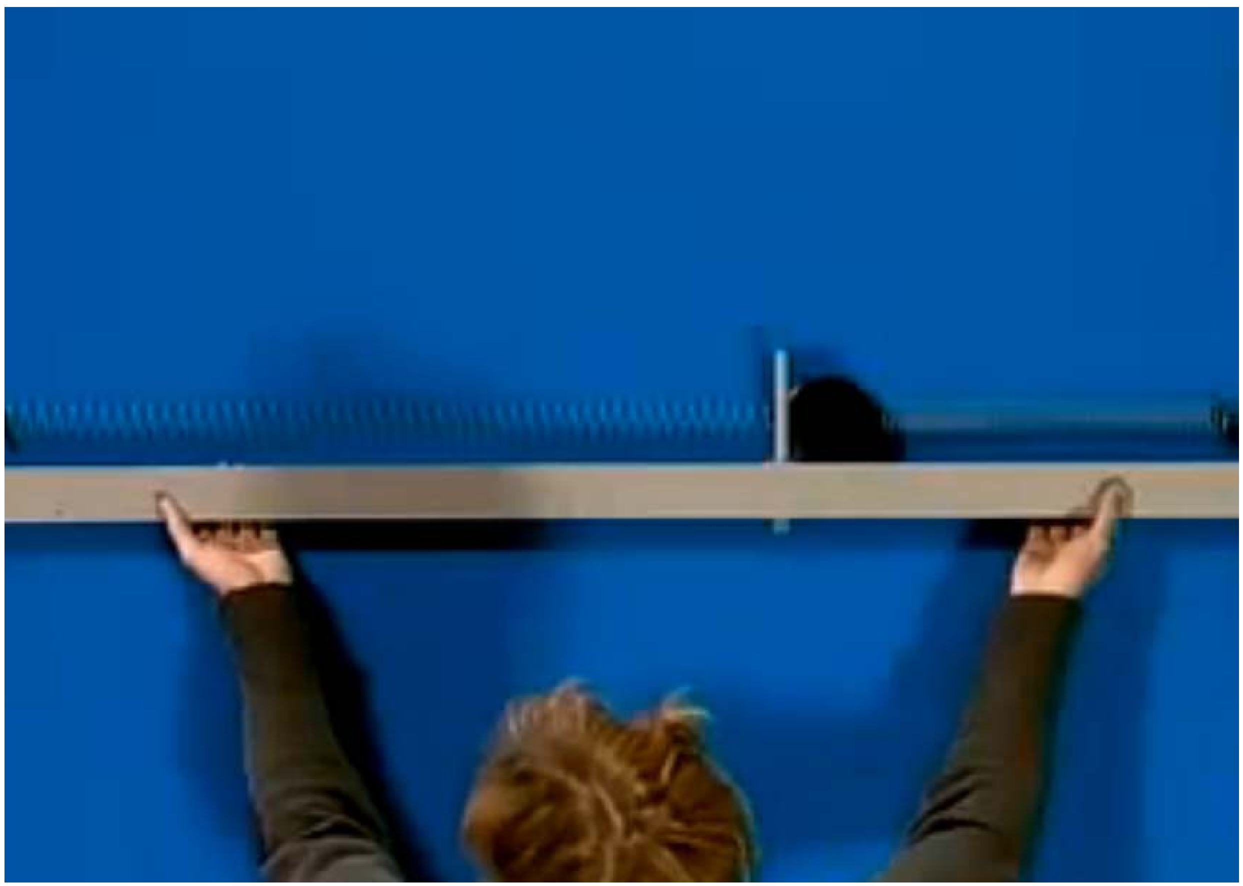


$$\omega_1 T - \omega_2 T = 2\pi$$

$$T = \frac{2\pi}{\omega_1 - \omega_2}$$

http://lamp.tu-graz.ac.at/~hadley/physikm/apps/resonance/schwebung.de.php

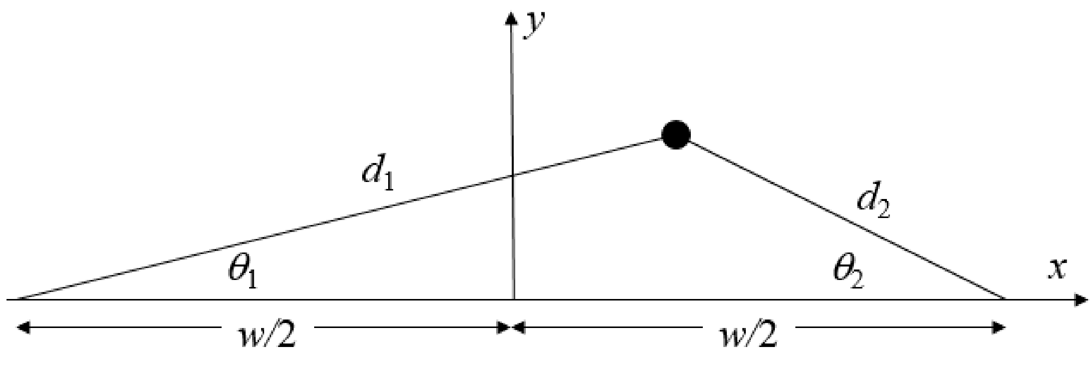




The Open University
https://www.youtube.com/watch?v=VQ5xRhNlQiY

A mass on a frictionless table connected to two springs

A mass m on a frictionless horizonal table is connected to the left and to the right by two springs each with a spring constant k. The length of the unstretched springs is l and the width of the table is $w > 2l$.



The equilibrium position for the mass is in the middle of the table ($x = 0, y = 0$) where the forces exerted by the springs cancel each other out. If the mass is moved away from its equilibrium position then the force on on the mass is,

$$\vec{F} = k\left[(d_2 - l)\cos\theta_2 - (d_1 - l)\cos\theta_1\right]\hat{x} - k\left[(d_2 - l)\sin\theta_2 + (d_1 - l)\sin\theta_1\right]\hat{y}.$$

Here $d_1 = \sqrt{(w/2 + x)^2 + y^2}$, $d_2 = \sqrt{(w/2 - x)^2 + y^2}$, $\theta_1 = \text{atan}\left(\frac{y}{w/2+x}\right)$, and $\theta_2 = \text{atan}\left(\frac{y}{w/2-x}\right)$.

The form below will calculate the trajectory of the mass when it is released from postion (x_0, y_0) at rest.

$m =$ 0.4 kg $k =$ 0.2 N/m $l =$ 0.15 m $w =$ 0.8 m $x_0 =$ 0.15 m $y_0 =$ -0.07 m

Load these equations into the form below

Numerical 6th order differential equation solver

$\frac{dx}{dt} = v_x$

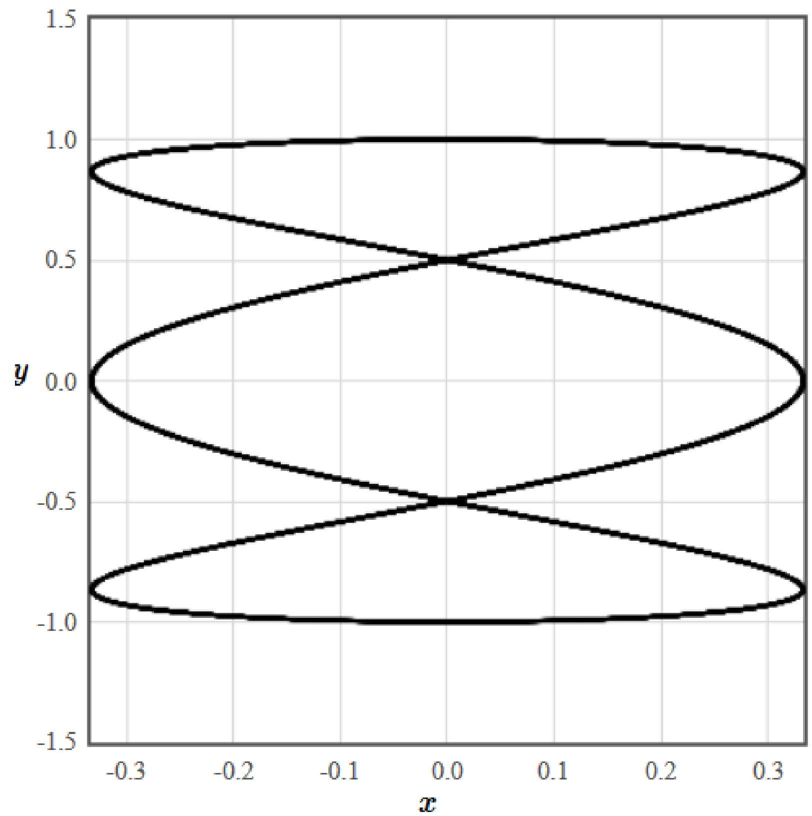
$\frac{dv_x}{dt} =$

```
(0.2*(sqrt((0.4-x)*(0.4-x)+y*y)-
0.15)*cos(atan(y/(0.4-x)))-0.2*(sqrt((0.4+x)*
(0.4+x)+y*y)-0.15)*cos(atan(y/(0.4+x))))/(0.4)
```

Überlagerung harmonischer Schwingungen mit ganzzahligem Frequenzverhältnis, die senkrecht zueinander schwingen (Lissajous-Figuren)

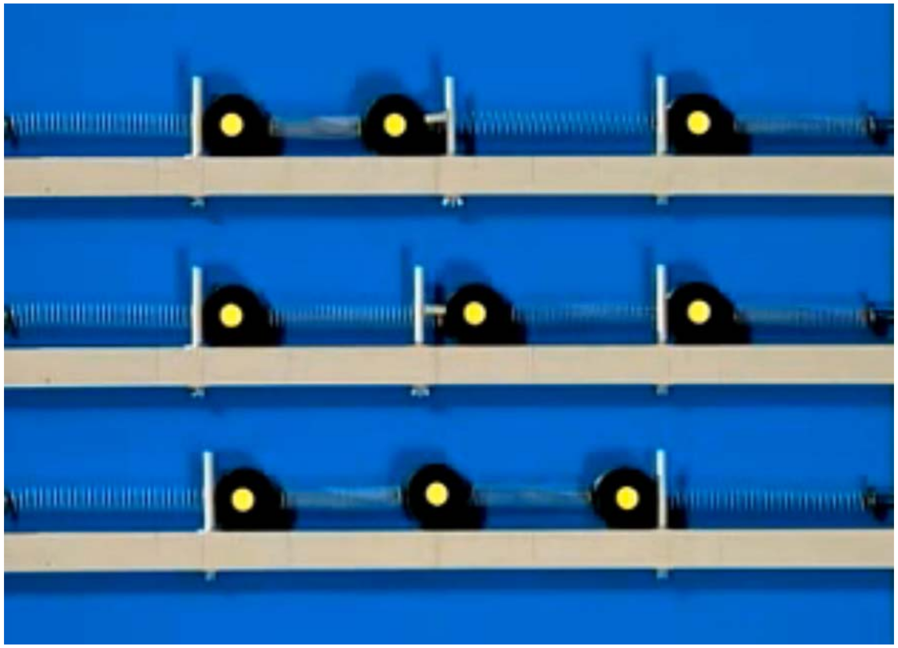
$$M\frac{d^2x}{dt^2} = -k_2 x$$

$$M\frac{d^2y}{dt^2} = -k_2 y$$

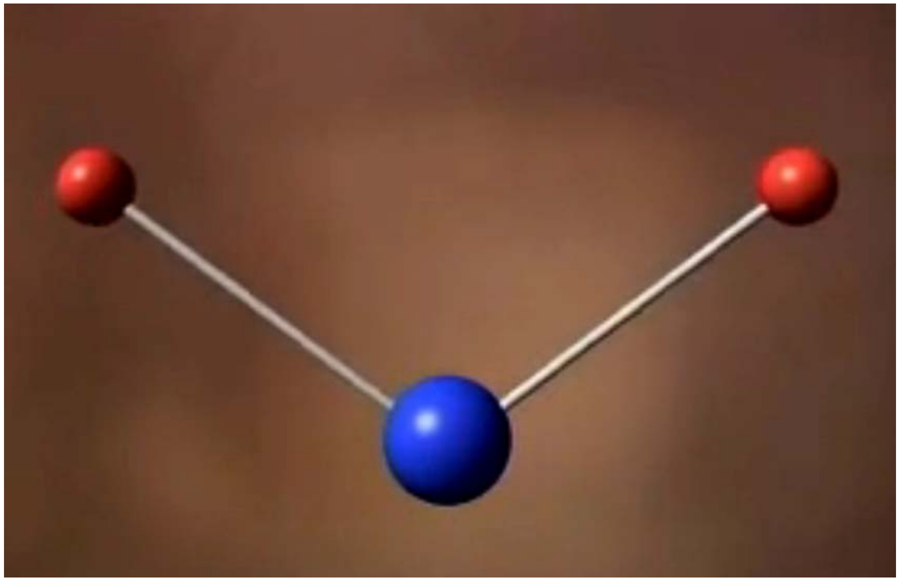


3 Eigenmoden

jede Bewegung kann als eine Superposition von Eigenmoden geschrieben werden.



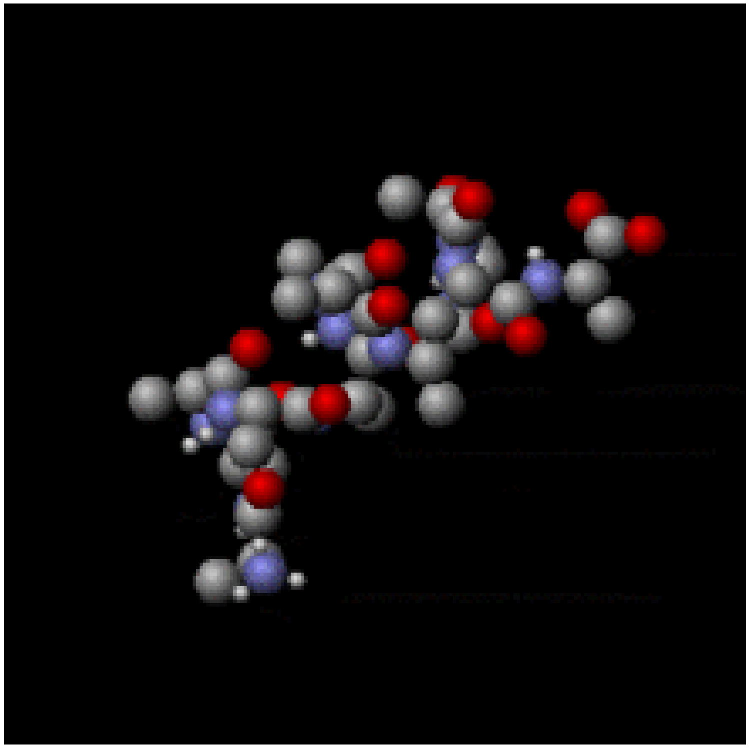
Eigenmoden:
harmonische Bewegung
Alle Teile schwingen mit der gleichen Frequenz



Open University
https://www.youtube.com/watch?v=jhGhSXAqXmU

Eigenmoden

jede Bewegung kann als eine Superposition von Eigenmoden geschrieben werden.



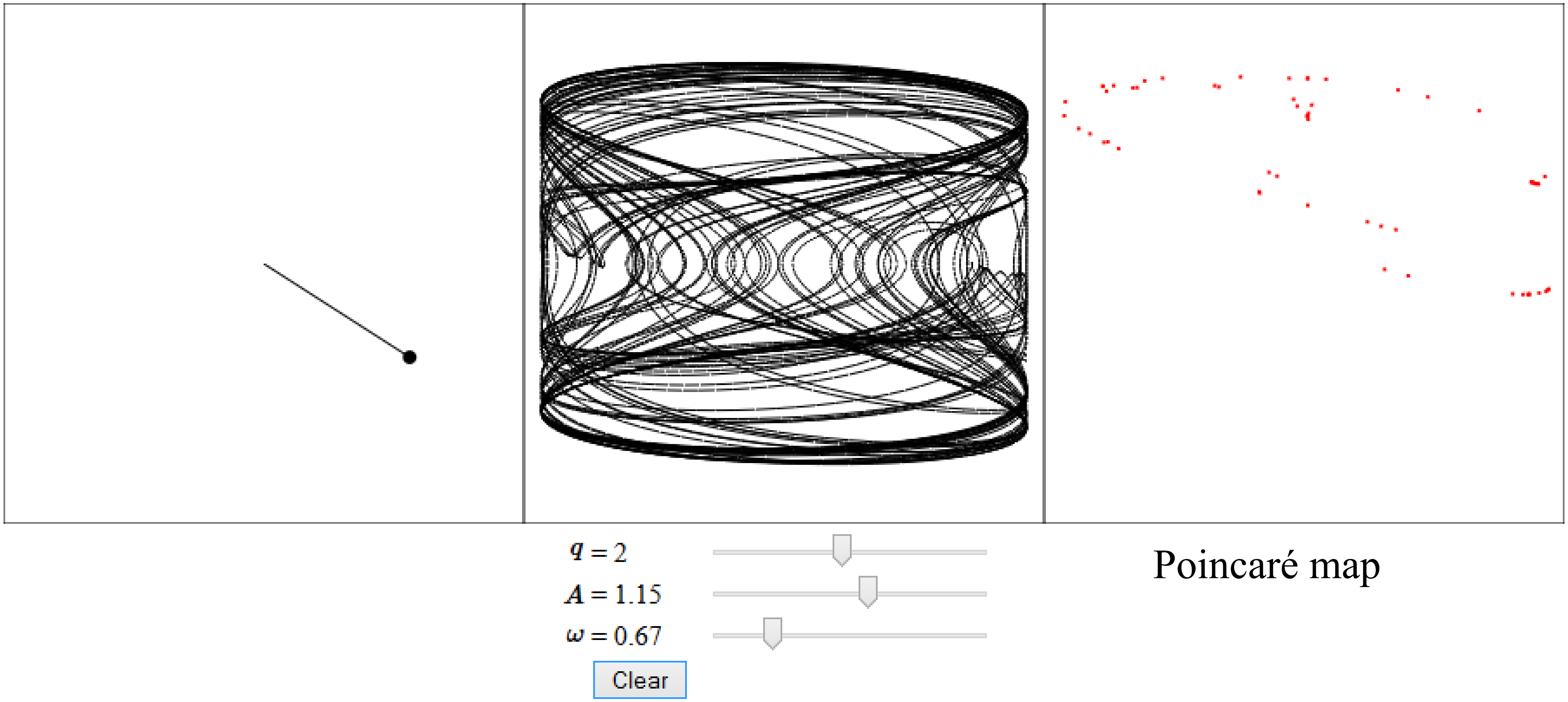
3 Translationsfreiheitsgrade
3 Rotationsfreiheitsgrade

$3N$ - 6 Eigenmoden (Schwingunen)

http://de.wikipedia.org/wiki/Freiheitsgrad

The driven pendulum

$$\frac{d^2\theta}{dt^2} + \frac{1}{q}\frac{d\theta}{dt} + \sin(\theta) = A\cos(\omega t)$$



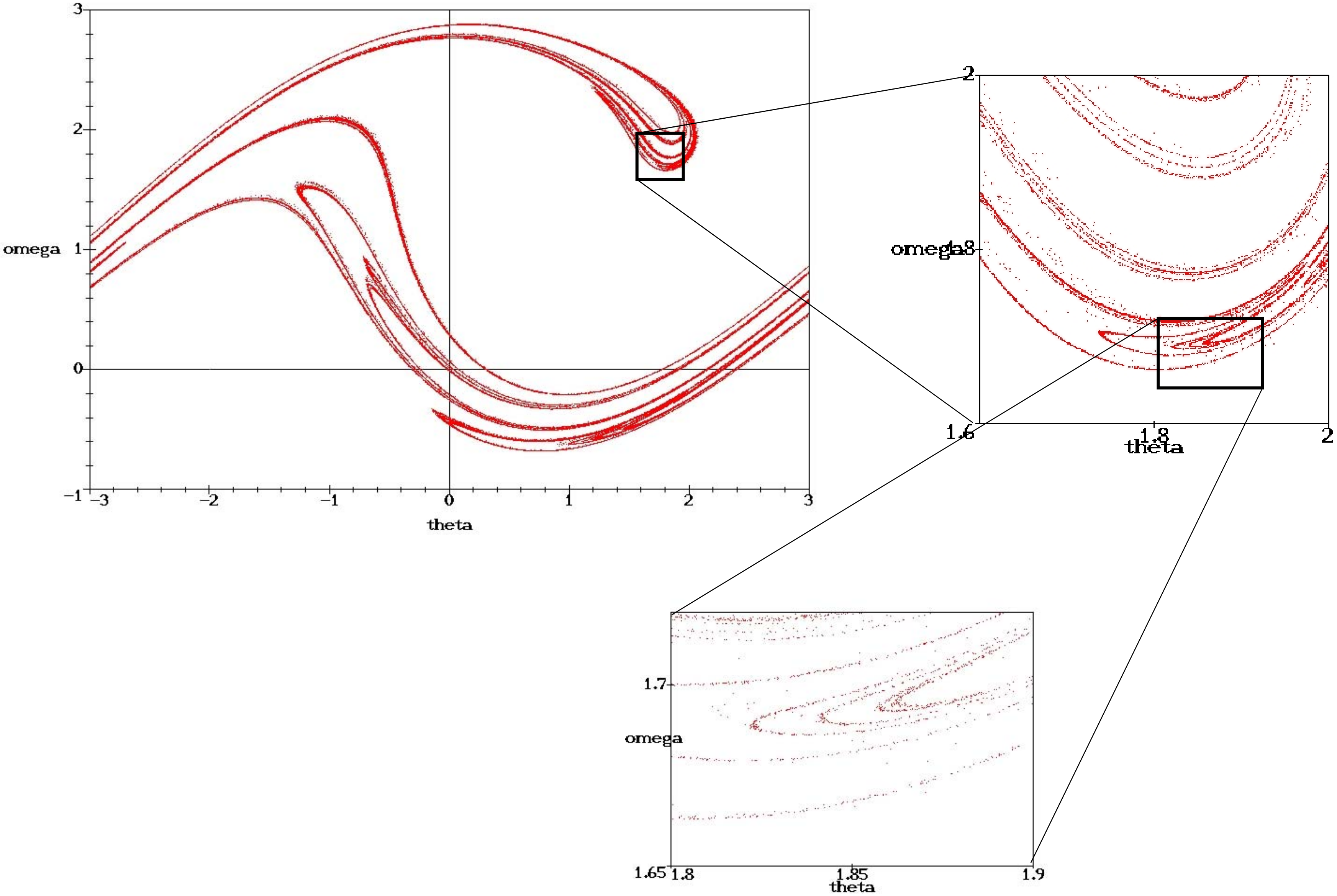
Poincaré map

$A = 0, 0.6, 1.04, 1.08, 1.1, 1.15$

Symmetriebrechung (→ 1.04) — Periodenverdopplung (→ 1.08) — Chaos (→ 1.15)

http://lamp.tu-graz.ac.at/~hadley/physikm/apps/numerical_integration/pendulum_ani.html

Seltsamer Attraktor



http://www.thphys.uni-heidelberg.de/~gasenzer/index.php?n1=teaching&n2=chaos