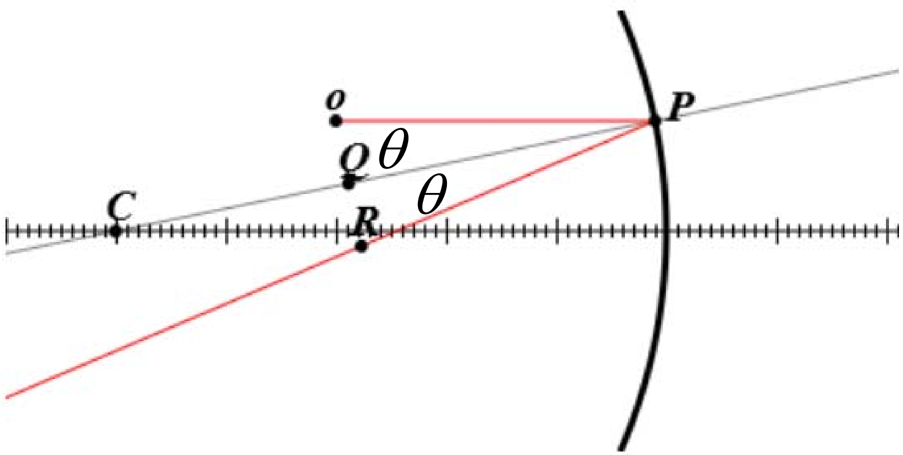
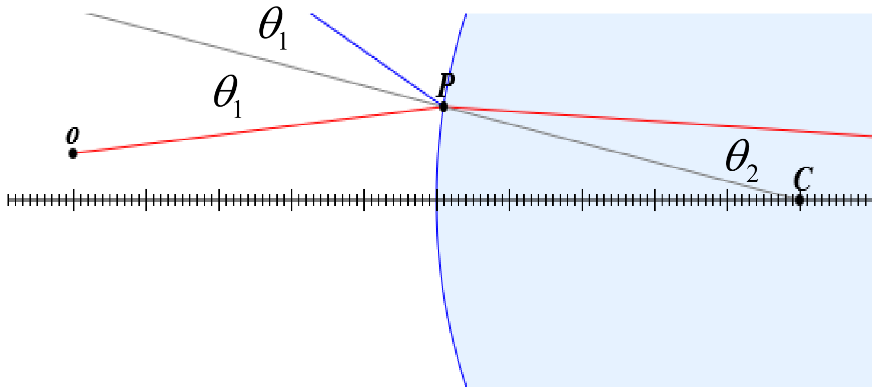
Optik

Geometrische Optik

Spiegel:
Einfallswinkel = Reflexionswinkel



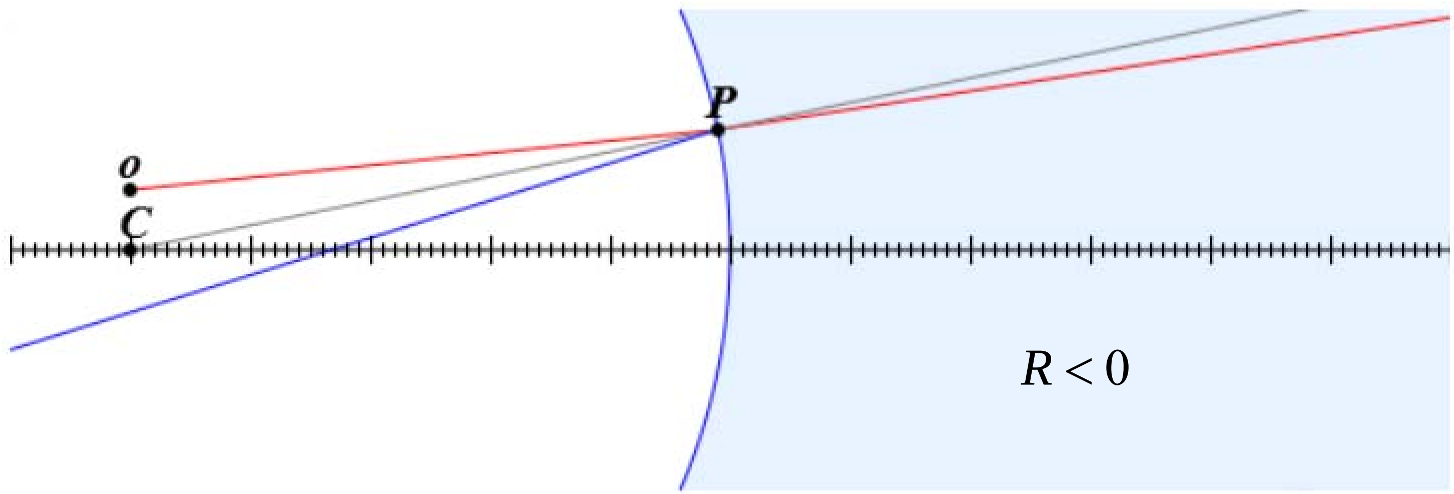
Linsen:
Snelliussches Brechungsgesetz

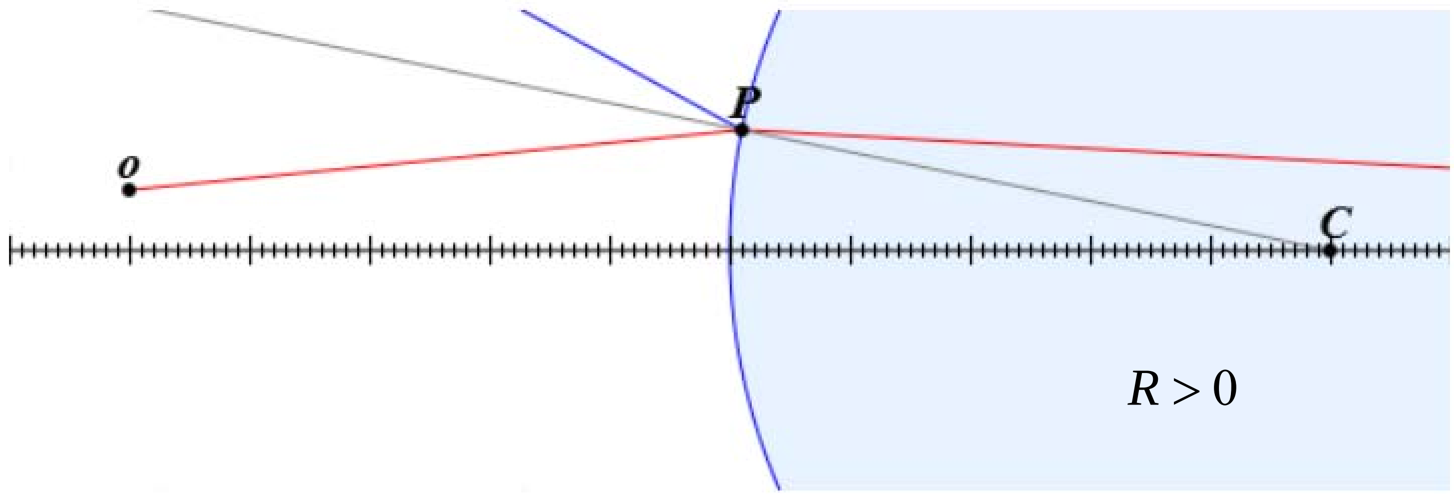


$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Wellenoptik: $L \sim \lambda$
Geometrische Optik: $L >> \lambda$

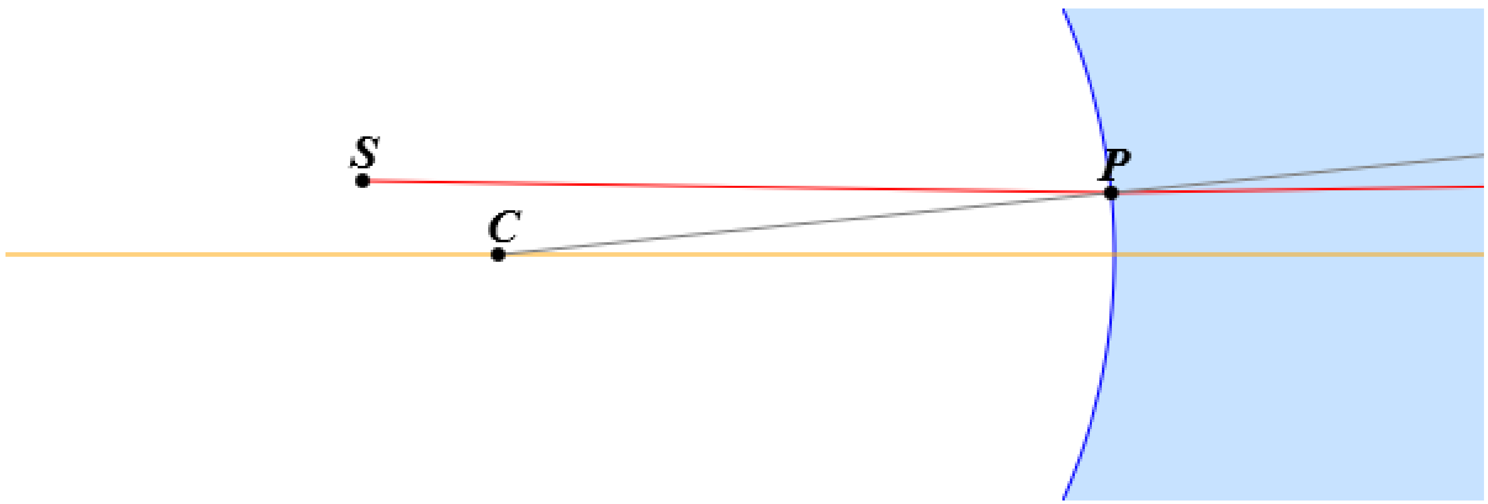
Brechung an einer gekrümmten Grenzfläche





Brechung an einer konkaven Grenzfläche

Eine konkave Grenzfläche sei durch einen Kreis mit dem Radius $R = 5$ cm und dem Mittelpunkt C an $(x_c = 0, y_c = 0)$ gegeben. Ein an der Position S ($x_0 =$ -1.1, $y_0 =$ 0.60) cm emittierter Lichtstrahl trifft auf diese Fläche am Punkt P in der Höhe $y_p =$ 0.50 cm. Der Brechungsindex ist $n_1 = 1$ links und $n_2 =$ 1.4 rechts der Grenzfläche. Wie groß ist der Winkel, welcher von der Normalen auf die Grenzfläche am Punkt P (die C und P verbindende graue Linie) und dem gebrochenen Strahl eingeschlossen wird?



Lösung

Dicke Linsen

$R_1 =$ 10 [cm]

$R_2 =$ -10 [cm]

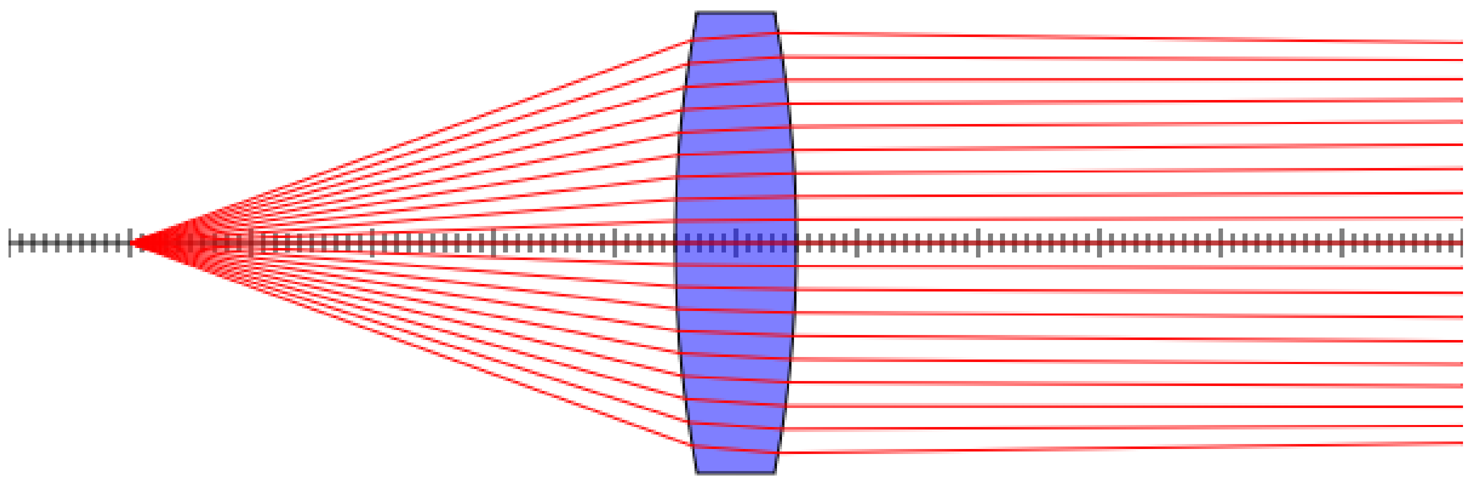
$d =$ 1 [cm]

$x_o =$ -5 [cm]

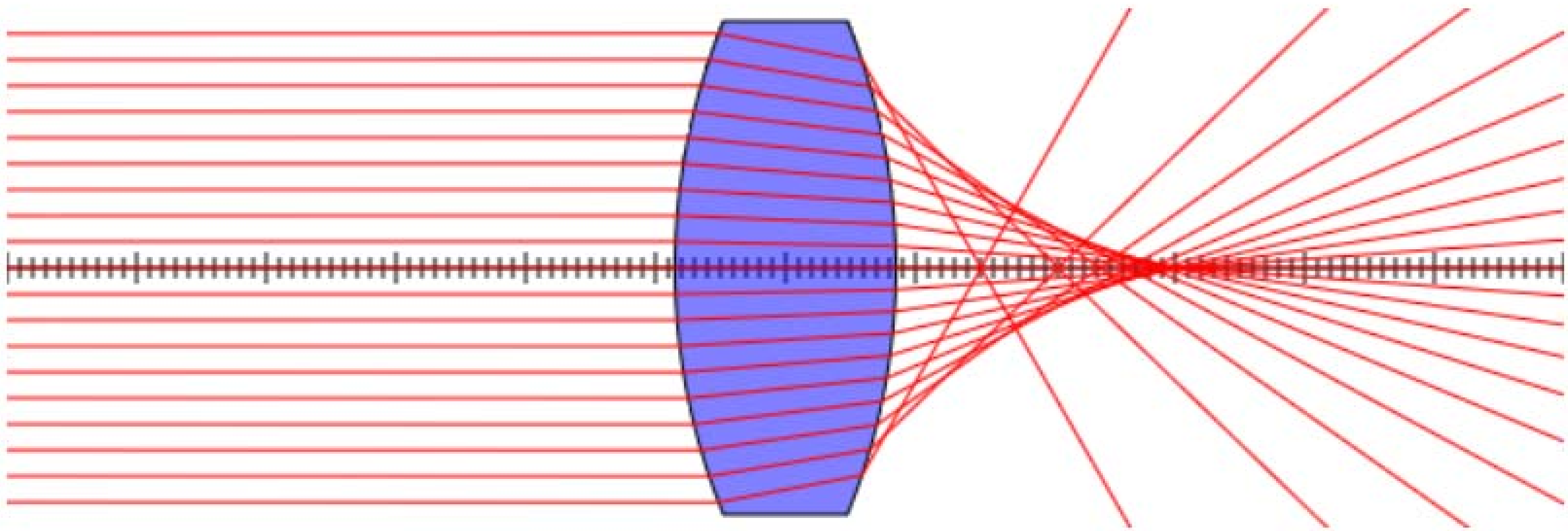
$y_o =$ 0 [cm]

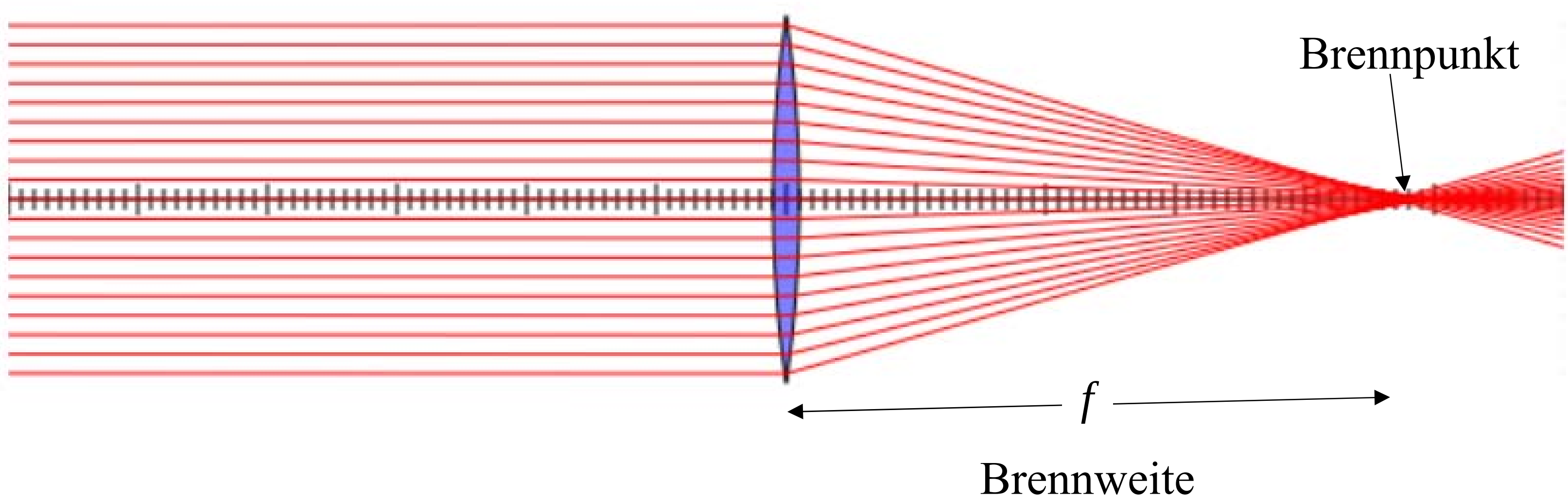
	Rot	Grün	Blau
$n_{\text{Umg}} =$	1	1	1
$n_{\text{Linse}} =$	2	2.5	3

show: ☑ Rot ☐ Grün ☐ Blau plot

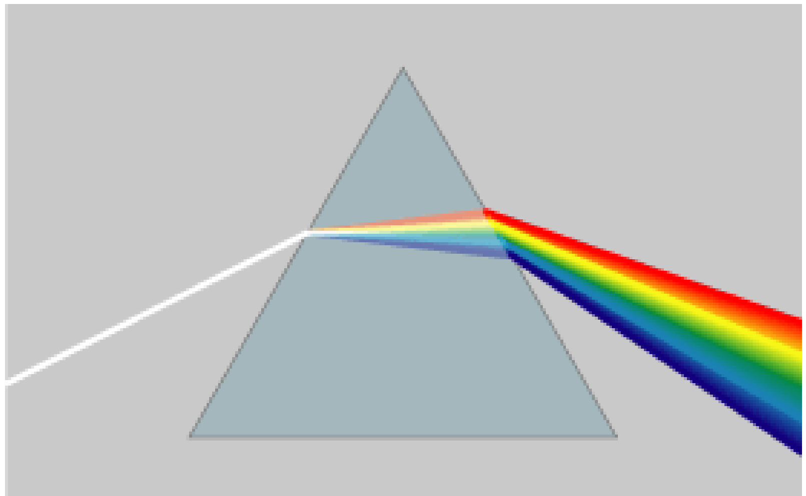


Sphärische Aberration





Dispersion

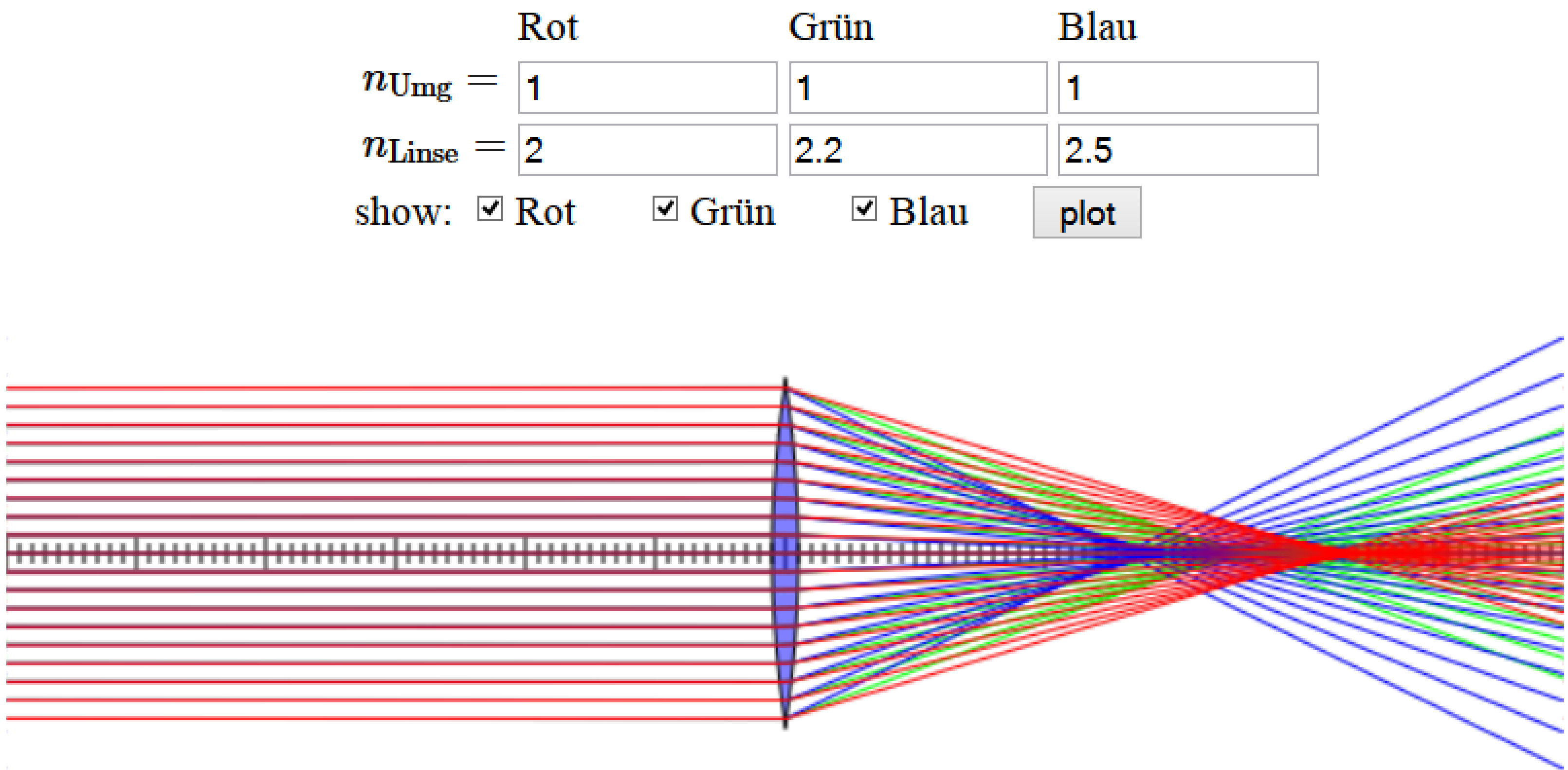


$$n(\lambda)$$

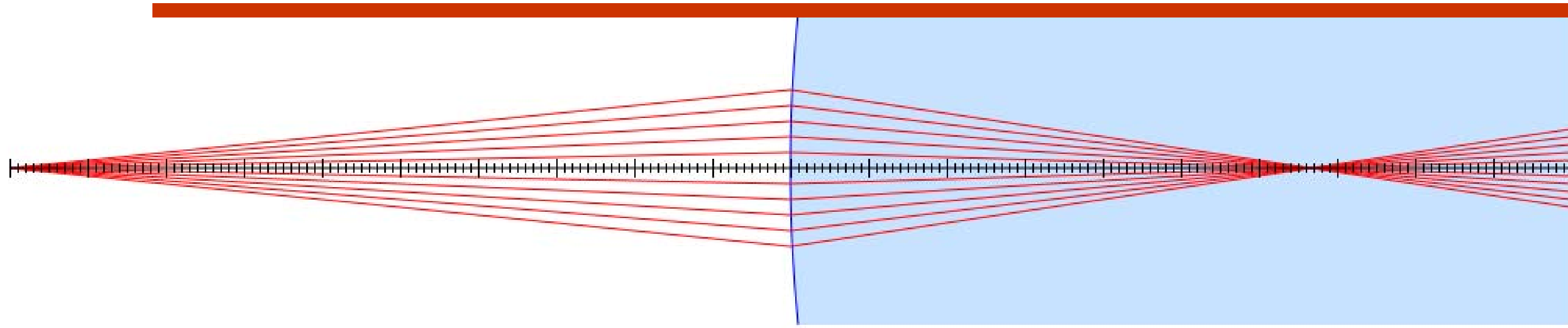
Brechungsindex

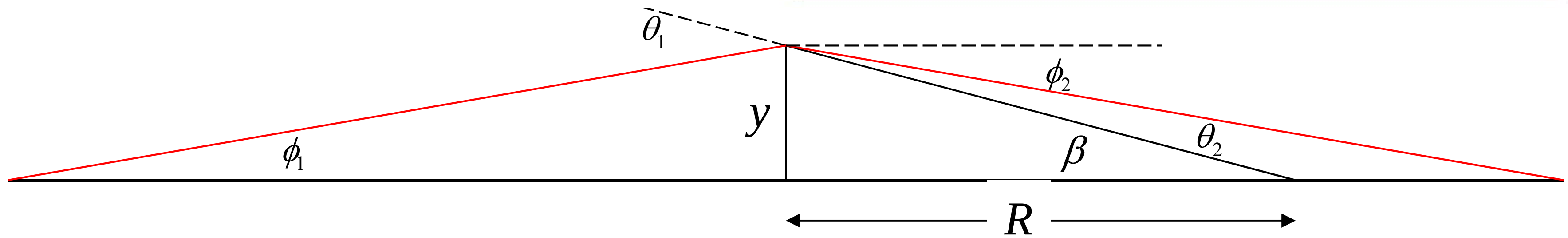
http://en.wikipedia.org/wiki/Dispersion_%28optics%29#mediaviewer/File:Prism_rainbow_schema.png

Chromatische Aberration



kleinen Winkeln zur optischen Achse





$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\downarrow$$

$$n_1\theta_1 \approx n_2\theta_2$$

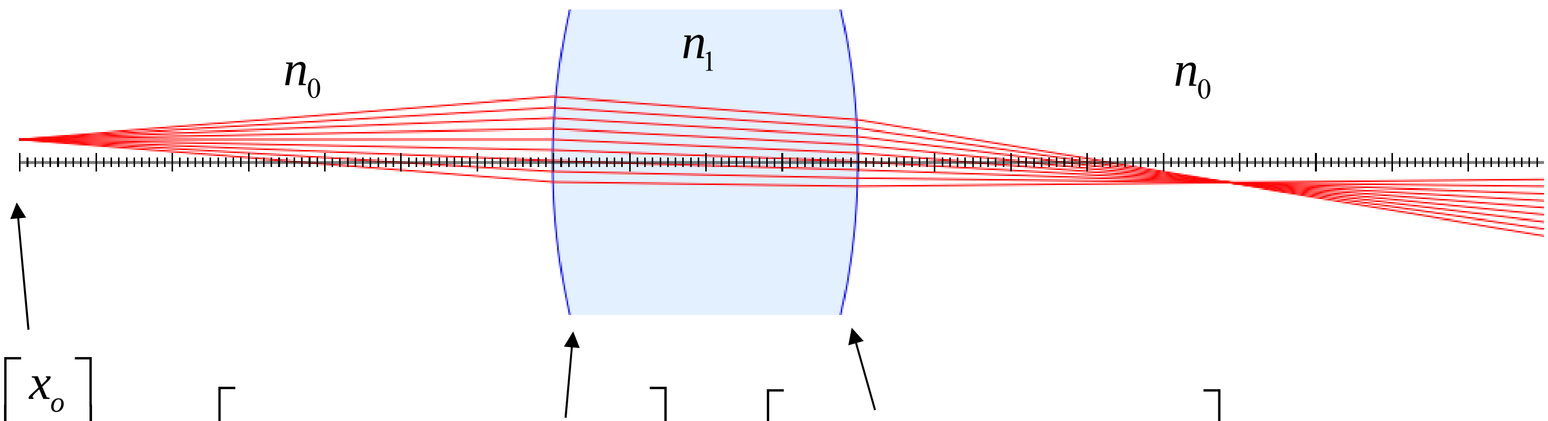
$$\theta_1 = \phi_1 + \beta$$

$$-\phi_2 + \theta_2 = \beta$$

$$\beta \approx \frac{y}{R}$$

$$\boxed{\phi_2 = \frac{n_1 - n_2}{n_2 R}\, y + \frac{n_1}{n_2}\phi_1}$$

Linse (kleinen Winkeln)

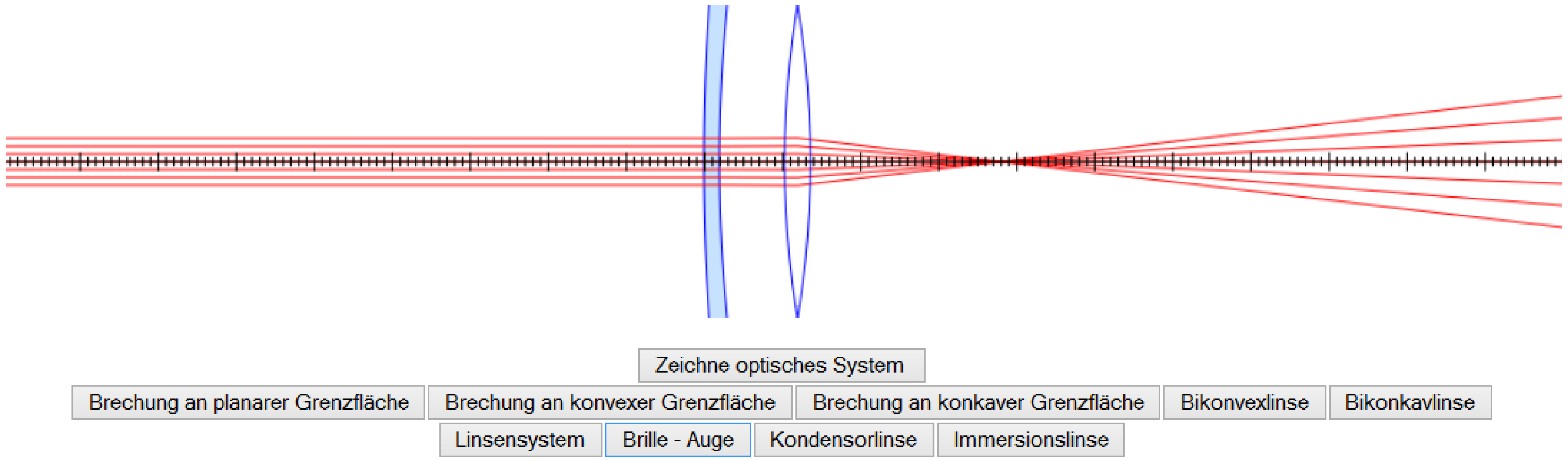


$$\begin{bmatrix} x_o \\ y_o \\ \phi_o \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 = y_o + \phi_o \left(x_1 - x_o \right) \\ \phi_1 = \dfrac{n_0 - n_1}{R_1 n_1} y_1 + \dfrac{n_0}{n_1} \phi_o \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 = y_1 + \phi_1 \left(x_2 - x_1 \right) \\ \phi_2 = \dfrac{n_1 - n_0}{R_2 n_0} y_2 + \dfrac{n_1}{n_0} \phi_1 \end{bmatrix}$$

Ray tracing mittels Transfermatrixmethode

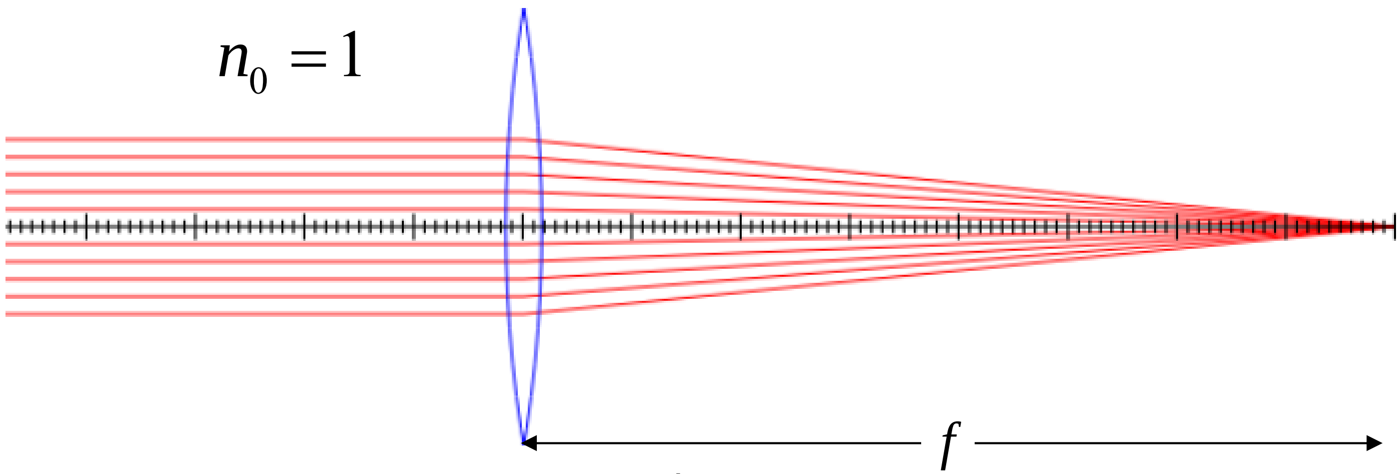


zwischen Grenzflächen $$y_{i+1} = y_i + \phi_i (x_{i+1} - x_i)$$

bei Grenzfläche $$\phi_{i+1} = \frac{n_1 - n_2}{n_2 R} y_i + \frac{n_1}{n_2} \phi_i$$

dünne Linsen (kleinen Winkeln)

$$n_0 = 1$$



Brennweite

$$\phi_1 = \frac{1-n_1}{R_1} y + \frac{\phi_0}{n_1}$$

$$\phi_2 = \frac{n_1 - 1}{R_2} y + n\left(\frac{1-n_1}{R_1} y + \frac{\phi_0}{n_1}\right)$$

$$\boxed{\phi_{i+1} = -\frac{y_i}{f} + \phi_i}$$

$$\frac{1}{f} = (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Abbildungsgleichung für dünne Linsen

$f =$ 2 [cm]

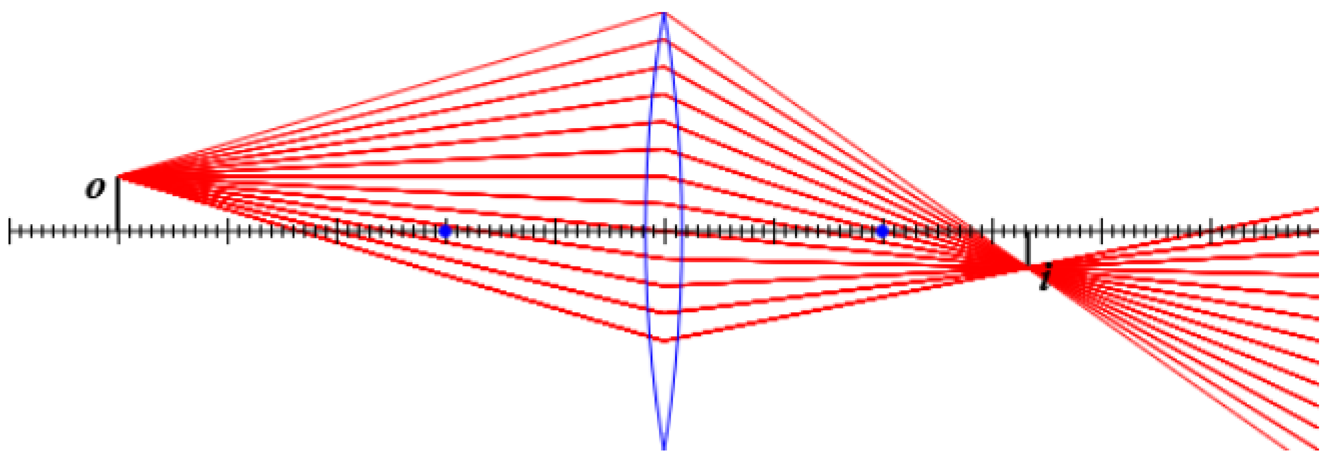
$x_o =$ -5 [cm]

$y_o =$ 0.5 [cm]

$x_i =$ 3.33333 [cm] $D =$ 50.0000 [m^{-1}]

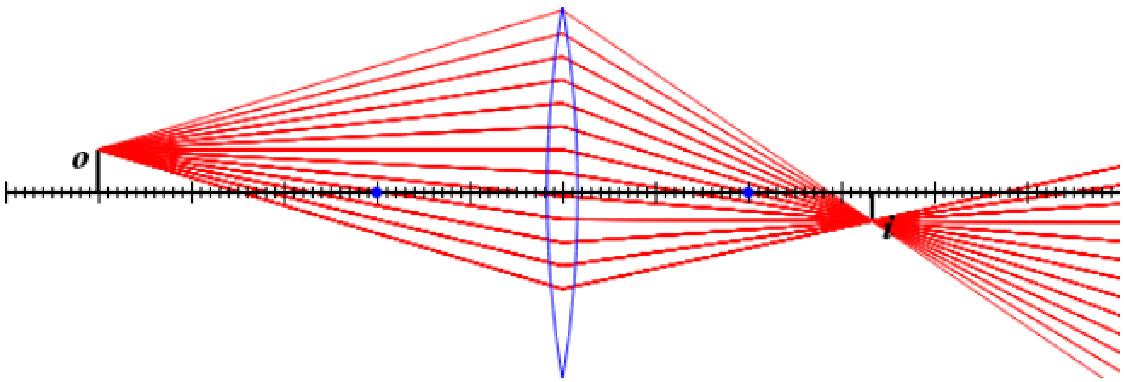
$y_i =$ -0.333333 [cm] $m =$ -0.666667

plot



$$-\frac{1}{x_o} + \frac{1}{x_i} = \frac{1}{f}$$

Abbildungsgleichung für dünne Linsen



$$x_i = \frac{f x_o}{f + x_o}$$

$$y_i = y_o \left(\frac{f}{f + x_o} \right)$$

↓

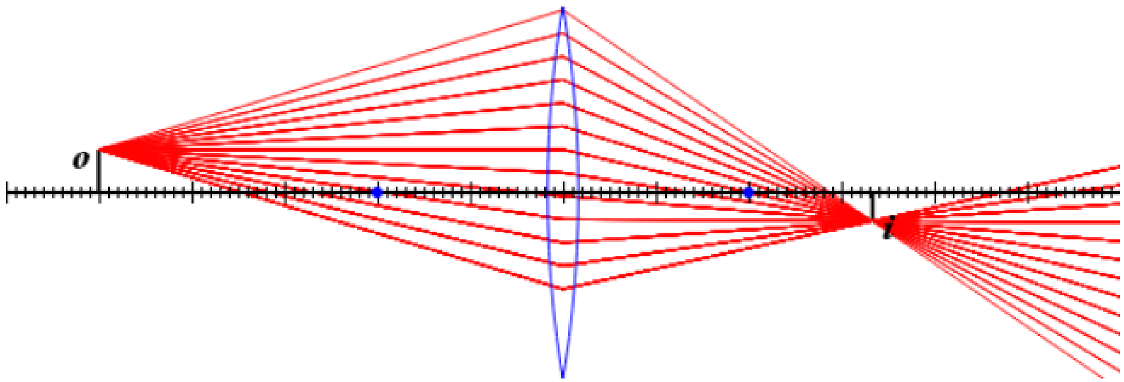
$$\frac{1}{x_i} - \frac{1}{x_o} = \frac{1}{f}$$

$$m = \frac{y_i}{y_o} = \left(\frac{f}{f + x_o} \right)$$

dünne Linsen

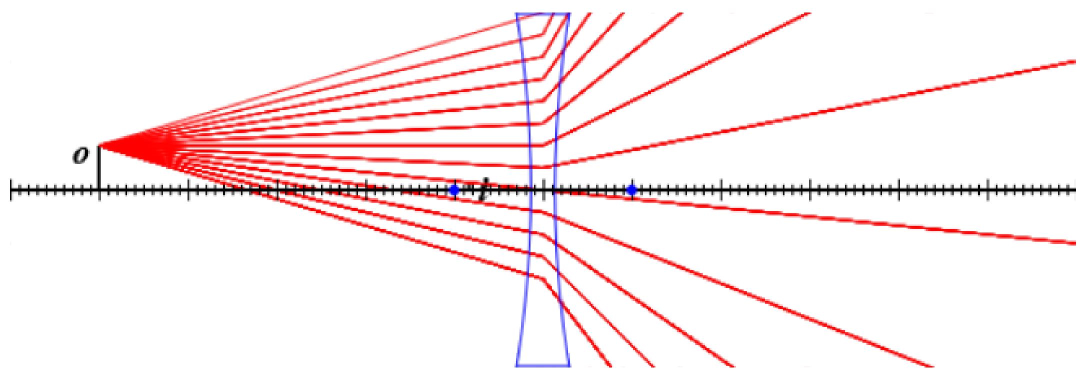
Sammellinse

$f > 0$



Zerstreuungslinse

$f < 0$



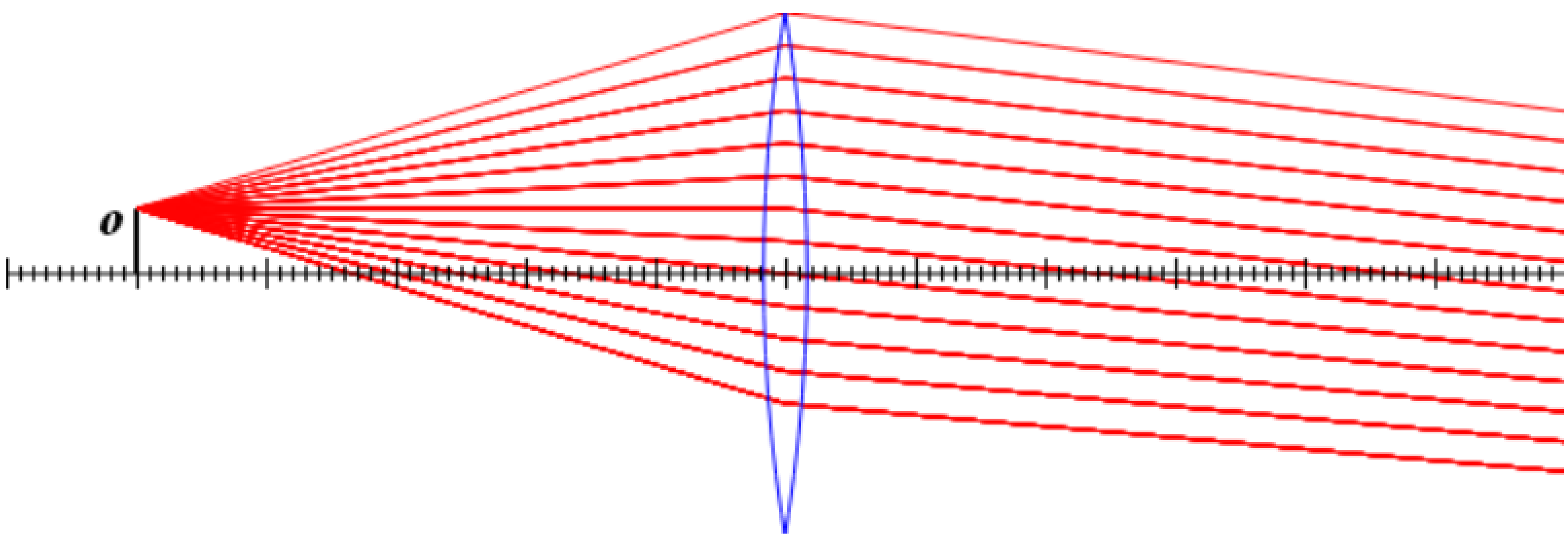
Brennweite

$x_o =$ -5 [cm] xo ← xo →

$y_o =$ 0.5 [cm] yo ↑ yo ↓

$x_i =$ 78.3333 [cm]

plot



Wie groß ist die Brennweite dieser Linse (in cm, gerundet auf die erste Nachkommastelle)?

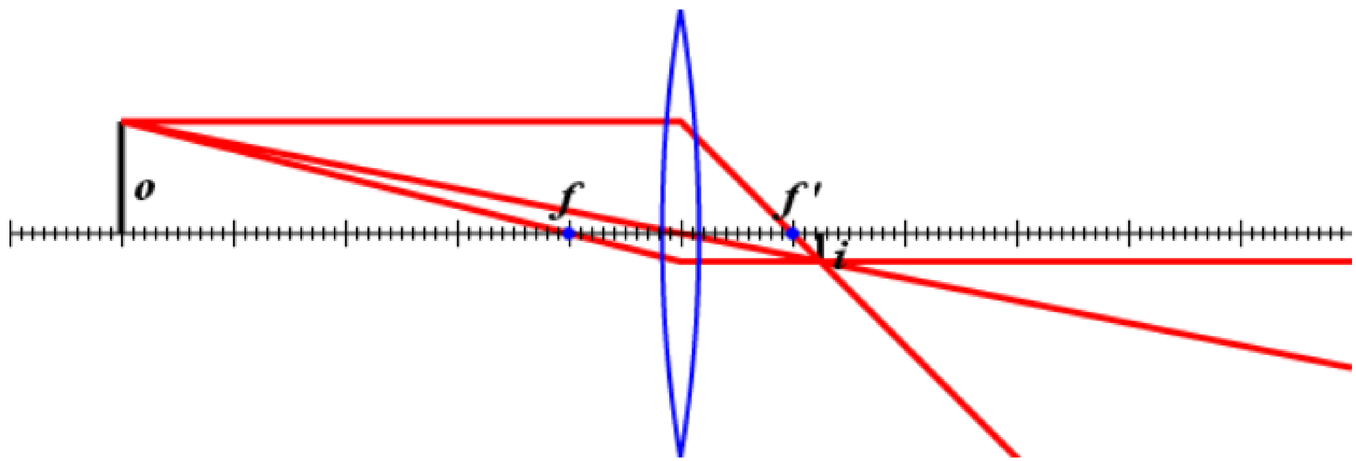
$f =$ [cm] Lösung

Reelle und virtuelle Bilder

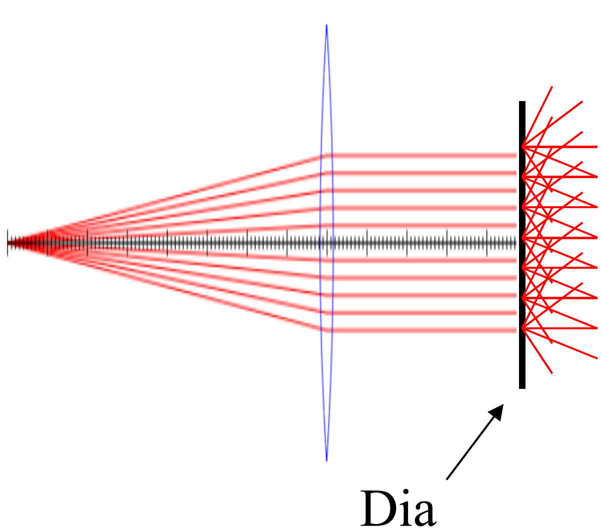
$f =$	1	[cm]	f↑	f↓		
$x_o =$	-5	[cm]	xo ←	xo →		
$y_o =$	1	[cm]	yo ↑	yo ↓		
$x_i =$	1.25000	[cm]	$D =$	100.000	$[m^{-1}]$	
$y_i =$	-0.250000	[cm]	$m =$	-0.250000		

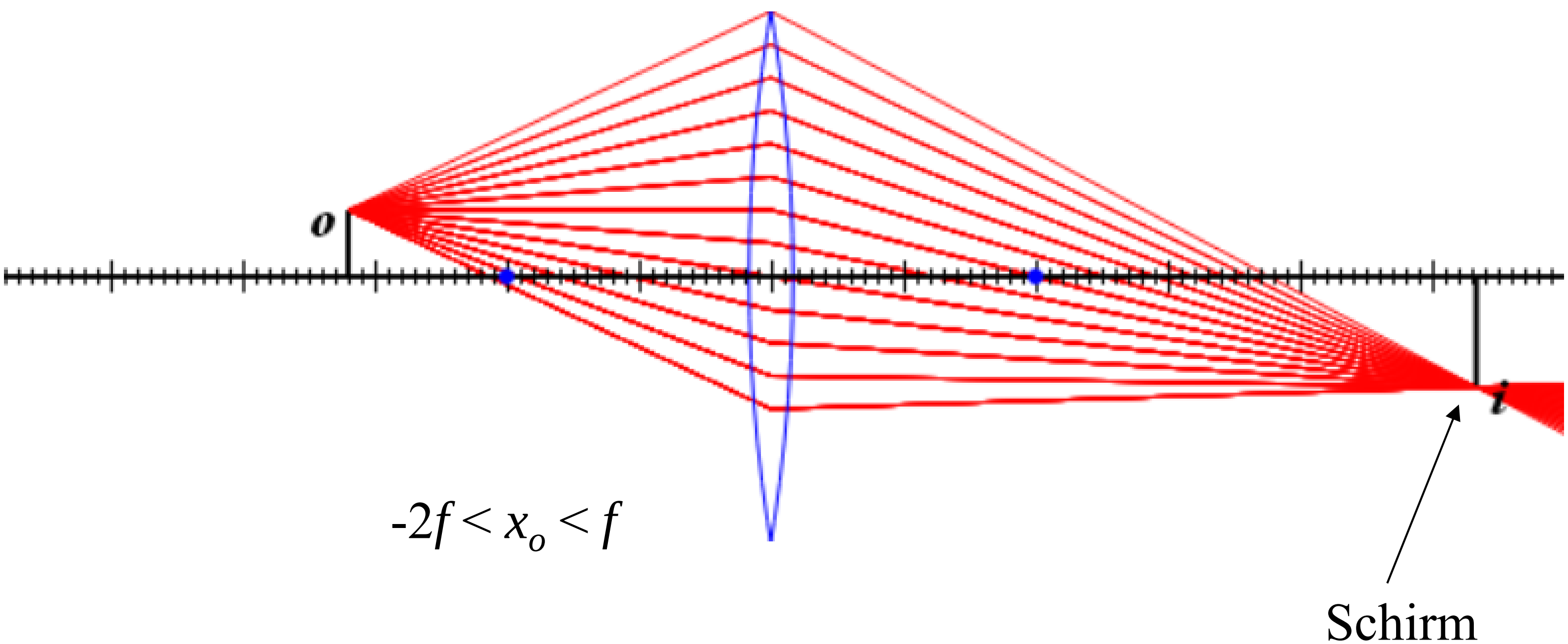
plot

The image is real and inverted.



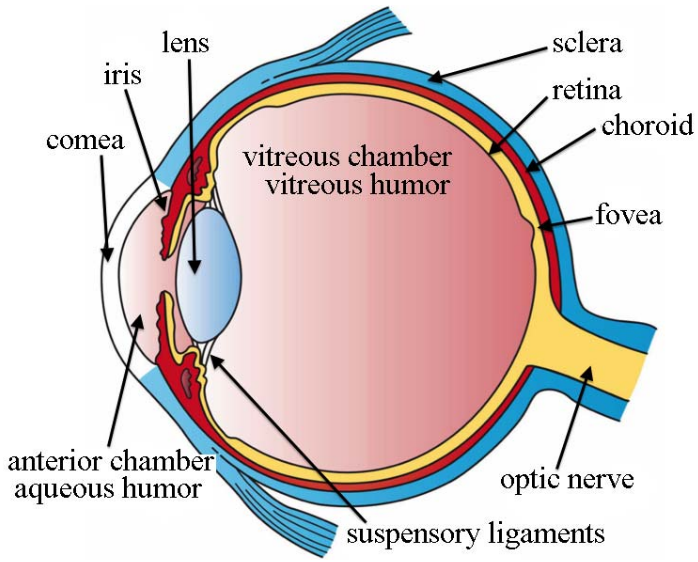
Diaprojektor

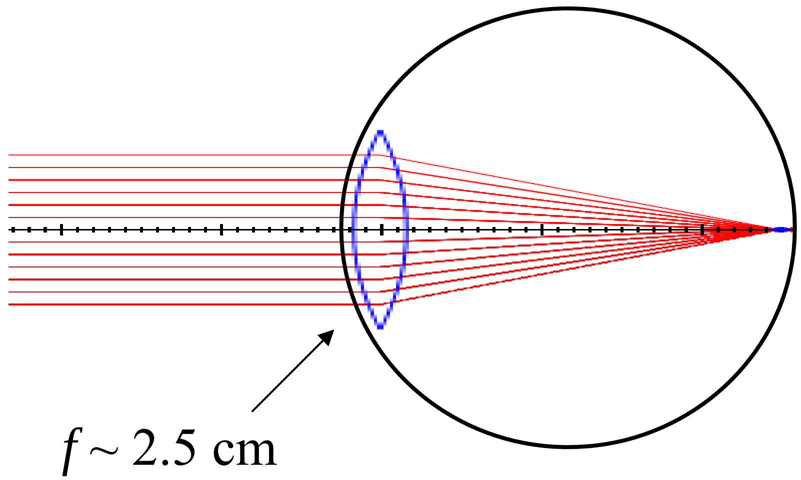




$-2f < x_o < f$

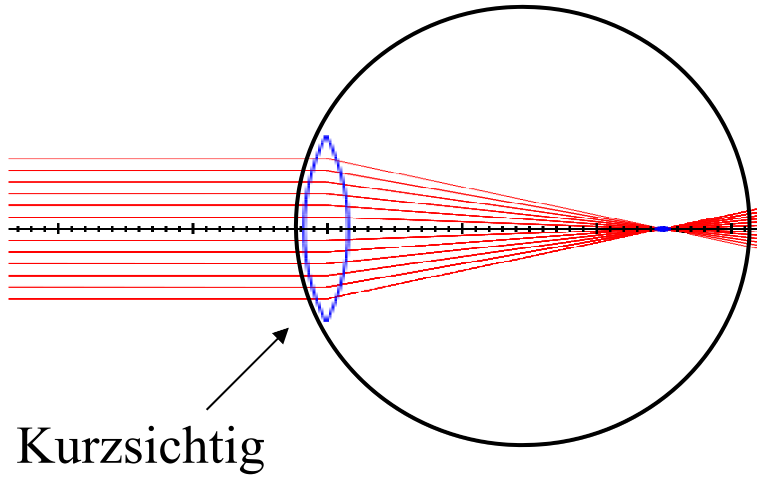
Auge

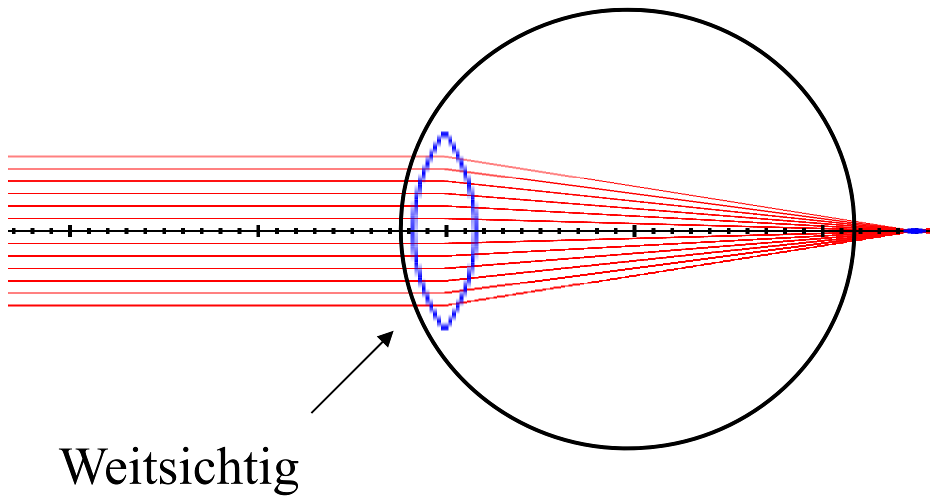




http://upload.wikimedia.org/wikipedia/commons/d/d0/Three_Main_Layers_of_the_Eye.png

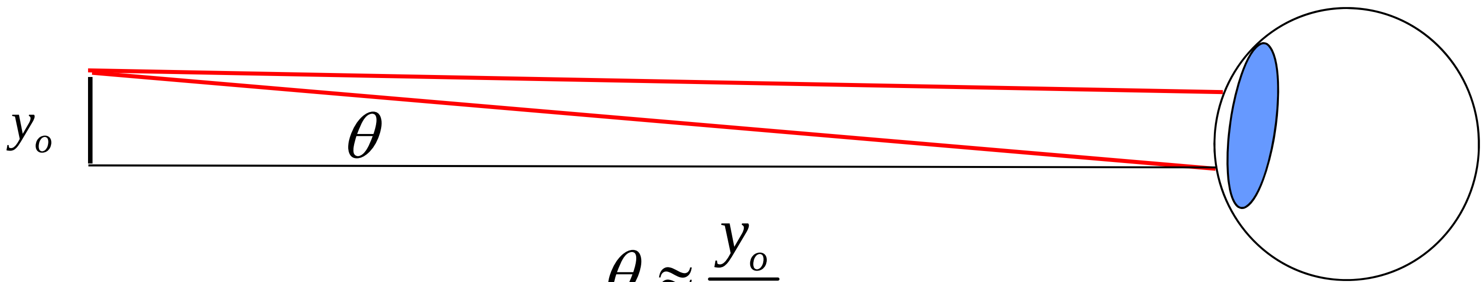
Auge



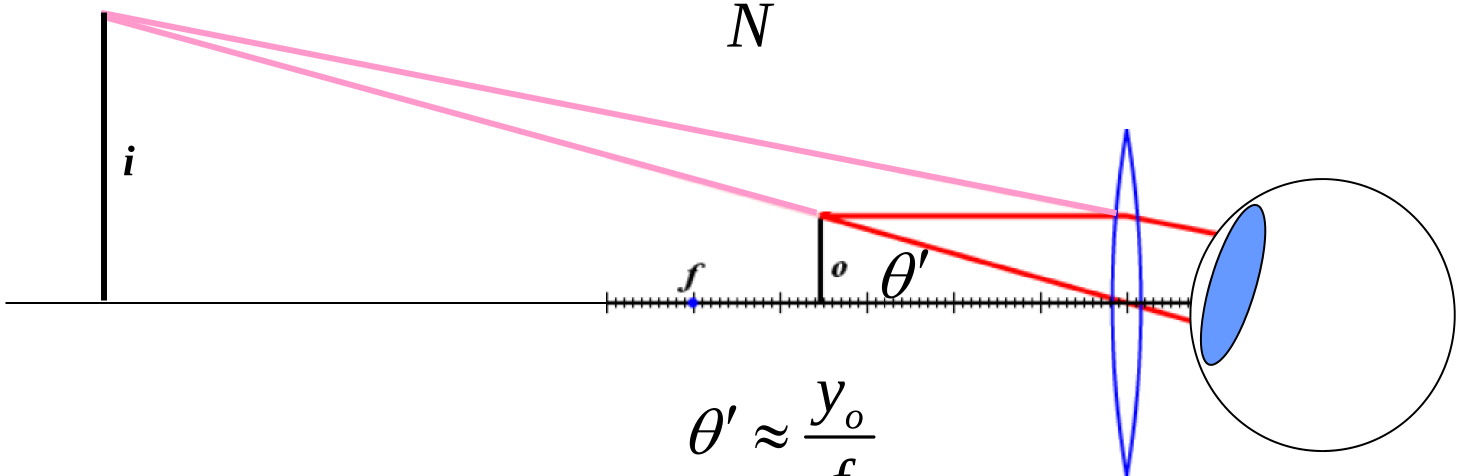


Lupe

Nahpunkt $N \sim 25$ cm



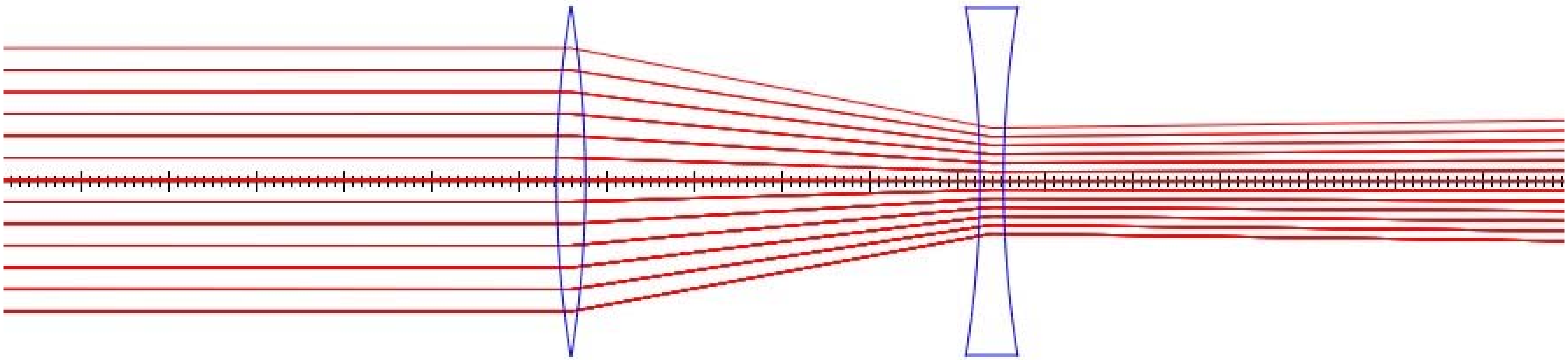
$$\theta \approx \frac{y_o}{N}$$



$$\theta' \approx \frac{y_o}{f}$$

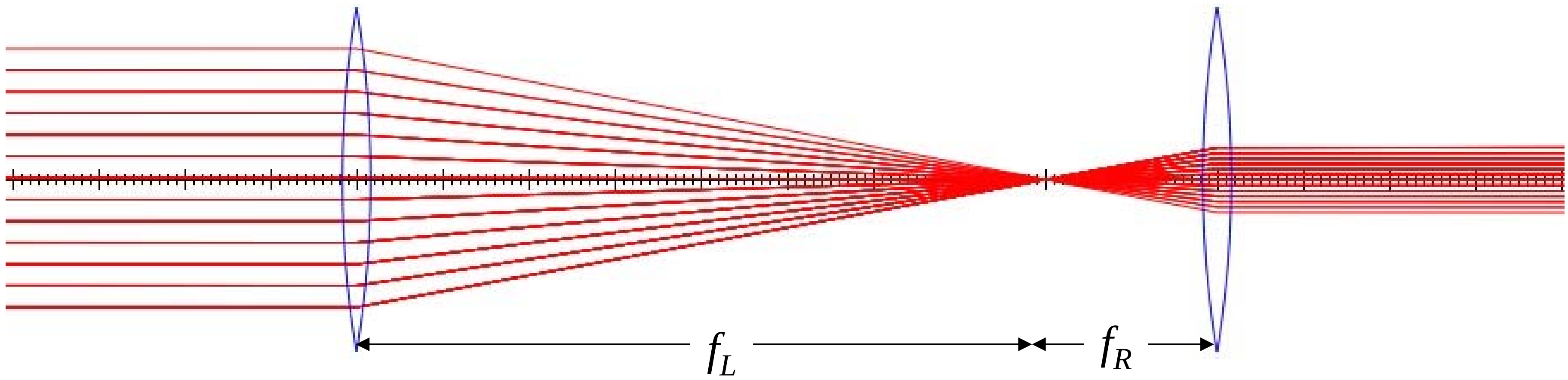
Vergrößerung $m \sim N/f$

Galilei'sches Teleskop



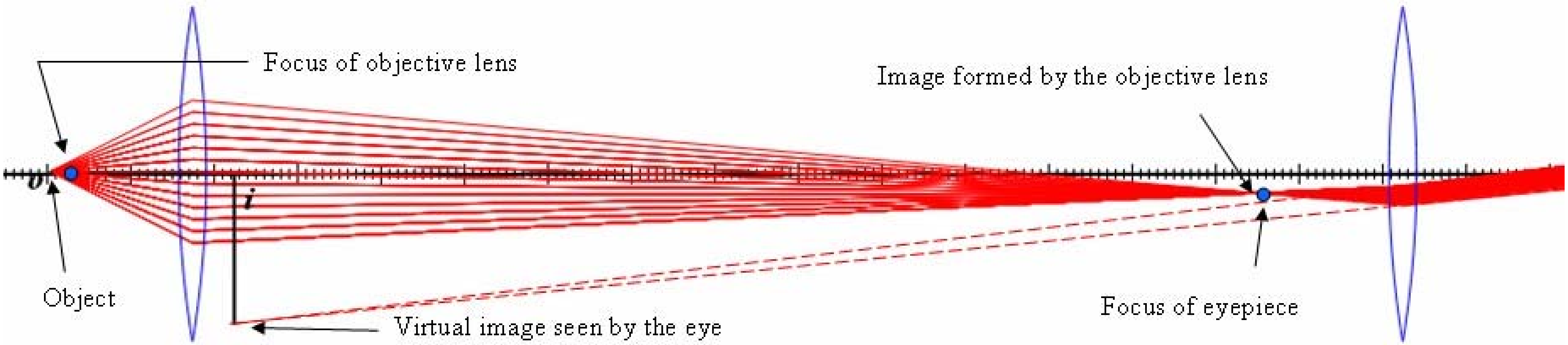
$$m = \frac{\theta_i}{\theta_o} = \frac{y_i x_o}{y_o x_i}$$

Keplersches Teleskop



$$m = \frac{\theta_i}{\theta_o} = \frac{y_i x_o}{y_o x_i}$$

Mikroskop



Focus of objective lens
Image formed by the objective lens
o
i
Object
Virtual image seen by the eye
Focus of eyepiece