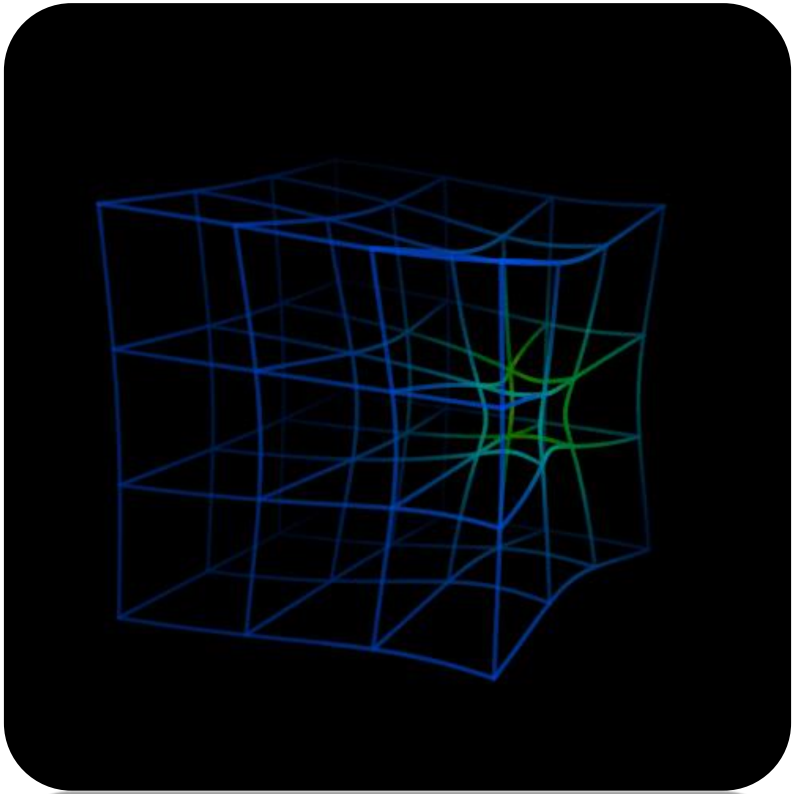
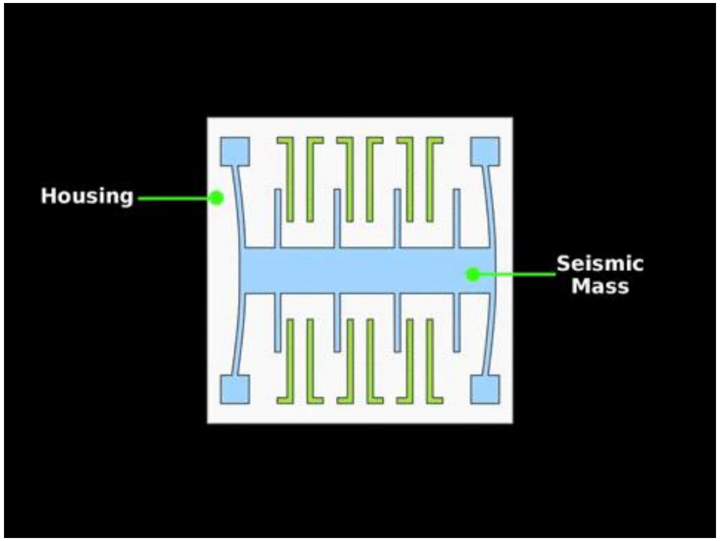
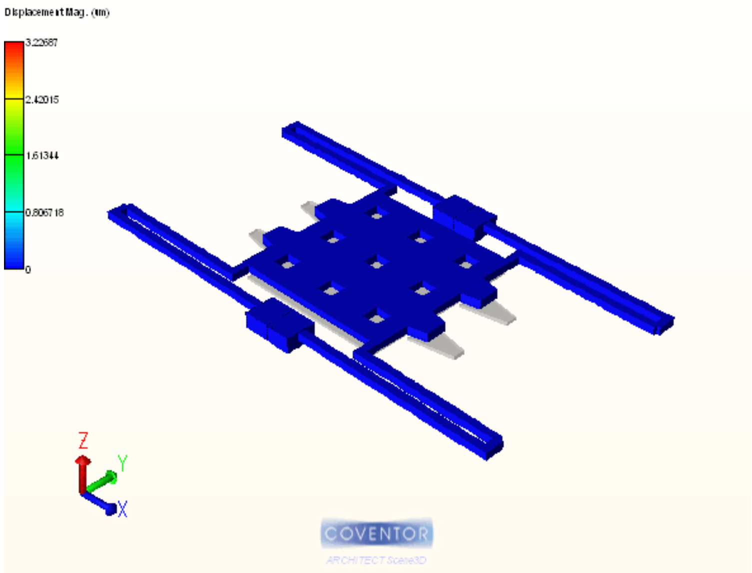
MICROMECHANICS



Damping: Pro and Contra

- In MEMS devices we always have MOVING PARTS
- When considering e.g. an accelerometer it is obvious that the movement should be DEFINED
- This means that two side effects should be avoided:
  1. Massive damping which reduces both, the sensitivity (image an accelerometer in honey) and the overall efficiency (power losses)
  2. Over-oscillations as wrong signals could appear (here damping would good)
- Both demands, however, are opposite (damping vs. no damping) and the solution lies somewhere in between depending on the application … so we have to have a closer look





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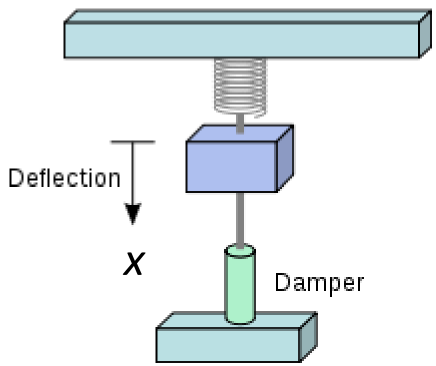
Damping: General

- We start with a simple, damped oscillating mass



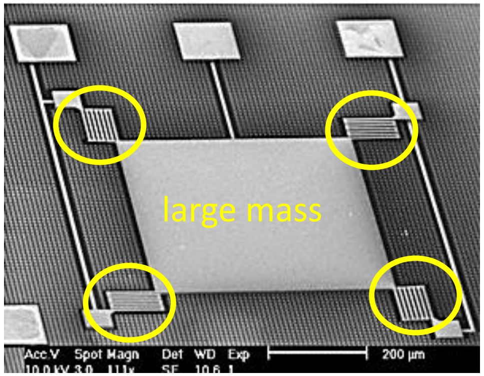
$$F = m\left(\frac{d^2x}{dt^2}\right) + c\left(\frac{dx}{dt}\right) + kx$$

x … displacement
c … damping coefficient
k … spring constant

- For an ideal case, the damping is zero → $c\left(\frac{dx}{dt}\right) = 0$

- With $F = 0$ it follows:

$$x = -\frac{ma}{k}$$



- **<u>Design Rule</u>**: high sensitivity (large x) follows if
  1. The mass is large (which explains the <u>large proof-masses</u>)
  2. Spring constants are low (which explains the <u>small connection parts</u>)

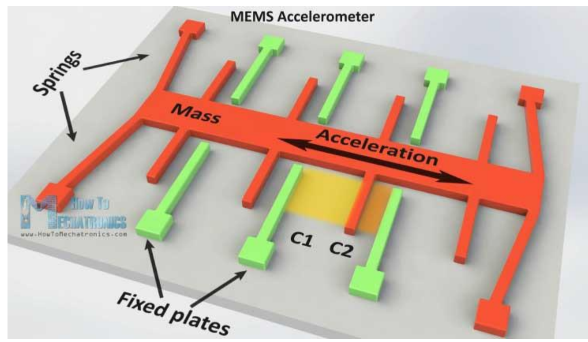
Sensitivity: Capacitive Accelerometer

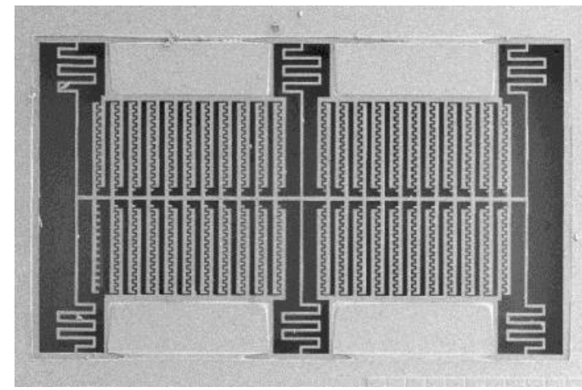
- Here we transfer the displacement Δx into a capacity change ΔC (transducing mechanism)

$$x = -\frac{ma}{k}$$

&

$$C = \frac{\varepsilon A}{d}$$

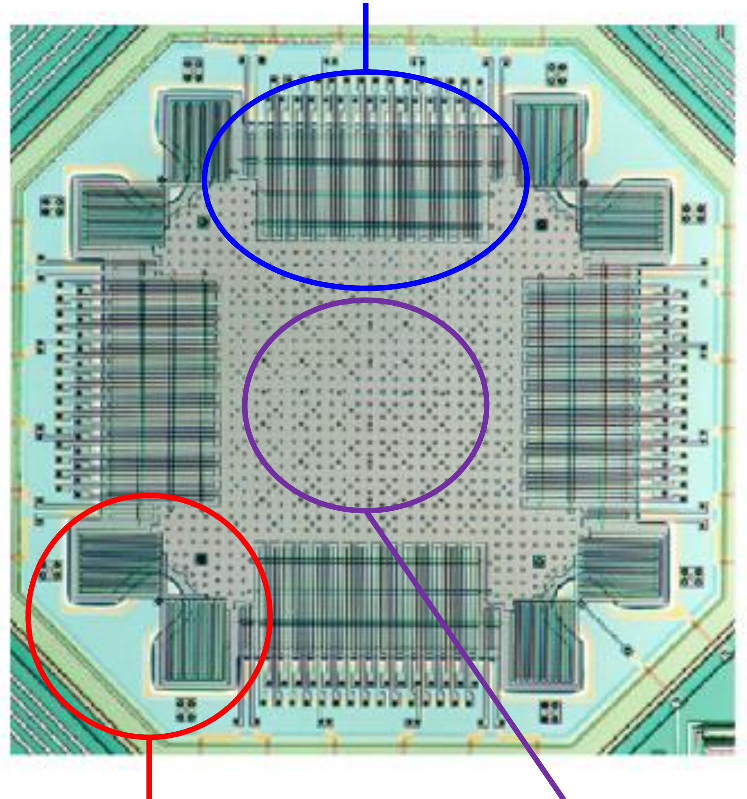




- From the latter equation we see that highest C changes are obtained if we use:
  - Large interface areas
  - Small interface distances d
- Taking the first equation into account as well we see that for smallest d's we again need :
  - Large displacements x …
  - … which require large masses and soft springs (small k)
- **<u>Additional Design Rules</u>**:
  1. Large interface areas (explaining multiple fingers with high structures
  2. Strong distance changes equivalent to large displacements (high mass, soft springs)

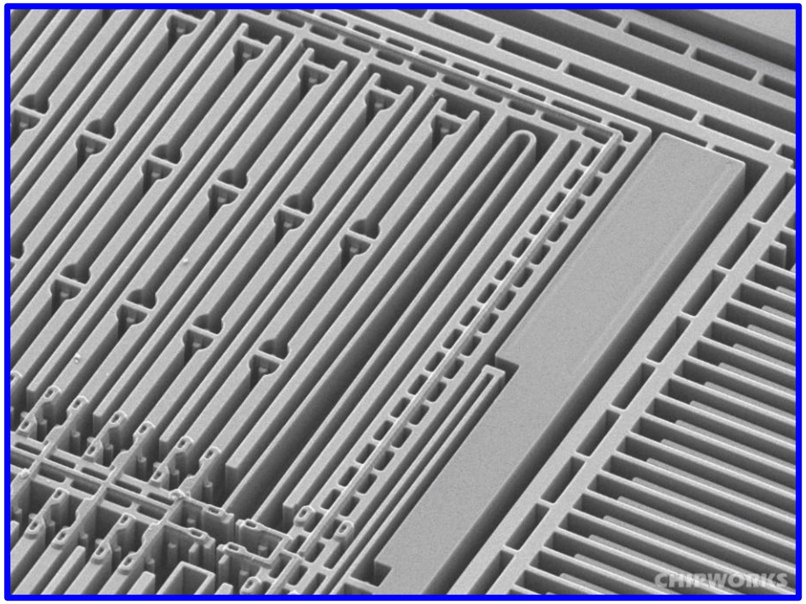
Sensitivity: Design Rules

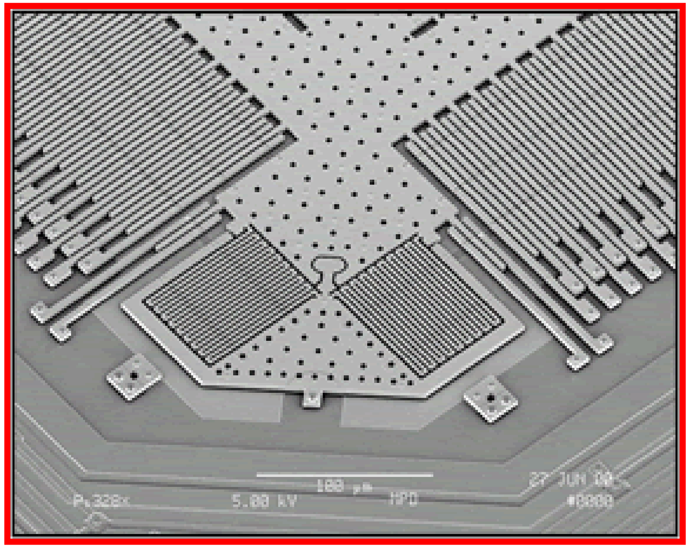
large area interfaces (detection)



soft springs
(high sensitivity)

high masses
(high sensitivity)



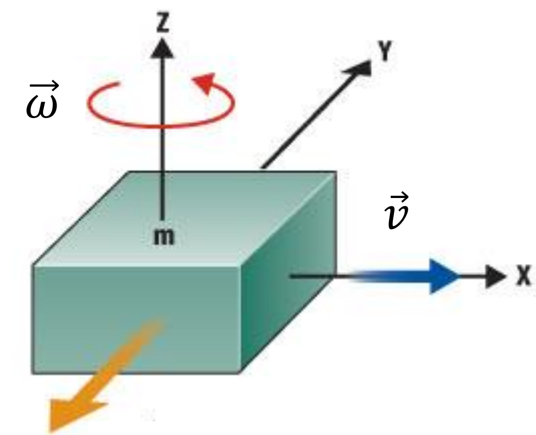


Sensitivity: Gyroscopes General

- The responsible force to measure the angular rate in gyroscopes is the Coriolis Force

- $\overrightarrow{F_c} = -2m\vec{\omega} \times \vec{v}$

  m … mass
  $\vec{\omega}$ … angular velocity
  $\vec{v}$ … linear velocity



$$\overrightarrow{F_c} = -2m\vec{\omega} \times \vec{v}$$

- This means that for an arising angular velocity in a moving system a force evolves perpendicular on the $\vec{v}$, $\vec{\omega}$ plane (and negative in direction)

- And we can immediately derive the requirement for highest sensitivity:
  1. Large mass (as before)
  2. High initial velocity $\vec{v}$ (which finally explains the high frequency operation)

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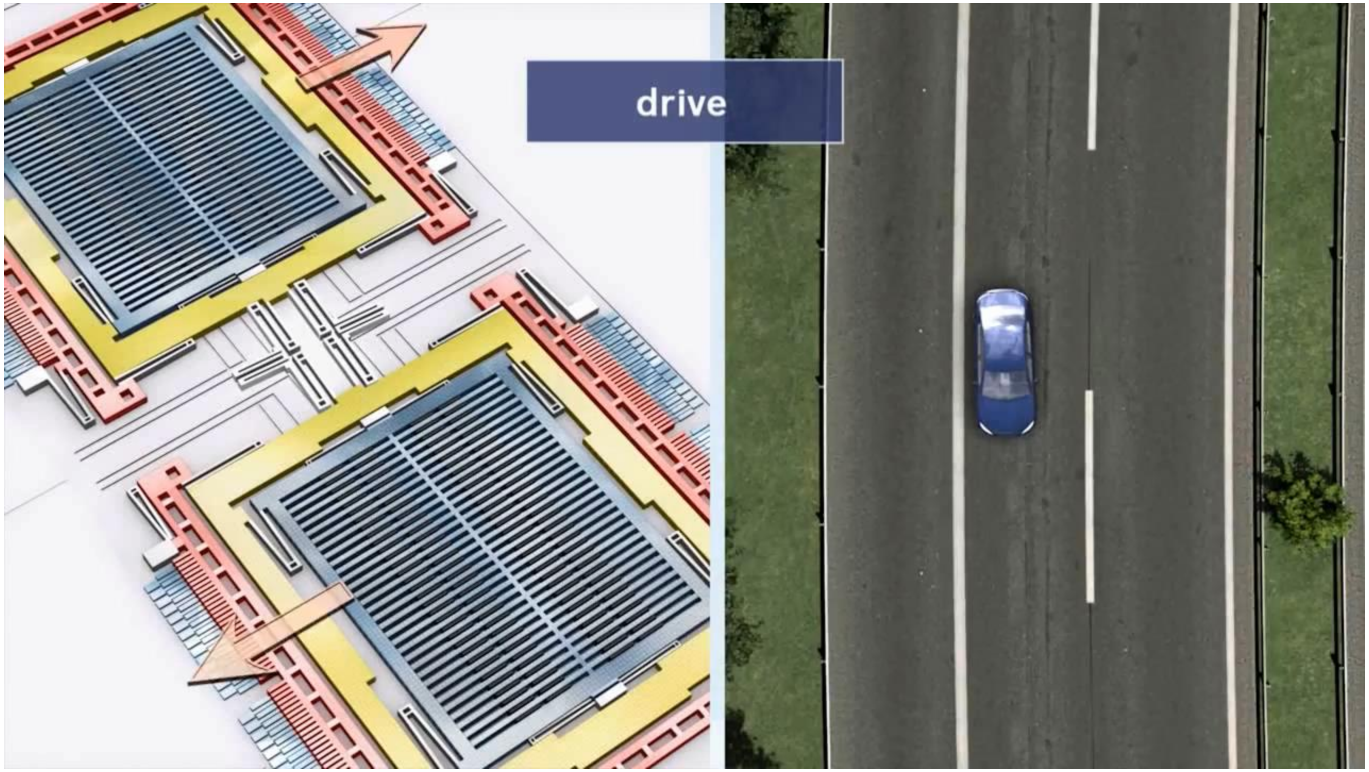
Linear Gyroscopes

- However, we have seen that most of the modern Gyroscopes use oscillation instead of rotation



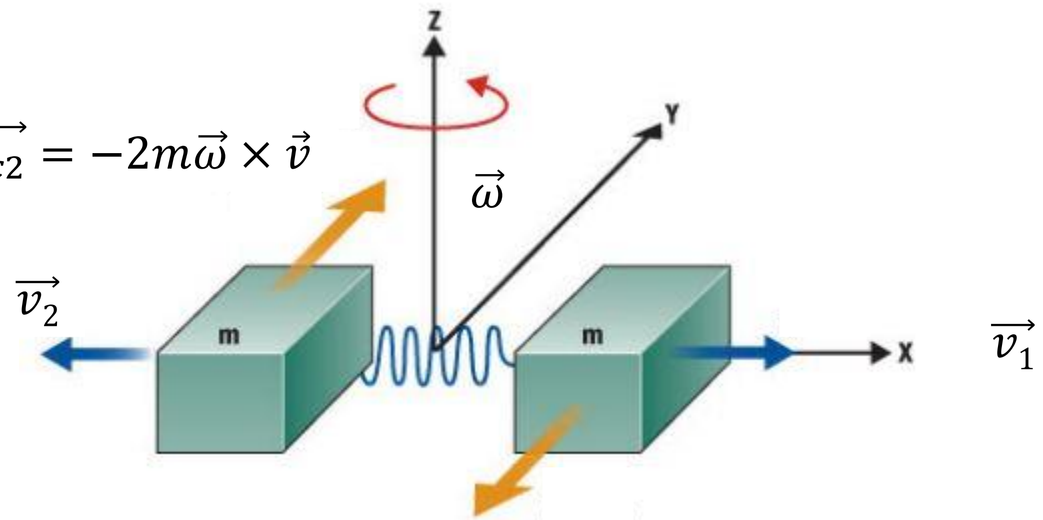
Linear Gyroscopes: Detail

- However, we have seen that most of the modern Gyroscopes use oscillation instead of rotation

Sensitivity: Gyroscopes Rotation vs. Oscillation

- Is that in conflict with the before discussed formalism?
- The trick is that we use a differential approach which is easier in detection
- This means that two masses are oscillated linearly but in OPOSITE directions
- What is detected is the capacity change DC which maximizes due to the differential operation

$$\overrightarrow{F_{c2}} = -2m\vec{\omega} \times \vec{v}$$



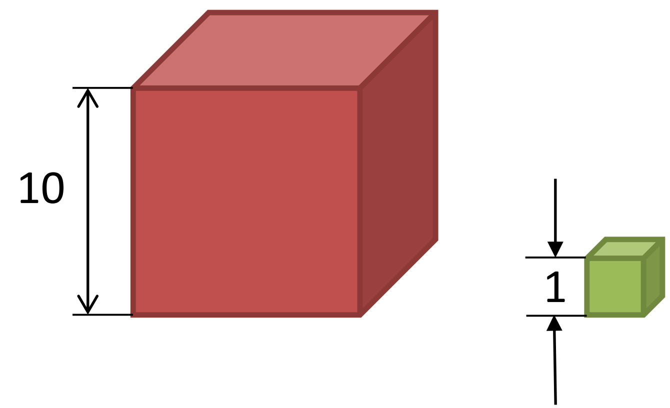
$$\overrightarrow{F_{c1}} = -2m\vec{\omega} \times \vec{v}$$

$$\Delta C = |\Delta C_1 - \Delta C_2|$$

Scalability – WHY and HOW?

We introduce a factor S which describes the scaling:

- $L_{new} = S*L_{old}$
- $W_{new} = S*W_{old}$
- $H_{new} = S*H_{old}$



For the right hand case it means that we have a scaling factor of 1/10 or 0.1

How do some quantities scale with?

Lenghts (s):
- Dimensions **l, w, h** scale via $\mathbf{S^1}$

Surface (A):
- $A = L_{new} * L_{new} = L_{old} * S * L_{old} * S = L_{old} * L_{old} * S^2$
- It follows that **A** scales with $\mathbf{S^2}$

Volume (V):
- $V = L_{new}^3 = L_{old}^3 * S^3$
- Hence, **V** scales via $\mathbf{S^3}$

- First derived quantity Surface to Volume → $S/_V = \frac{6*L_{old}^2*S^2}{L_{old}^3*S^3} = S/_{V_{old}} * \frac{1}{S}$ → for S = 1/10 → x **10!**

Scalability: More Quantities and an Example

Further quantities of interest:

- Mass (weight) → $m = \rho * V$ → $m \sim L^3$ → $\sim S^3$
- Moment of inertia → $I \sim m * R^2 = S^3 * S^2$ → $\sim S^5$
- Torque → $\tau \sim R \times m * g \sim S * S^3$ → $\sim S^4$

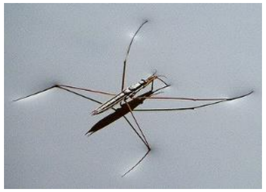
Lets have a look on the water bug and why he can move on water (γ_{H2O} is 72 mN/m)

Real person

- Mass: 50 kg
- Weight: F = m.g = 50*9.8 = 490 N
- Required perimeter to “float”: L = F / (2.γ) = 490/(2.72.10^{-3}) = 3403 m (☹)

Water bug (S = 1/1000)

- Mass: $m_{HUMAN} * S^3 = m_{HUMAN} * (10^{-3})^3 = m_{HUMAN} * 10^{-6} = 50 * 10^{-6}$ kg
- Weight: $50 * 10^{-6} * 9.8 = 4.9 * 10^{-7}$ N
- Required perimeter to “float”: $4.9 * 10^{-7}$ /(2.72.10^{-3}) = 3.4 µm (☺)



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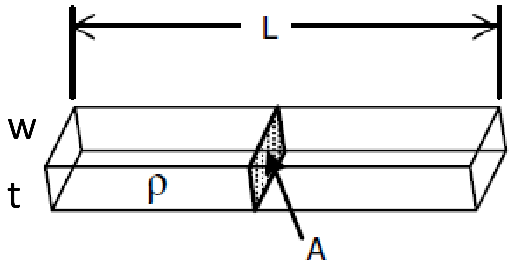
Scalability: Other Quantities

Electric resistance - R:

$$R = \frac{\rho L}{A} = \frac{\rho L}{wt} \propto \frac{1}{S}$$

ρ ... electrical resistivity

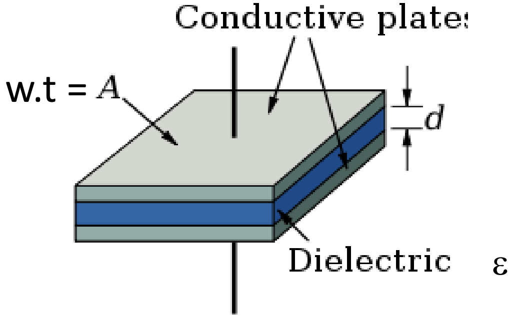


– Smaller dimensions increase R

Electric capacitance – C:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{\varepsilon wt}{d} \propto S$$

ε_0 ... electric constant
ε_r ... dielectric constant (rel. permittivity)

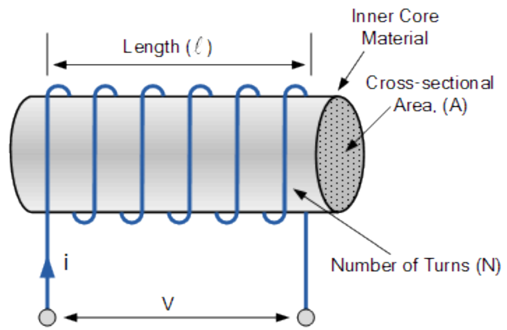


– Smaller dimensions decrease C (area related)

Electric Inductance – L:

$$L = \frac{\mu N^2 A}{l} = \frac{\mu N^2 wt}{l} \propto S$$

μ_0 ... magnetic constant
μ_r ... relative permeability
N ... number of windings
l ... coil length



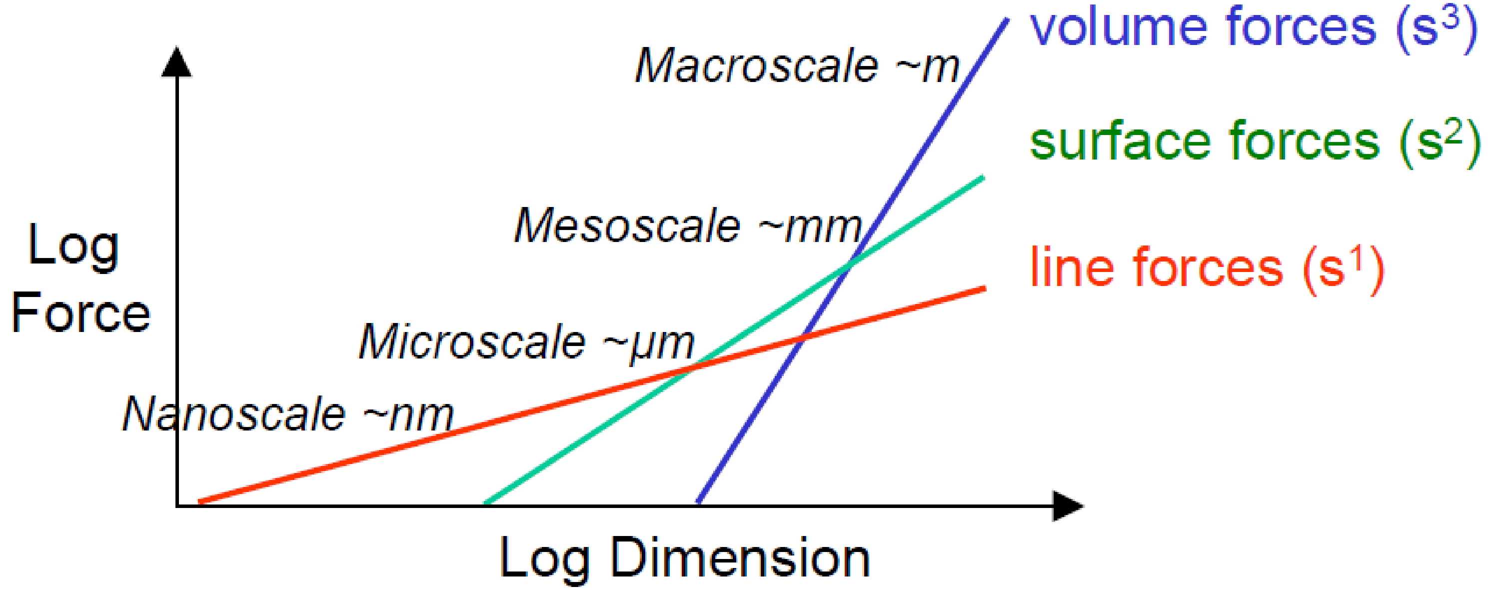
– Smaller dimensions decrease L

Force Behaviour During Downscaling

Considering MEMS relevant forces in general, we can summarize

- Surface tension → S^1
- Electrostatic 2D, pressure → S^2
- Magnetic 2D, gravitational → S^3
- Magnetic 3D → S^4

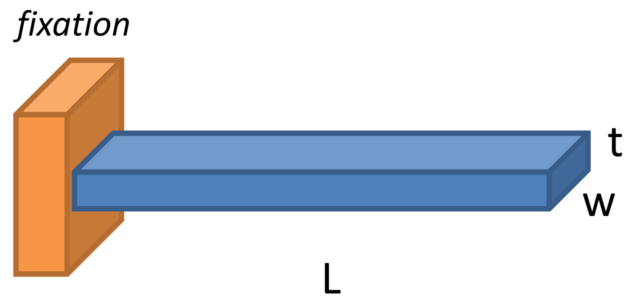
- That means that all forces can be downscaled but some of them get VERY small for VERY small scales which is why the micron-scale is fine for most applications (NEMS can get complicated)



Scalability: Stiffness

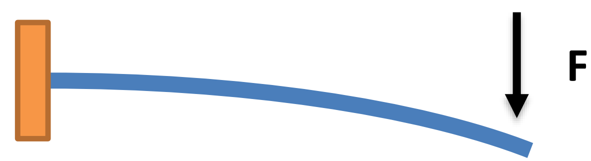
In many cases MEMS work with resonant systems … lets have a closer look on simple systems

Single beam stiffness (one side fixed Euler buckling):



$$k_{bending} \propto \frac{EI}{L^3} \propto \frac{Ewt^3}{L^3} \propto S$$

E … Young's modulus
I … second moment of area

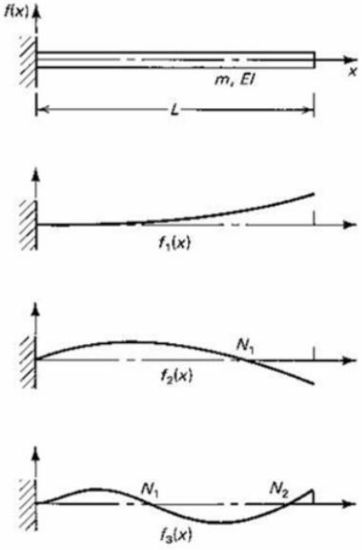


– Which means that smaller dimensions makes the beam softer

$$k_{axial} \propto \frac{EA}{L} \propto \frac{Ewt}{L} \propto S$$

– Which means that smaller dimensions makes the beam softer

Scalability: Natural Frequencies



As a result, the natural frequencies f_n change as follows:

$$k \propto \frac{Ewt^3}{L^3} \propto S \;\rightarrow\; f_n = n\frac{1}{2\pi}\sqrt{\frac{k}{m}} \propto \sqrt{\frac{k}{m}} \propto \sqrt{\frac{S}{S^3}} \propto \frac{1}{S}$$

– This means that e.g. a reduction by a factor 10 means an increase by a factor of 10!

– However, we here always assumed a HOMOGENEOUS downscaling of ALL dimensions!

– If we change e.g. only the width it follows

$$m = \rho V = \rho L t w \propto w \propto S \;\rightarrow\; f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \propto \sqrt{\frac{k}{m}} \propto \sqrt{\frac{S k_{old}}{S m_{old}}} \propto \sqrt{\frac{S}{S}} = 1$$

– Which means that changing the thickness t only will NOT change the natural frequency

This is bad and good together as designers have more degree of freedoms!

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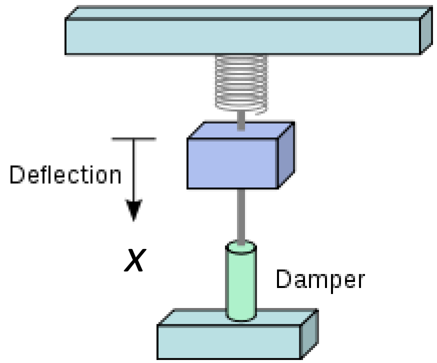
Damping … Finally

- We started with a general consideration of moving elements, discussed sensitivity related design rules and finally focused on the scalability …

- … however, all this cases did IGNORE damping as it happens in real situations!



$$F = m\left(\frac{d^2x}{dt^2}\right) + c\left(\frac{dx}{dt}\right) + kx$$

x … displacement
c … damping coefficient
k … spring constant

- So lets consider what happens if the damping is NOT zero …

Damping Quantities

- First we introduce the "**damping ratio ξ**" which is a dimensionless quantity describing the oscillation behaviour after an input (e.g. sudden movement in an accelerometer)

$$\xi = \frac{c}{2m\omega_0}$$

c ... damping coefficient

m ... mass

ω_0 ...natural base frequency

- The damping coefficient is of additive character if multiple mechanism take place

$$c_{tot} = \sum_i c_i$$

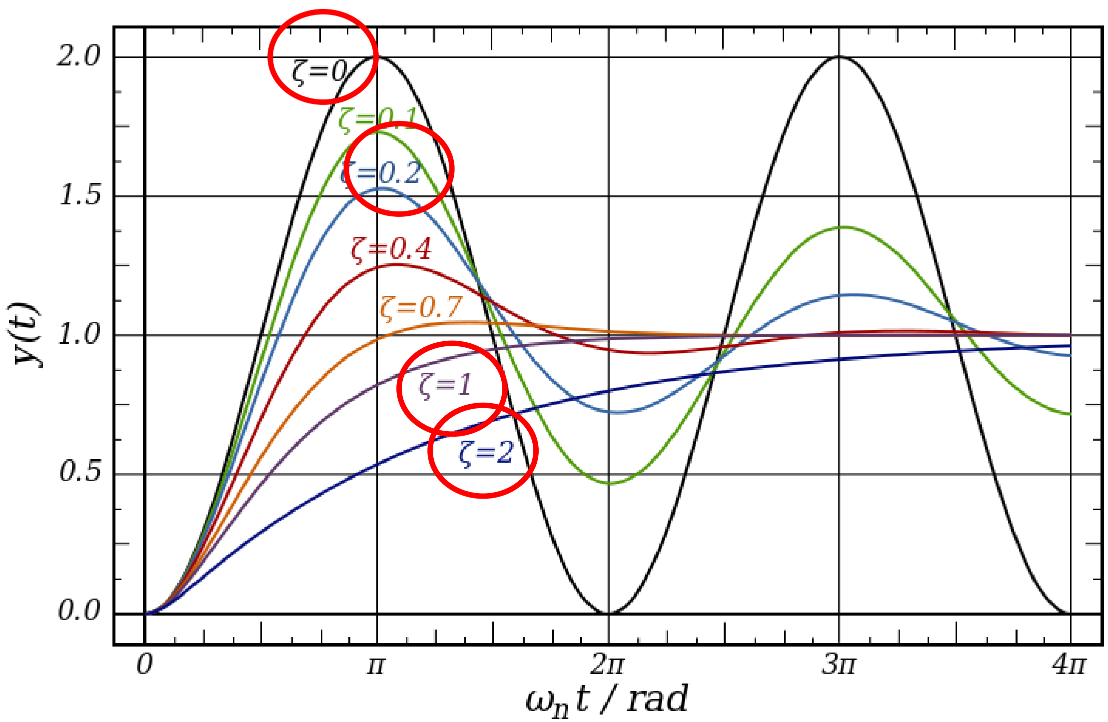
- An often found quantity of oscillating systems is the quality factor Q which relates to ξ via

$$Q = \frac{1}{2\xi} = \frac{m\omega_0}{c}$$

Damping Behaviour

ξ is a very practical quantity to describe the oscillation behaviour

- ξ = 0 → undamped → ideal (unreal) case
- ξ < 1 → underdamped → amplitude decays exponentially
- ξ > 1 → overdamped → no further oscillations with exponential approach
- ξ = 0 → critically damped → no first over-oscillation effect with fastest possible approach



$$\xi = \frac{c}{2m\omega_0}$$

Overdamping occurs for:

- Higher damping
- Low frequencies
- Smaller masses

Damping Behaviour

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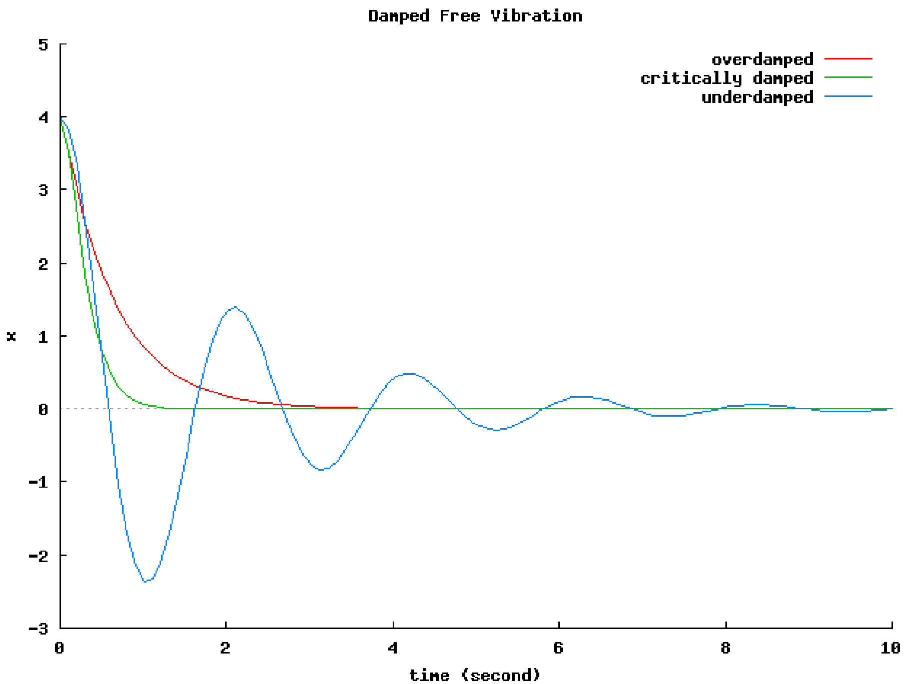

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Damping Behaviour

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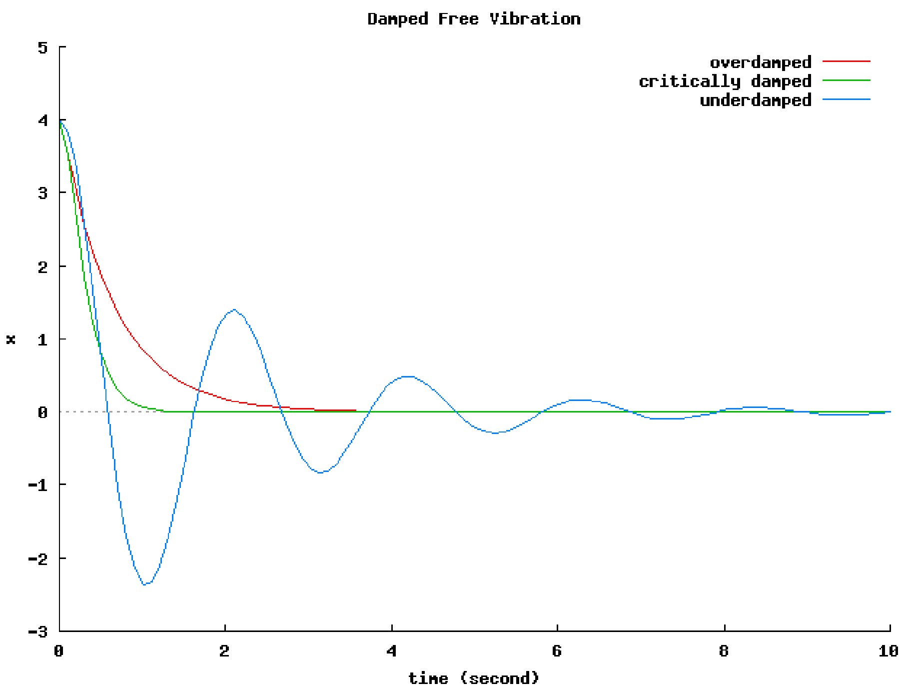
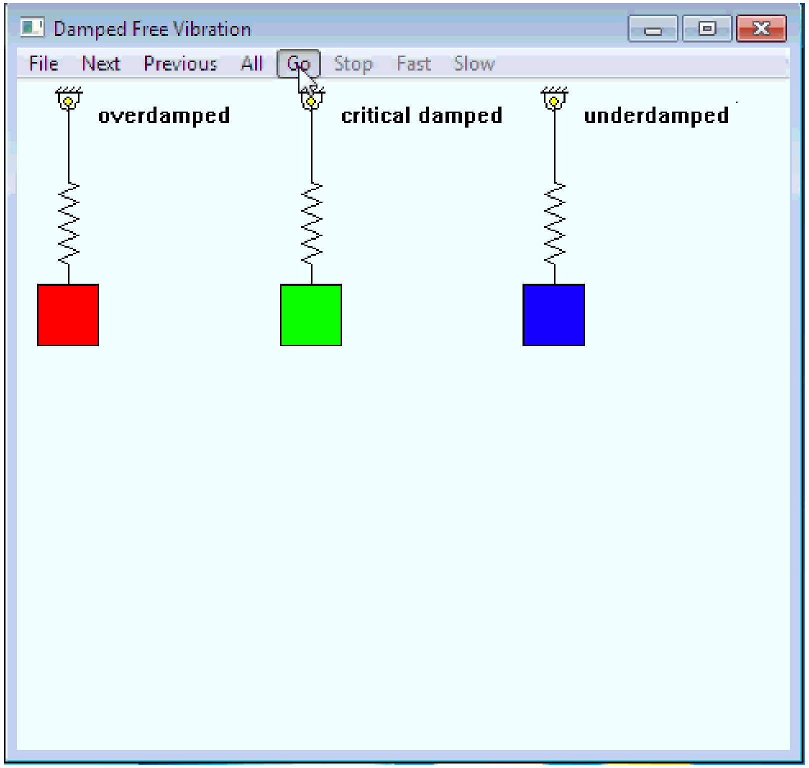
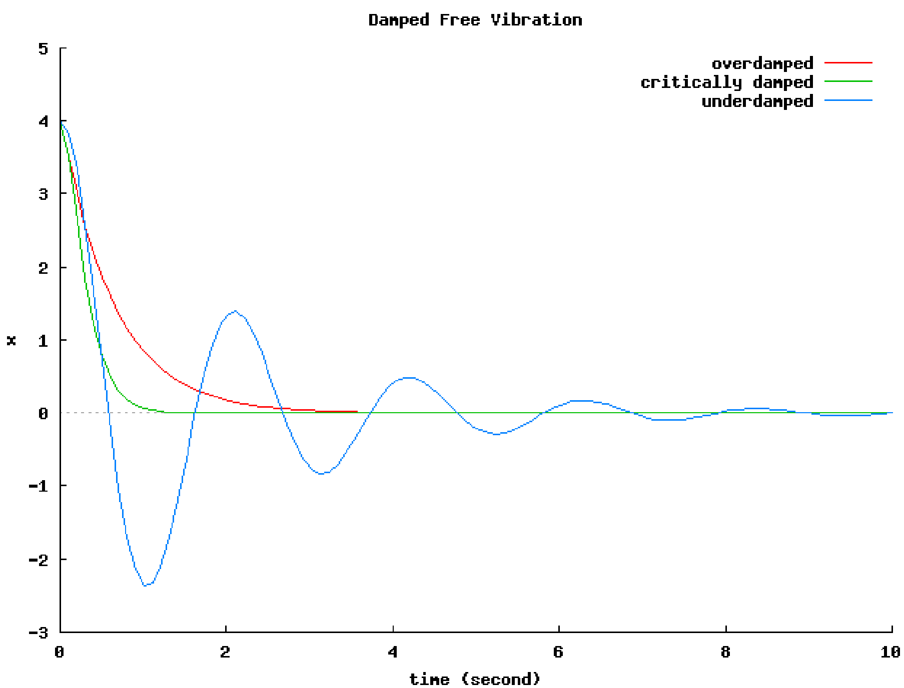


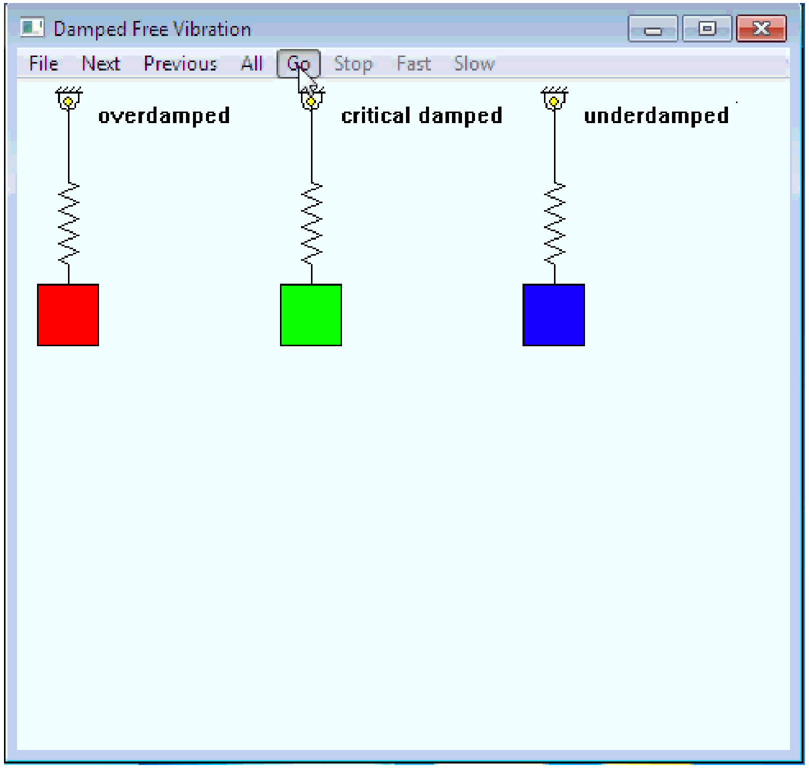


Damping Behaviour in Q Notation

In Q notation the central value is ½ and the tendency is inverted!

- $Q \rightarrow \infty$ → undamped → ideal (unreal) case
- Q > ½ → underdamped → amplitude decays exponentially
- Q < ½ → overdamped → no further oscillations with exponential approach
- Q = ½ → critically damped → no first over-oscillation effect with fastest possible approach

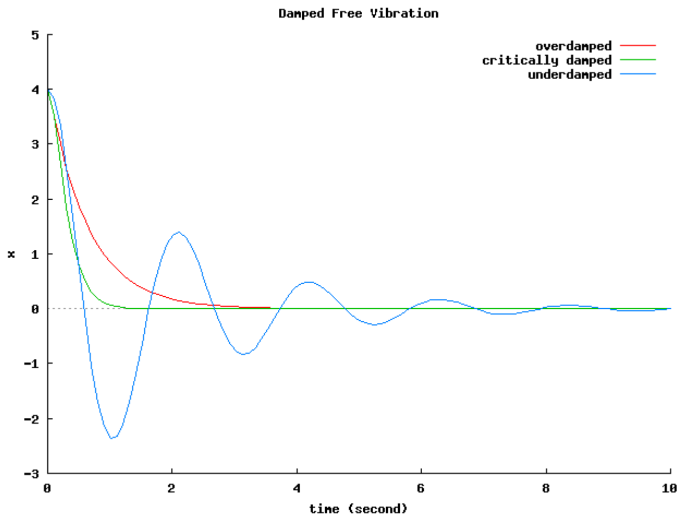




Damping Ssources

For applications both oscillation regimes can be beneficial

- Accelerometers, fluid pumps, speakers → over damped (no over-oscillations should appear)
- Resonators, gyroscopes → under damped (oscillation essentially required)



Next, we need to understand the nature of damping effects which can be classified into:

- Intrinsic → material related
- Anchor → design induced
- Fluid / quasi-fluid → surrounding media related

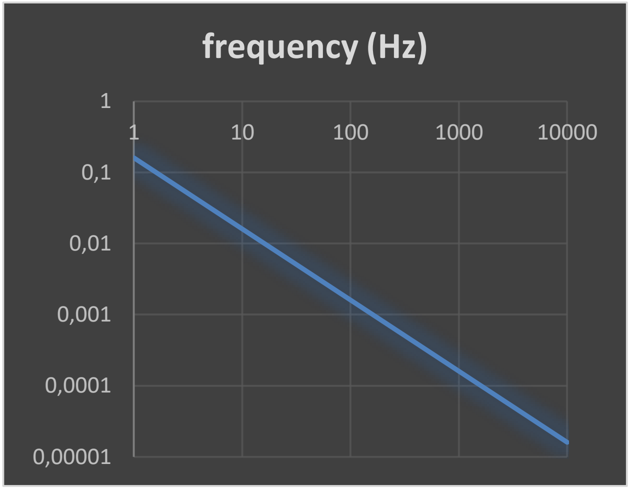
Although each of these effects are always evident, the dominating influence varies in dependency on the material, operation, design and environmental conditions

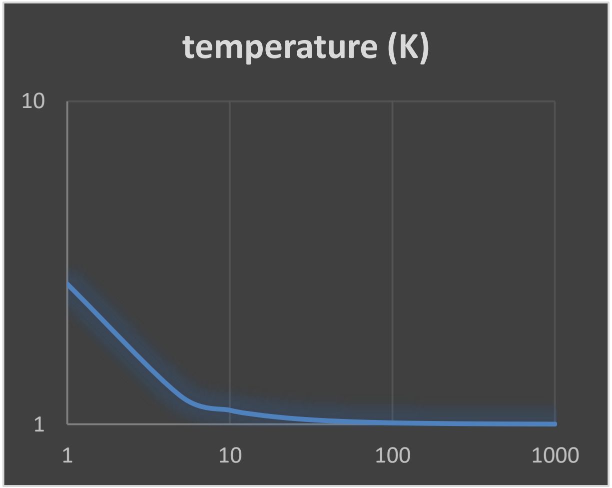
Damping: Intrinsic Losses

- These losses are related to the material itself and mainly refers to atom / molecule movements
- These effects, however, often depend on the temperature which is called **thermoeleasticity**

- The Q factor then changes in dependency on the frequency (also in dependency on the mode)

$$Q = \frac{E}{\mu_\gamma \omega} = \frac{E}{\omega} \frac{1}{Ae^{\frac{W_A}{RT}}}$$





E … Youngs modulus

μ_γ … material viscosity

ω … frequency

W_A … activation energy (splits into bond-crack energy and motion enthalpie)

R … molar gas constant

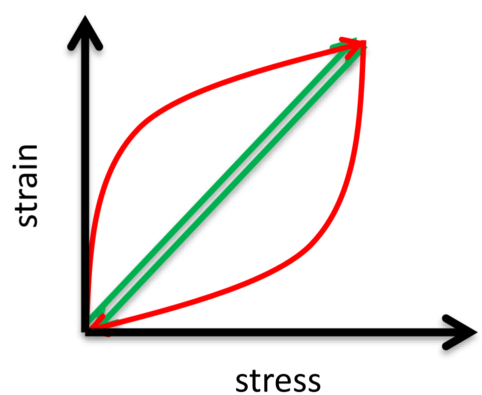
T … temperature

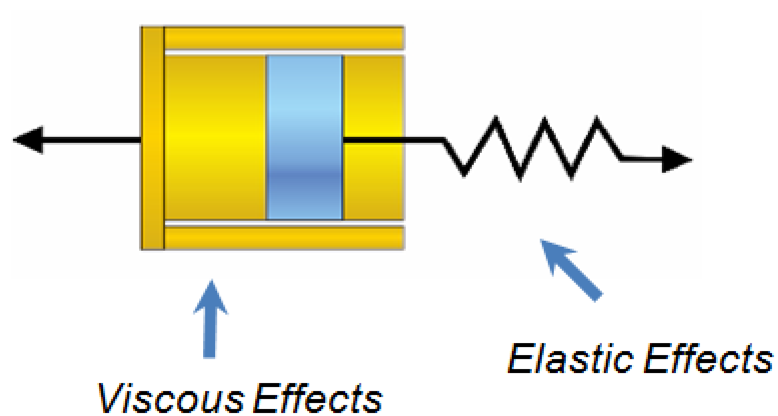
- Practically spoken: **higher temperatures and higher frequencies lead to lower Q**

- This, in turn, means higher damping and less defined oscillation behaviour → lower performance

Damping: Intrinsic Losses

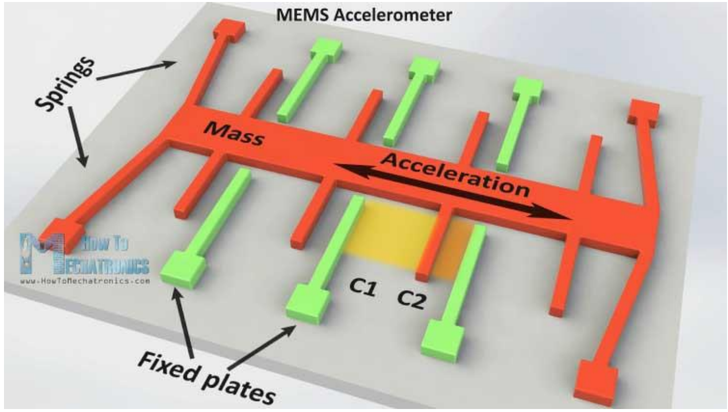
- Another intrinsic loss can be a frequency and movement dependent change of mechanical properties called viscoelasticity
- In elastic materials the arising strain due to an applied stress is linear and reversible
- In viscoelastic materials, a pseudo-plastic trend can be seen in both load and unload cycles but they are still reversible
- The effect stems from molecular / atomic movements and is often observed in amorphous materials and depends on temperature, frequency and the extend of applied stress
- The border to real plastic deformation (irreversible) is often very small …

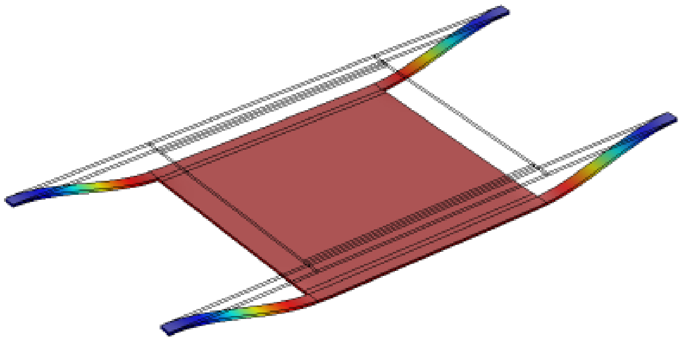


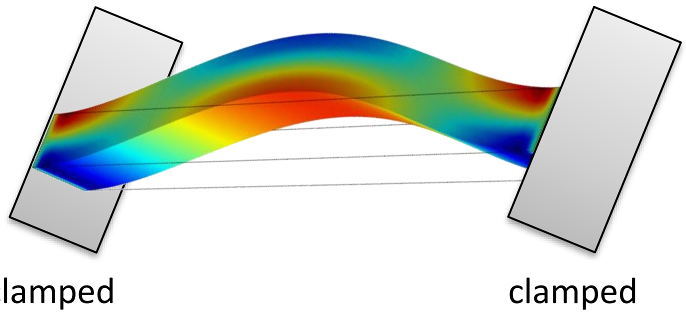


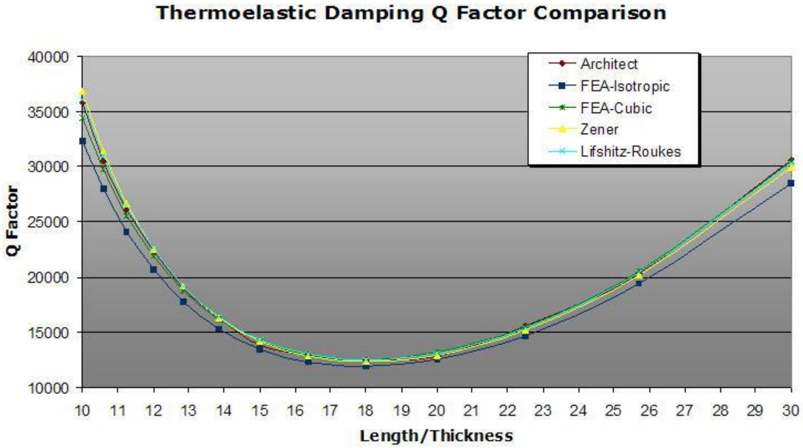
Damping: Anchor Losses

- As implied by the name, anchor losses describe the loss of (vibration) energy by mounting
- The losses strongly depend on the design (see bottom left) and result from increased temperatures which induce both, general losses (low efficiency) and also thermoelastic effects which further worsen the performance



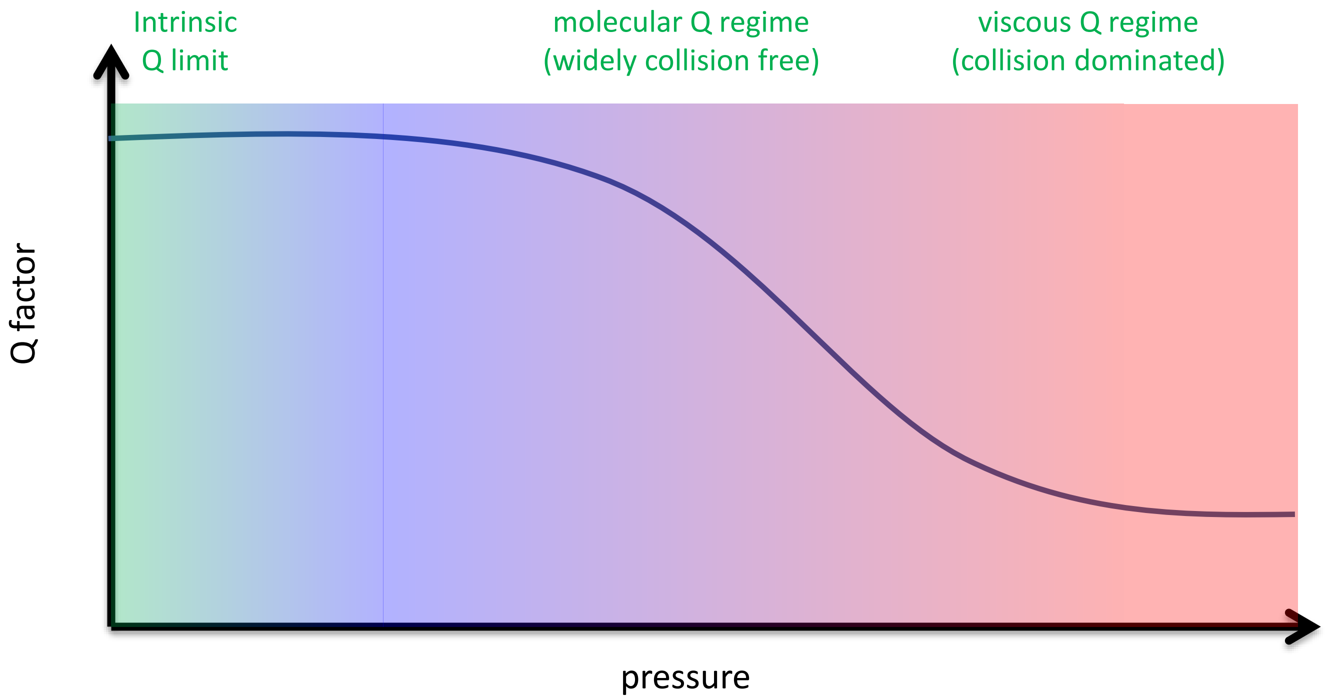






Damping: Fluid / Semi-Fluid Losses

- As long as no media is around a resonating element (vacuum conditions) the Q factor is determined by its intrinsic and anchor losses
- Once the pressure is increasing the regime changes
  - Molecular → gas molecules are considered to move on straight lines
  - Viscous → gas molecules undergo collisions with other gas molecules



Damping: Fluid / Semi-Fluid Losses

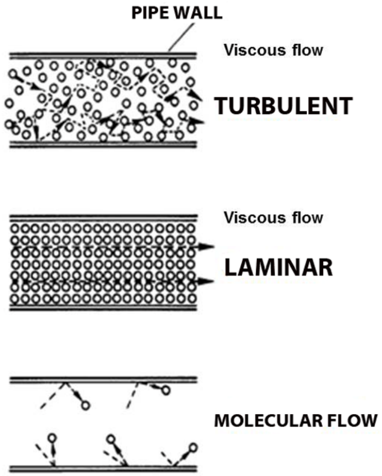
- For a more clear description the Knudsen number K_n gives is introduced

$$K_n = \frac{\lambda}{d_c} = \frac{1}{d_c}\frac{RT}{\sqrt{2}\pi d^2 p}$$

λ … mean free path
d_c … characteristic length
R … universal gas constant
T … temperature
d … effective gas molecule diameter
p … pressure

- Based on the calculated value, the Knudsen number K_n allows estimation of the regime:

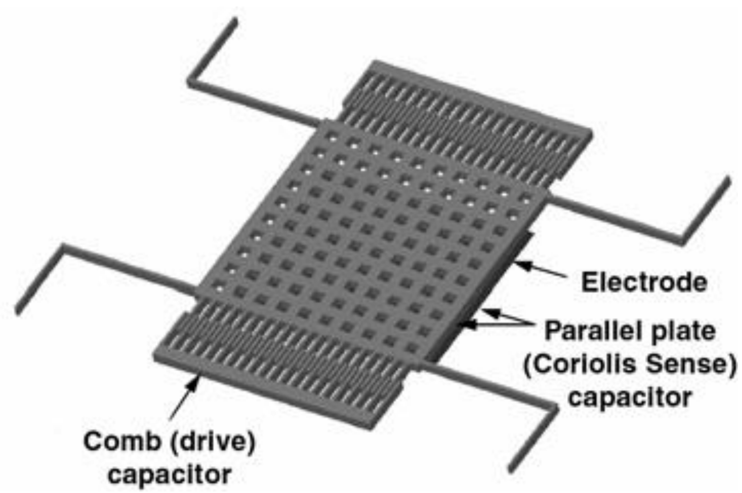
1. $K_n < 0.01$ turbulent (viscous)
2. $0.01 < K_n < 0.1$ laminar (viscous) → gas collision dom. but collective
3. $0.1 < K_n < 2$ Kundsen flow ($\lambda \approx d_c$) → gas / wall balanced
4. $K_n > 2$ molecular regime → wall collision dominated

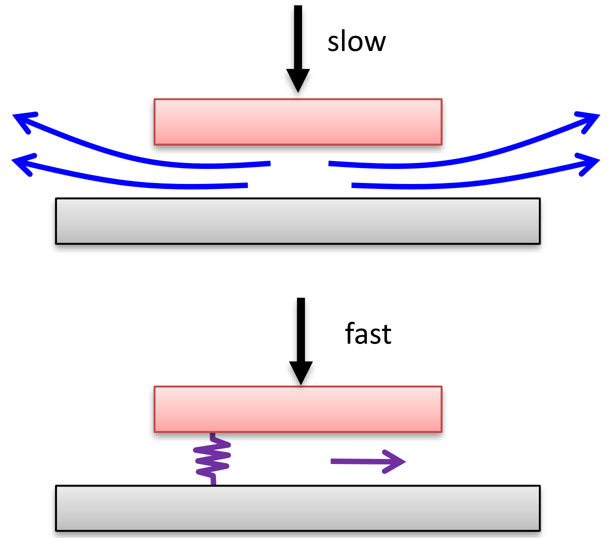


- Most of the MEMS systems work in the Kundsen flow regime as a result of the characteristic lengths on the micro-scale

Damping: Fluid / Semi-Fluid Losses

- The importance of these considerations lies in the additional environm. damping of moving parts
- As example we consider a 3-axes accelerometer which not only moves laterally but also vertically
  - Slow Z movement → gas is squeezed out → additional losses (c_{GAS})
  - Fast Z movement → gas behaves like a spring as it cannot leave fast enough ($c_{GAS} + k_{GAS}$)
- If we expand the considerations to gyroscopes operating at high frequencies, the inertia effects of the gas has also to be taken into account …
- Additional holes are a very powerful method to reduce these additional losses … but complex in calculation and design



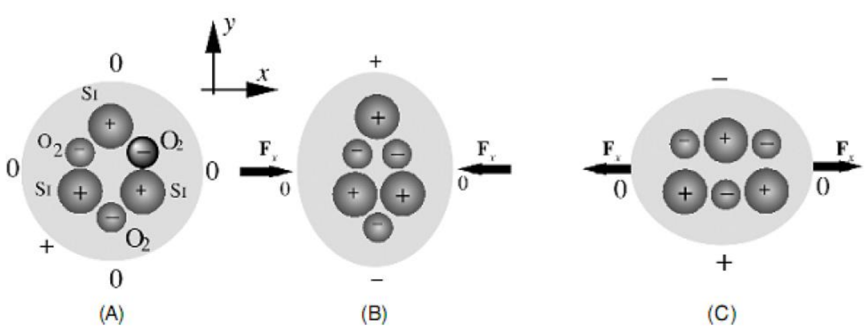


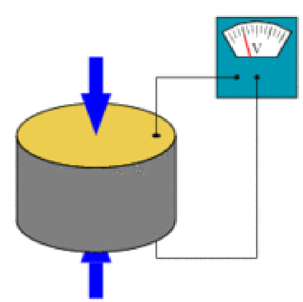
Piezoelectric Actuation

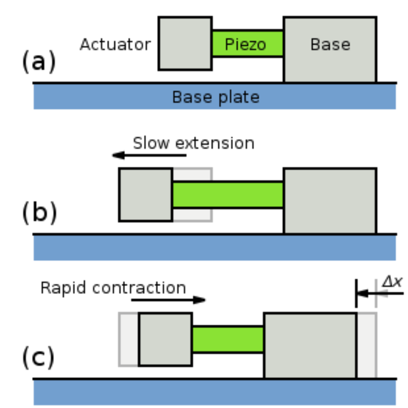
- In principle the piezoelectric effect is the generation of electric charges due to dimensional changes (compression – expansion) for solid, crystalline materials

- The important detail is that the effect can be used in two different ways:
  - Applying a stress → generation of electric charges (can be measured for sensing)
  - Applying a voltage → dimensional change (actuation purpose)

- There are natural and artificial materials
  - Natural: **quartz (SiO_2)**, phosphate materials (e.g. Berlinite – $AlPO_4$), lead titanate ($PbTiO_3$), silk, wood (!), DNA, …
  - Artificial: crystals (quartz analogous, Lithium tantalite - $LiTaO_3$, ...), ceramics (Barium titanate - $BaTiO_3$, Zinc-Oxide – ZnO, Bismuth ferrite – $BiFeO_3$, …), and some polymer (polyvinylidene fluoride - PVDF).







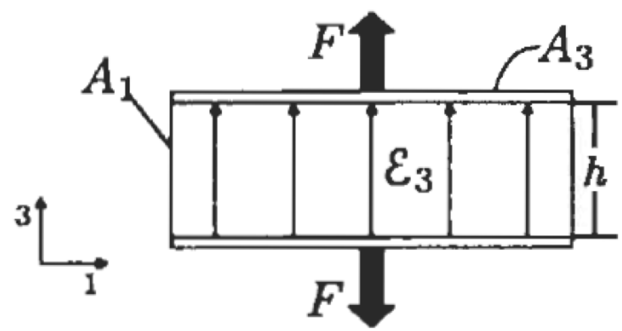
Piezoelectric Actuators

- We start with the equilibrium equation

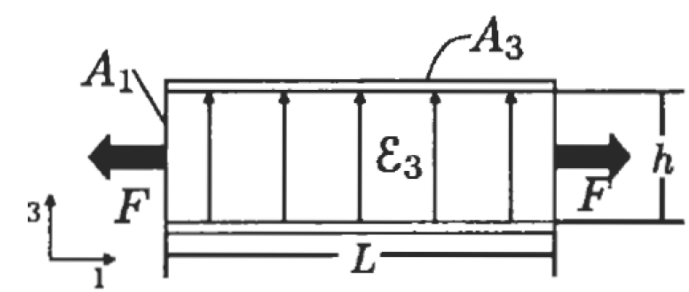
$$T = Es - e\xi$$

T … mechanical stress
E … Young's modulus
s … mechanical stress
e … piezoelectric coefficient (which is a tensor)
ξ … electric field

- **Longitudinal** → force parallel to electric field

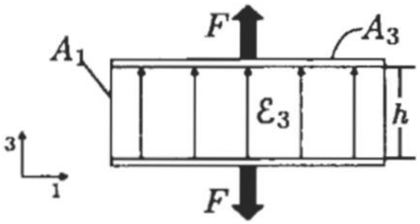


- **Transverse** → force perpendicular to electric field



FELMI-ZFE

Piezoelectric Actuators



- Now we index the quantities in accordance to the relevant directions (E field and movement)

$$T_3 = E s_3 - e_{33} \xi_3$$

- If no mechanical constraints are given (spatially free deformation) we can use $T_3 = 0$ which gives

$$s_3 = \frac{e_{33}}{E} \xi_3 \rightarrow \text{ with } U = \xi h \text{ it follows } \rightarrow s_3 = \frac{e_{33}}{E} \frac{U}{h}$$

- Now rewriting the mechanical stress into $s_3 = \frac{\Delta h}{h}$ we obtain

$$\Delta h = \frac{e_{33} h}{E} \frac{U}{h} = \frac{e_{33}}{E} U$$

- Which gives the displacement Δh in dependency on the material parameters *e* (piezoelectric coefficient) and the Youngs modulus *E* and the operation voltage *U*

FELMI-ZFE

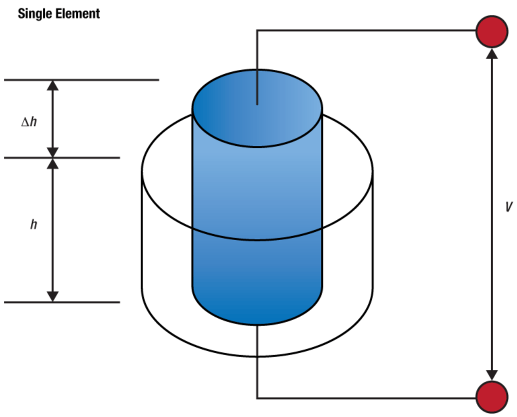
Piezoelectric Actuators

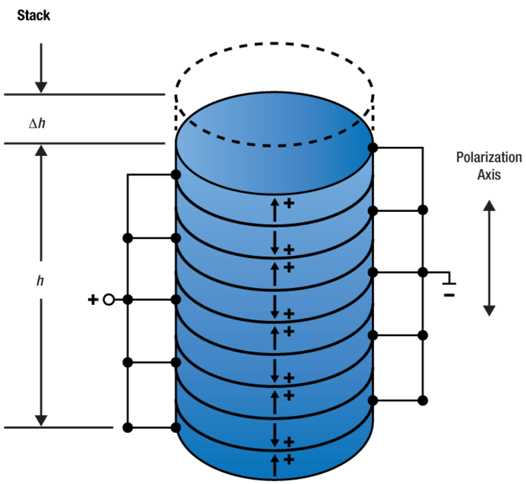
- The important finding, however, is the fact that the displacement is independent on the actuator dimension (only the piezoelectric coefficient e_{ii} is of relevance from the material side)

$$\Delta h = \frac{e_{ii}}{E} U$$

- The only way to increase the absolute movements is to increase the operation voltage (not always applicable) and the application of multiple stacks N:

$$\Delta h_{tot} = N \frac{e_{ii}}{E} U$$



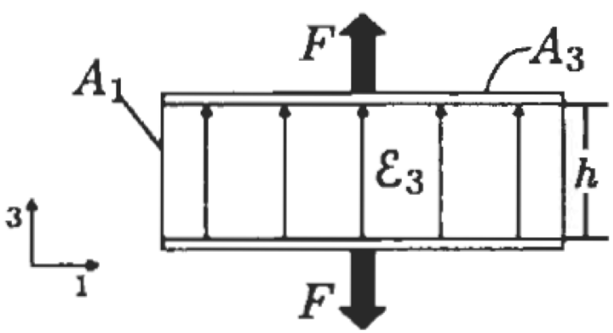


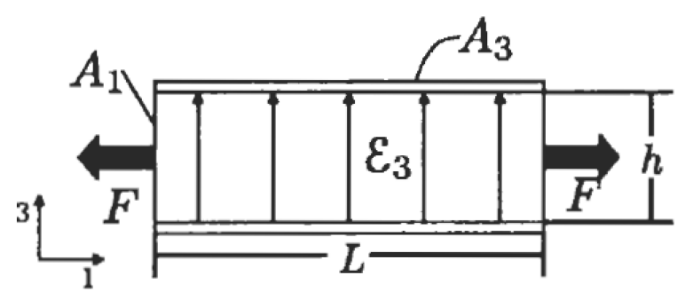
Piezoelectric Actuators

- Another degree of freedom is the different design by means of longitudinal and transverse
- The formalism changes slightly into:

	Longitudinal	Transverse
F_{piezo}	$F_{piezo} = \frac{e_{33} A_3}{h} v$	$F_{piezo} = \frac{e_{31} A_1}{h} v$
Displacement (Δh)	$\Delta h = \frac{e_{33}}{E} v$	$\Delta L = \frac{e_{33} L}{E\ h} v$

- A evident, there is a size dependency of *ΔL* on *L* in the transverse case as well as a thickness dependency! Here the design is slightly more flexible!





Piezoelectric Actuators – Smart Design

- Smart design then allows for spatially controlled movements ☺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